ANALYSIS OF UNREINFORCED MASONRY STRUCTURES USING ELASTIC/VISCOPLASTIC MODELS

ABSTRACT

This paper presents two elastic/viscoplastic constitutive models for analyzing the nonlinear behaviour of unreinforced masonry structures. One is a continuum model to represent the behaviour of brick units and the other is an interface model to simulate the fracture behaviour of mortar joints. It is demonstrated that the use of the viscoplastic formulation in the continuum model can effectively remove the mesh-size dependency problem that is commonly encountered in the finite element analysis of concrete and masonry structures. A numerical example is presented to demonstrate the capability of the models in capturing the detailed failure behaviours of an URM wall before and after being retrofitted with reinforcing steel.

1. INTRODUCTION

Unreinforced masonry (URM) buildings, many of which have historic and cultural importance, constitute a significant portion of existing buildings around the world. In general, these structures do not conform to modern engineering standards and suffer from deteriorations caused by various environmental loads. The performance of these structures under seismic loads has been a major concern and various retrofit methods have been developed in recent years to improve their seismic resistance. The ability to assess their seismic performance before and after retrofit is of primary importance.
A number of finite element models have been developed in recent years (Ewing et al. [1], Ali and Page [2], Rots [3], Lotfi and Shing [4,5], and Lourenco [6] to name a few) to analyze the failure behaviour of reinforced as well as unreinforced masonry structures. The behaviour of URM structures is governed to a large extent by the strengths of masonry units and mortar joints. The failure of these structures is often dominated by cracks propagating through mortar joints and brick units. This may lead to an abrupt softening behaviour. Masonry units may also exhibit compressive softening under extreme loads. Hence, the analysis of these structures presents a challenging problem. Previous experience in the finite element analysis of concrete and masonry structures that often exhibit severe softening behaviour has indicated the strong dependency of the numerical solutions on the mesh size and mesh orientation [7,4]. One solution to circumvent this problem is the use of viscoplasticity formulations to convert an ill-posed problem to a well-posed problem and, thereby, restore the objectivity of the numerical solutions [8,9,10,11].

In addition to the computational advantage, a viscoplastic formulation can also be used to account for the rate-dependent phenomenon of a material. Experimental evidence indicates that the behaviour of most materials is sensitive to the rate of loading. In cementitious materials, the rate-of-loading effect is pronounced. For example, under a slow loading rate, the fracture of concrete is often governed by the failure of the bonds between the aggregate particles and the hardened cement paste. Under a high rate of loading, however, the cracks are more likely to intersect the aggregate particles [12]. As a result, the fracture strength of concrete under higher loading rates will be higher. One important aspect in the modelling of cementitious materials is the post-peak behaviour. Experimental results indicate that the softening branch in the stress-strain curve is more abrupt under high strain rates than under low strain rates [13]. Furthermore, the size of the fracture process zone tends to decrease with increasing loading rates, which leads to a more brittle response [12].

To address the aforementioned issues, viscoplasticity-based continuum and interface finite element models have been developed at the University of Colorado for analyzing the nonlinear behaviour of masonry structures. The continuum model is used to represent the nonlinear behaviour brick units, while the interface model is used to simulate the fracture behaviour of mortar joints. This paper presents the model formulations and numerical examples that demonstrate their performance. Furthermore, the capability of the models in capturing the detailed failure behaviour of an URM wall panel subjected in-plane lateral loads is examined using the experimental results of Manzouri et al. [14].

2. ELASTIC/VISCOPLASTIC CONTINUUM MODEL FOR MASONRY UNITS

A continuum element is developed in this study to model the nonlinear behaviour of brick units. Continuum finite element models which exhibit strain softening always experience localization of deformation into a narrow band of elements [9,15]. The width and orientation of such bands depend on the mesh size and mesh orientation. Consequently, the resulting numerical solutions are not objective [7,8]. Mathematically speaking, spurious mesh sensitivity is a symptom of the ill-posedness of a boundary value problem [16]. A number of remedies have been proposed to eliminate the mesh-dependency problem. Among the various options, we have selected the viscoplasticity approach in this study. Sandler [11] has shown that introducing rate-dependency
can convert an ill-posed problem into a well-posed problem. Sluys [10] has pointed out that
generalized viscoplasticity can regularize the ill-conditioning related to material softening and thus produce a
unique solution. He has also demonstrated that the use of viscoplastic softening models
eliminates the sensitivity of numerical solutions to the mesh orientation in the case of shear band
formation. Besides their computational advantages, rate-dependent models provide a better
description of the physical behaviour of most materials as mentioned previously. For this reason,
an elastic/viscoplastic continuum model is developed in this study. The model formulation is
presented below.

2.1. Yield Surface and Viscoplastic Flow Function

The elastic/viscoplastic continuum model developed here is based on the Drucker-Prager failure
criterion. For the elastic/viscoplastic formulation, the total strain rate is decomposed into an
elastic part and a viscoplastic plastic part as follows.

\[ \dot{\varepsilon}_{ij} = \dot{\varepsilon}_{ij}^E + \dot{\varepsilon}_{ij}^{VP} \]  

in which the superposed dot represents differentiation with respect to time. The stress rate is
related to the elastic strain rate via an elastic constitutive relation:

\[ \sigma_{ij} = D_{ijkl} \varepsilon_{kl} = D_{ijkl} (\varepsilon_{kl} - \varepsilon_{kl}^{VP}) \]  

in which \(D_{ijkl}\) represents a fourth-order elasticity tensor. The viscoplastic strain rate is expressed
[17] as

\[ \dot{\varepsilon}_{ij}^{VP} = \gamma (\dot{\phi}(F)) \tilde{m}_{ij} \]  

in which \(\gamma\) is a fluidity parameter, \(\phi\) is a function defining the condition of overstress, \(F\) represents
the loading function, and \(\tilde{m}_{ij}\) is a vector defining the direction of viscoplastic flow. The function
\(\phi(F)\) is a non-dimensionalized function and the term with brackets \(\langle \ast \rangle\) is defined to have the
following properties.

\[ \langle \phi(F) \rangle = 0 \quad \text{when} \quad F \leq 0 \]  

\[ \langle \phi(F) \rangle = \phi(F) \quad \text{when} \quad F > 0 \]  

The second equation represents an overstress condition under which viscoplastic flow occurs.
The value of \(\phi\) increases monotonically with the level of overstress. In this study, it is defined by
the following expression.
in which \( F_0 \) is a nonzero positive function of the stress state for non-dimensionalizing the function \( \phi \). The yield and loading surfaces defined by the Drucker-Prager criterion can be expressed in terms of the deviatoric stress and hydrostatic stress as follows.

\[
F(\sigma, \chi) = \sqrt{J_2} + \alpha(\chi)I_1 - \kappa(\chi) = 0
\]  

in which \( I_1 \) is the first invariant of the stress tensor, which reflects the level of hydrostatic pressure, and \( J_2 \) is the second invariant of the deviatoric stress. The variables \( \alpha \) and \( \kappa \) are material parameters expressed as functions of an equivalent viscoplastic strain (or damage parameter) \( \chi \). The surface defined in Eq. (6) is shown in Fig. 1.

2.2. Plastic Potential and Flow Rule

A non-associated flow rule is adopted here with the plastic potential defined as

\[
Q = \sqrt{J_2} + \beta(\chi)I_1
\]  

in which \( \beta \) is a plastic dilatation parameter. Hence,

\[
m_{ij} = \frac{\partial Q}{\partial \sigma_{ij}} \text{ and } \bar{m}_{ij} = \frac{m_{ij}}{\|m_{ij}\|}
\]
The above plastic potential provides a flow direction that is independent of the hydrostatic stress. This may not exactly reflect experimental observations in that a higher hydrostatic stress often leads to a lower plastic dilatation for cementitious materials. However, this phenomenon is important only under a high hydrostatic compression [18], which is not likely to happen for problems investigated in this paper.

2.3. Evolution Laws

The evolution of the loading surface is governed by the equivalent viscoplastic strain (or damage parameter) $\chi$ as follows.

$$\kappa = A\chi^2 + B\chi + C \quad \text{and} \quad \alpha = \bar{A}\chi^2 + \bar{B}\chi + \bar{C} \quad \text{for} \quad 0 < \chi < \chi_2$$

$$\kappa = De^{-E\chi} \quad \text{and} \quad \alpha = \bar{D}e^{-E\chi} + \alpha_r \quad \text{for} \quad \chi_2 \leq \chi$$

in which $\chi_2$ is the point at which the functions change from a quadratic to an exponential form. These functions are shown in Figs. 2 and 3. Parameters $A$, $B$, and $C$ are determined with the conditions that $\kappa = \kappa_0$ at $\chi = 0$, and $\kappa = \kappa_{\text{max}}$ and $d\kappa/d\chi = 0$ at $\chi = \chi_1$, where $\chi_1$ is a user-determined value. Parameters $D$ and $E$ are determined with the continuity conditions at $\chi = \chi_2$. Parameters $\bar{A}$, $\bar{B}$, $\bar{C}$, $\bar{D}$ and $\bar{E}$ are determined in the same way, while $\alpha_r$ is a residual value of $\alpha$. To enforce the mathematical constraint that two consecutive loading surfaces must not intersect, it is necessary that $d\kappa/d\chi > \kappa/\alpha$ along the descending branches of the two functions, and $d\kappa/d\chi < \kappa/\alpha$ along the ascending branches. Hence, the parameters for $\kappa$ and $\alpha$ are selected in such a way that these conditions apply.

In order to have different normal strains associated with the peak tensile stress and peak compressive stress as shown in Fig. 4, it is assumed that viscoplastic tensile strain will induce more damage than viscoplastic compressive strain. Hence, the damage parameter is defined in the following manner.
\[ \dot{\chi} = \int_0^t \sqrt{\dot{\omega}_i \dot{\omega}_i} \, dt \]  

(11)

in which \( \dot{\omega}_i = \xi e_i^p \) where \( e_i^p \) is a principal viscoplastic strain and \( \xi \) is a factor that, as defined in Fig. 5, depends on the stress state the material is subjected to. In the figure, \( k \) is a positive value defined as the ratio of the inelastic strain associated with the uniaxial peak compression stress to the inelastic strain associated with the uniaxial peak tensile stress. A simple evolution law of this sort has a drawback that the loss of the tensile resistance and compressive resistance is simultaneous and cannot be decoupled. This implies that tensile fracture will result in the loss of the compressive strength of the material as well.

\[ v_p = \varepsilon^p \]  

where \( v_p \) is a principal viscoplastic strain and \( \varepsilon^p \) is a factor that, as defined in Fig. 5, depends on the stress state the material is subjected to. The figure, \( k \) is a positive value defined as the ratio of the inelastic strain associated with the uniaxial peak compression stress to the inelastic strain associated with the uniaxial peak tensile stress. A simple evolution law of this sort has a drawback that the loss of the tensile resistance and compressive resistance is simultaneous and cannot be decoupled. This implies that tensile fracture will result in the loss of the compressive strength of the material as well.

For the plastic potential, the evolution of the plastic dilatation parameter is given by

\[ \beta = \beta_0 e^{-\eta \chi} + \beta_1 \]  

(12)

in which \( \beta_0, \eta, \) and \( \beta_1 \) are parameters to be determined by users. This equation reflects the fact that plastic dilatation is generally reduced as damage increases. With the above relation, the direction of the viscoplastic flow changes as the material undergoes damage.

In brittle materials, once fracture or damage propagates, the viscosity effect diminishes due to the loss of cohesion. Hence, the fluidity parameter \( \gamma \) is assumed to be a function of the damage parameter \( \chi \) as shown in Fig. 6. To prevent numerical problems, the fluidity parameter is assumed to remain finite even after severe damage.
2.4. Implementation

The above constitutive model has been implemented in an isoparametric quadrilateral plane-stress element. The rate-dependent material law is solved with the generalized midpoint rule.

2.5. Numerical Examples

The following examples are used to demonstrate the performance of the elements.

2.5.1 Objectivity with Respect to the Mesh Size

A set of uniaxial compression analyses are conducted on a square panel using both an inviscid model and the elastic/viscoplastic model. The panel is discretized into a single element as well as regular 4x4 and 8x8 meshes with four-node elements as shown in Fig. 7. The nodal displacements at the top are prescribed in order to trace the post-peak response and progressive failure of the panel. The results are shown in Fig. 8. The inviscid model used in this example has a gentler slope in the post-peak regime than the elastic/viscoplastic model to avoid numerical problems. In spite of this, it is evident that the result provided by the inviscid model in the post-peak regime is heavily influenced by the mesh size, while the elastic/viscoplastic model results in objective solutions that are unaffected by the mesh size.

![Figure 7: Mesh and boundary conditions for mesh-dependency study](image1)

2.5.2 Third-Point Loading Test

A third-point loading test is simulated to show the performance of the elastic/viscoplastic continuum model in diffusing damage and removing the mesh-size dependency of the numerical results. For the finite element analyses, the middle third of the beam is discretized with two different mesh sizes, a 6x10 mesh and a 12x20 mesh. The results of the analyses performed with a displacement rate of 10 in./sec (0.254 m/sec) are shown in Fig. 9. The damage patterns obtained with the two meshes are shown in Fig. 10. It can be seen that the localization of damage into a vertical band, which would have been expected from an inviscid model, does not occur in this case. However, it must be pointed out that while the mesh-size dependency is removed here, the resulting damage pattern differs from the true behaviour of a brittle beam subjected to third-point loading, where a localized vertical crack is more likely to occur. Hence, the use of a viscoplastic formulation for regularization is not without compromise. For tensile fracture, the use of a discrete
crack approach with interface elements is probably a more viable approach.

![Figure 8: Mesh-dependency analysis: (a) inviscid model; (b) elastic/viscoplastic model](image)

(1 ksi = 6.89 MPa; 1 psi = 6.89 kPa)

![Figure 9: Third-point loading test](image)

(1 lb. = 4.45 N; 1 in. = 25.4 mm)

![Figure 10: Damage at 0.04-in (1-mm) displacement](image)

3. ELASTIC/VISCOPLASTIC INTERFACE MODEL FOR MORTAR JOINTS

Modelling of discontinuities introduced by the tensile or shear fracture of mortar joints is best handled in a discrete fashion using interface elements. To this end, a plasticity-based dilatant interface formulation developed by Lofti and Shing [5] is adopted here. The model has been extended to an elastic/viscoplastic formulation using the same approach as that for the continuum model. However, it must be pointed out that interface elements do not have the mesh-size...
dependency problem suffered by continuum elements. The use of the rate-dependent formulation here is to enhance the robustness of the numerical solution, and to provide a tool to model the rate-dependent behaviour of mortar joints for future studies.

The details of the interface model are already reported elsewhere [5] and, therefore, will not be elaborated here. To simulate mixed-mode fracture, the failure and loading surfaces are represented by a hyperbolic function as follows.

\[ F(\sigma, q) = \tau^2 - \mu^2 (\sigma - s)^2 + 2r(\sigma - s) = 0 \]  

(13)

in which \( q = [r, s, \mu]^T \), where \( r \) is the radius of curvature at the apex of the hyperbolic curve, \( s \) is tensile strength of the interface, and \( \mu \) is the slope of the asymptotes of the hyperbola. These internal variables determine the interaction between the normal and shear strengths. The higher the normal compression is, the higher will be the shear resistance. It is interesting to note that when \( r \) is zero, the Mohr-Coulomb failure criterion is recovered from Eq. (13). An interface is elastic if the stress state remains within the yield surface. Fracture can occur in tension (mode I), shear (mode II), or a mixed mode once the initial yield surface is reached. The decrement of cohesion and frictional resistance after fracture depends on the plastic work related to decohesion and friction.

4. ANALYSIS OF URM WALL PANEL

Manzouri et al. [14] tested a series of URM walls to investigate the effectiveness of using grout injection and steel reinforcement for the repair and retrofit of URM structures. The grout injection technique and grout mixes developed in this study were evaluated with masonry mortar joint tests as well as wall panel tests. Masonry mortar joints were tested in a direct shear machine under various levels of compressive loads. One set of tests were conducted on intact mortar joints and the other on grout injected joints. Eleven tests were conducted on four wall panels in their original condition and also after they were damaged and repaired with different methods, including grout injection and steel reinforcing. The walls were subjected to in-plane cyclic lateral displacements with constant axial compressive loads. Each wall was loaded to a displacement of 0.6 in. in the first cycle, and the displacement amplitude was reduced to 0.2 in. in the second cycle and increased gradually with each subsequent cycle. The retrofit scheme used for one of the wall specimens that had an opening is shown in Fig. 11. The specimen was tested in its virgin state and then repaired with grout injection and strengthened with reinforcing bars.

The above walls are analyzed with the finite element models presented previously. For the finite element mesh, each brick unit is modelled by two four-node elements. The mortar joints between the bricks are modelled by two-double-node interface elements. The positions of the mortar joints in the masonry walls are exactly modelled. The reinforcing bars used for the retrofit are modelled with two-node bar elements with elastic-perfectly plastic behaviour. The nodal connectivity in a finite element mesh is shown in Fig. 12.
The interface model is calibrated with the results of the direct shear tests conducted on mortar joints. The comparison of the initial and residual yield surfaces with test data is shown in Fig. 13 together with the values of the material parameters. The comparison of the shear force-vs.-relative shear displacement curves obtained from the simulations and tests under different compressive loads is shown in Fig. 14. However, the head joints are assumed to be much weaker with a tensile strength that is ¼ of that of the bed joints to reflect the actual construction. The
continuum model is calibrated with the results of the compression tests conducted on three-course prisms as well as the wall tests. The static compressive strength of the masonry is selected to be 1,700 psi (11.7 MPa), which is about 85% of the average compressive strength of the prisms and is found to provide the best match with all the wall test results. This adjustment is needed for two reasons. One is the additional strength that can be introduced by the viscosity of the model and the other is that prism tests often led to a slightly higher compressive strength due to the end constraints. The tensile strength of the brick units is assumed to be 170 psi (1.2 MPa). The material parameters used for the brick units are shown in Table 1.

The material properties that are not directly available from the material tests are so selected to provide a best match of the results obtained from the prism tests and wall tests. The same set of material parameters are used for all the walls.

The analyses are conducted with a monotonically increasing lateral displacement. The rate of the lateral displacement is 1 in./sec (25.4 mm/sec). As in the tests, the vertical load is kept constant during each analysis.

The numerical results obtained for Wall #4 that had an opening, as shown in Fig. 11, are presented here. The numerical and experimental load-displacement curves for the virgin wall are compared in Fig. 15. The deformed mesh is compared to the
experimentally observed damage pattern in Fig. 16. The diagonal crack in the compression pier and the damage near the corner of the opening, which were observed in the experiment, are reproduced in the analysis. Wall #4 was later repaired and strengthened with steel as shown in Fig. 11. The strengthened wall is analyzed with the same finite element model but with reinforcing bars represented by elastic-perfectly plastic truss elements. The numerical and experimental results are compared in Fig. 17. It must be mentioned that this test was conducted on a wall that was damaged previously without repair. The only change in this test is that the two sides of the wall were restrained from uplifts. This can possibly be responsible for the fact that the strength obtained in the analysis is higher than the test result because the model does not account for the prior damage. The deformed mesh is compared to the experimentally observed damage pattern in Fig. 18. The two show reasonable similarity.

Table 1: Material parameters for brick units
(1 ksi = 6.89 MPa; 1 psi = 6.89 kPa)

<table>
<thead>
<tr>
<th>$E_n$ (ksi)</th>
<th>$v$</th>
<th>$\chi_1$</th>
<th>$\chi_2$</th>
<th>$\alpha_0$</th>
<th>$\alpha_{max}$</th>
<th>$\alpha_r$</th>
<th>$\kappa_0$ (psi)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1200</td>
<td>0.29</td>
<td>0.001</td>
<td>0.0012</td>
<td>0.4</td>
<td>0.41</td>
<td>0.3</td>
<td>147.5</td>
</tr>
<tr>
<td>$\kappa_{max}$ (psi)</td>
<td>$\beta_0$</td>
<td>$\beta_r$</td>
<td>$\eta$</td>
<td>$\gamma_{in}$ (1/sec)</td>
<td>$\gamma_{fmax}$ (1/sec)</td>
<td>$f_0$ (psi)</td>
<td></td>
</tr>
<tr>
<td>251</td>
<td>0.4</td>
<td>0.1</td>
<td>500</td>
<td>1.0</td>
<td>4.0</td>
<td>1000</td>
<td></td>
</tr>
</tbody>
</table>

Experiment (cyclic load)                Analysis (uni-directional load)

Figure 16: Comparison of damage patterns of Wall #4
5. CONCLUSIONS

This study has demonstrated the effectiveness of the elastic/viscoplastic continuum model in removing the mesh-size dependency problem that is commonly encountered in the finite element analysis of concrete and masonry structures. It has been shown with numerical examples that the continuum and interface models developed in this study can capture the nonlinear behaviours of an URM wall before and after retrofit with good accuracy. However, further improvements of the models are needed to simulate the cyclic behaviour of URM structures. These include the capability to model the degradation of the elastic modulus of masonry due to damage during unloading and reloading cycles and the improvement of the plasticity formulation for the continuum model to distinguish tensile damage from compressive damage.

6. ACKNOWLEDGMENTS

This study was supported by the US National Science Foundation under Grant No. MSM-9017149. However, opinions expressed in this paper are those of the writers and do not necessarily reflect those of the sponsor.
7. REFERENCES


