IV–6. A Model for the In-Plane Behaviour of Masonry and a Sensitivity Analysis of its Critical Parameters

A.W. Page  
Lecturer, Department of Civil Engineering, The University of Newcastle, N.S.W., Australia.

ABSTRACT

A finite element model for the in-plane behaviour of clay masonry is presented. The model reproduces the nonlinear characteristics of masonry caused by material nonlinearity and progressive joint failure. Masonry is considered as an assemblage of elastic brick continuum elements acting in conjunction with linkage elements simulating the mortar joints. These joint elements are assumed to have nonlinear deformation characteristics with limited shear and tensile capacity. The model is applied to a masonry deep beam, and analytical and experimental results are presented. Sensitivity analyses of the model parameters are carried out with reference to the deep beam test. Accurate definition of joint failure is shown to be the most important characteristic in a model of this nature, with joint and brick deformation characteristics being less significant.

INTRODUCTION

The analysis of clay masonry walls subjected to in-plane loading is a problem commonly encountered in the design of masonry structures. Masonry is a two-phase material which typically consists of elastic, brittle bricks set in inelastic mortar joints. These joints impart directional properties to the masonry assemblage, and act as planes of weakness due to their low bond strength. Depending upon the degree of compression present, failure can occur in the joints alone, or as a combined brick-joint failure. The mechanism of failure is not fully understood, and no completed failure criterion has yet been developed, although significant results for concrete masonry have been recently presented by Hegemier et al.

Most previous analyses have considered masonry to be an assemblage of bricks and mortar with average properties. Isotropic linear elastic behaviour has usually been assumed. Models of this nature may be satisfactory at low stress levels, but will be inadequate at higher stress levels when extensive stress redistribution will occur. This redistribution is caused by nonlinear material behaviour and progressive joint failure.

The emphasis in this investigation is on the determination of stress distributions in masonry, rather than the accurate prediction of final failure which will occur after progressive failure and/or slip in a number of joints. In modeling masonry, an analogy has been drawn with the behaviour of jointed rock as modeled by Goodman, Taylor and Brekke using the finite element technique. Masonry is considered as an elastic continuum of brick elements, with a regular array of joint (linkage) elements embedded between them. These joint elements have low tensile capacity, high compression capacity, and a shear capacity which is a function of the bond strength and superimposed compression. The nonlinear characteristics of the modeled masonry thus result from the nonlinear deformation characteristics of the joints under shear and compression, and the local failure and slip that occurs in the joints. The detailed derivation of this model has been previously described by the author.

This paper describes a typical application of a model of this type. A test on a masonry deep beam is used as the basis of comparison between predicted and observed performance. Sensitivity analyses of the critical parameters of the model are also carried out to determine their relative importance.

FINITE ELEMENT MODEL

A typical finite element subdivision is shown in Figure 2(b). Bricks are modeled using conventional eight parameter rectangular plane stress elements with four internal degrees of freedom and isotropic elastic properties. The joint element concept developed by Goodman et al has been adapted to simulate the mortar joints. Due to its elongated nature the joint element is assigned different characteristics to the typical continuum element. Its deformation is limited to normal and parallel to the joint direction, so that only normal and shear stresses can be transmitted across the joint. Failure occurs when a shear or tensile bond criterion is exceeded, with residual shear capacity remaining in some cases. The material characteristics required to define the behaviour of the joint element are therefore the deformation characteristics of the mortar joints and a suitable failure criterion. These characteristics are summarized in Figure 1. The values shown are for the tests on the sample of masonry described later in this paper.

Joint failure occurs when the criterion shown in Figure 1(c) is violated. If the combination of shear and tensile stress lies outside Region 1, a tensile bond failure is simulated by assuming zero joint stiffness. If failure occurs under a combination of shear and compressive stress, (outside Regions 2 or 3), a shear bond failure is simulated. In this case, some residual capacity will remain in the joint. The stiffness of the joint in the normal direction is assumed to remain unchanged, and a reduced shear stiffness is allocated to simulate the frictional resistance of the joint after an initial shear bond failure.

The above element types have been incorporated into an incremental finite element computer program which simulates masonry behaviour at various levels of applied...
load. Two major iterations are involved at each load level. The first iteration reproduces the nonlinear response of the mortar, and the second allows for the changing structure stiffness as the joints progressively fail in tension or shear.

**EXPERIMENTAL EVALUATION OF MASONRY PROPERTIES**

The parameters required to define the analytical model are the elastic properties of the bricks, the deformation characteristics of the mortar joints, and a criterion of failure for the joints. Since the experimental evaluation of these properties has been previously described in detail, only the main features will be outlined. Medium strength, half scale masonry was used for all tests. The mean value of brick elastic modulus and Poisson's ratio were 6740 MPa and 0.17 respectively.

Mortar properties were obtained indirectly from tests on panels of masonry constructed from bricks with known average elastic properties. If brick deformations are subtracted from total measured masonry deformations, the remainder can be attributed to the mortar. A series of uniaxial compression tests were carried out on panels of masonry with varying bed joint angles to the applied load, producing varying ratios of shear stress (τ) to normal stress (σ) on the joints. From these tests, average mortar stress-strain curves were derived. A failure criterion for the mortar joints in terms of the ultimate shear stress (τ₀) and ultimate normal stress (σ₀) can also be obtained. These curves are shown in Figure 1. The strength of the joint under pure tension was obtained from a separate series of uniaxial tension tests on masonry couplets.

### DEEP BEAM VERIFICATION TEST

To check the adequacy of the analytical model, tests were performed on a half scale masonry deep beam with restrained supports subjected to a central load. The testing arrangement is shown schematically in Figure 2(a). These tests have also been described in more detail elsewhere.

To allow comparison with the analytical model, the vertical stress distribution along A-A was measured by means of electrical resistance strain gauges attached to previously calibrated bricks. The beam was loaded to failure. The ultimate load was 110 kN.

The finite element model simulating the behaviour of the beam is shown in Figure 2(b). The load was applied in 10 kN increments. For purposes of comparison a conventional finite element analysis was also carried out assuming masonry to be an isotropic linear elastic continuum.

An elastic modulus of 4700 MPa, (the initial tangent value to the masonry stress-strain curve of Fig. 1) and a Poisson's ratio of 0.2 were assumed in this analysis.

The calculated and observed vertical stress distributions along the fifth course of bricks, (plane A-A, Figure 2), are compared in Figure 3 for various levels of applied load. It can be seen that good agreement is exhibited between experiment and analytical model throughout the load range, particularly in view of the inherent variability of the material. The increasing difference between the isotropic elastic analysis and observed values as the applied load increases, illustrates the degree of stress redistribution taking place.

The pattern of progressive joint failure as predicted by the computer simulation is shown in Figure 4. Joints failing in tension or slipping in shear are indicated. The large number of joints that have progressively failed illustrates the significant influence of joint properties on masonry behaviour. Progressive cracking patterns during the test were difficult to observe due to small crack size. Final collapse of the panel involved both brick and joint failure. For accurate prediction of ultimate load, a criterion for brick failure would have to be included in the model.

### SENSITIVITY ANALYSIS OF THE MODEL PARAMETERS

#### Brick Elastic Properties

Variations in brick stiffness were obtained by holding the joint properties constant and varying the elastic properties of the bricks. Values of 0.5, 3 and 6 times the original brick elastic modulus were used. The analyses were performed with one 10 kN increment of load being applied in each case. At this low load level, no joint failures occurred, ensuring that brick stiffness was the only variable.

It was found from these analyses that variations in stresses throughout the panel were quite small even for large variation in brick elastic modulus. Typical values are summarized in Table 1.

It can be concluded therefore, that brick stiffness is not a sensitive parameter for this model, and the experimental determination of its values need not be particularly precise.

#### Joint Deformation Characteristics

Two sensitivity analyses were carried out. The first was to determine the influence of mortar stress-strain characteristics on the nonlinear behaviour of masonry. The second was to test the sensitivity of the assumed mortar stress-strain curves used in the original model.

#### Mortar Deformation Properties

Comparision was made between the original analysis incorporating nonlinear mortar properties and an analysis assuming linear elastic properties for the mortar (initial tangent values to the mortar curves shown in Figure 1(a) and 1(b)). In both cases, the capacity for progressive joint failure was maintained. Ten load increments of 10 kN were applied.

The vertical stress distributions along the fifth brick course for a load level of 100 kN, (when nonlinear effects will be greatest), are shown in Figure 5. The stress distribution for the isotropic elastic case, with no provision for joint failure, is also included.

It can be seen that there is a significant difference between the isotropic elastic analysis and the two analyses performed using the proposed model. However, the difference between the analyses with elastic and inelastic joint deformation characteristics is not great, indicating that the
predominate cause of masonry nonlinearity is progressive joint failure rather than inelastic mortar characteristics.

**Sensitivity of Mortar Stress-Strain Characteristics**

Since the mortar stress-strain curves are nonlinear, their influence on masonry behaviour will depend on local stress level. To determine the significance of this influence, two additional analyses of the deep beam test were carried out. The first assumed the elastic and shear moduli of the mortar joints to be twice the original value at all stress levels, and the second assumed them to be half the original value. In both cases, the joint failure criterion and deformation characteristics after initial failure were left the same as in the original model. Ten load increments of 10 kN were again applied.

From the analyses it was found that changing joint stiffness did not significantly influence the solution unless the joint flexibility increased to such an extent that premature joint failures occurred with accompanying redistribution of stress.

The two foregoing sensitivity analyses indicate that linear elastic properties could be assigned to the mortar joints without unduly affecting the performance of the model. This would result in significant savings in computer time, since one of the two major iterations at each load level would be eliminated. Certainly, small inaccuracies in estimates of joint stiffness caused by the simplifying assumptions made in the derivation of joint properties should not be significant.

**Joint Failure Criterion**

To test the sensitivity of the joint failure criterion, four alternatives were considered as shown in Figure 6. Emphasis was placed upon the lower regions of the $\tau_{m} - \sigma_{m}$ range (Region $\Lambda$), since the stress state in the joints will lie in this region for most of the load range.

Criteria #1 and #2 correspond to joints constructed from the same basic materials but with differing standards of workmanship achieving varying bond strengths. An extreme case was also investigated (Criterion #3), corresponding to zero tensile and shear bond strength. The deep beam problem was analysed for each case with all other properties being held constant. Ten load increments of 10 kN were applied.

The analyses revealed that with the exception of Criterion #4, joint failure patterns were significantly influenced by varying joint failure criterion. For the case of zero shear and tensile bond strengths, solution failed to converge even at the 10 kN load level due to excessive joint cracking and slip, indicating that some nominal bond strength is essential. For the other cases, cracking became less excessive as the joint strength was increased, with failures following a similar pattern but at different load levels. As would be expected, the amount of stress redistribution taking place correlated with the degree of cracking. A significant improvement in performance, (compared to Criterion #3), was achieved by supplying some nominal shear and tensile bond capacity (as for Criterion #2). In this case, despite substantial joint failure, the beam was capable of carrying the load for the full load range in marked contrast to Criterion #3.

It can be concluded therefore, that bond strength is extremely important, and the in-plane behavior of masonry is quite sensitive to its value. The failure criterion used in the model should be evaluated with care, particularly for the lower regions of the criterion which define tensile and shear bond failure.

**CONCLUSION**

This paper has described a method for the simulation of the in-plane behaviour of masonry using the finite element technique. Masonry is considered as a continuum of isotropic, elastic bricks acting in conjunction with inelastic mortar joints with limited shear and tensile bond capacity. A model of this nature is capable of reproducing the nonlinear response of masonry for most of the load range. Final failure can only be predicted if a criterion for brick failure is included in the analysis. A criterion of this nature is difficult to determine due to the complex triaxial stress state produced by mortar-brick interaction, the inherent variability of the material and the local effects of bonding pattern.

The use of the joint element concept enables the nonlinear characteristics of masonry to be reproduced from properties defined from relatively simple uniaxial tests producing various combinations of shear and normal stress on the mortar joints. The need for biaxial testing, with its associated difficulties, is therefore eliminated.

From a sensitivity analysis of the parameters defining the model, it appears that of the three characteristics that must be determined, the accurate definition of joint failure is the most important. Some inaccuracies in the experimental evaluation of the brick and joint deformation characteristics can be tolerated. It has also been shown that the major cause of masonry nonlinearity is progressive joint failure rather than inelastic mortar deformation characteristics.

The accuracy of the analysis will be influenced by the degree of variability inherent in the material properties. Stress history, creep and workmanship effects will also exert some influence in actual wall panels.

**ACKNOWLEDGEMENTS**

The work described in this paper was carried out in the Department of Civil Engineering at the University of Newcastle. The assistance of P.W. Kleeman, Senior Lecturer and the staff of the Civil Engineering Laboratory is gratefully acknowledged.

**REFERENCES**


### TABLE 1—Variation of Stress Within Masonry Deep Beam For Varying Brick Stiffness

<table>
<thead>
<tr>
<th>Types of Stress</th>
<th>Stress (MPa)</th>
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<tr>
<td>Vertical Stress on 5th Brick Course*</td>
<td>0.5E 0.187</td>
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<tr>
<td>(Largest Variation)</td>
<td>E 0.191</td>
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<tr>
<td>Joint Shear Stress Adjacent to 5th Brick Course</td>
<td>0.048 0.045</td>
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<tr>
<td>(Largest Variation)</td>
<td>E 0.035</td>
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<tr>
<td>Maximum Brick Principal Stress*</td>
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<tr>
<td>(Adjacent to Support)</td>
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<tr>
<td></td>
<td>6.0E 0.804</td>
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*All stresses compressive

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![Figure 1](image-url) **Figure 1.** Material Characteristics to Define Joint Element
Figure 2. Deep Beam Test

Figure 3. Vertical Stress Distribution Along A-A Deep Beam Test
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Figure 4. Joint Cracking Patterns, Deep Beam Test

![Joint Cracking Patterns](image)

Figure 5. Vertical Stress Distributions Along Fifth Brick Course for Varying Joint Properties

![Stress Distributions](image)

Figure 6. Joint Failure Criteria Used in Sensitivity Analysis.