IV–17. Torsional Analysis of Coupled Shear Walls

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ABSTRACT

In a previous publication the authors have developed an approach to the problem of torsion in multi-storey structures and compared their results with those obtained by other methods both theoretical and experimental. For the example considered it was shown that better agreement was obtained between theoretical and experimental results if the slab effect was not included.

In this paper the authors illustrate the application of the method to the solution of a simple cross-wall brickwork structure which was previously investigated by Kalita and Hendry, Coull and Irwin and Tso. The theoretical results for rotation obtained by the authors and other researchers are compared with the experimental values obtained by Kalita and Hendry.

INTRODUCTION

In previous publications[5,6] the authors have described the experimental results obtained for rotations and deflections due to eccentric loading on a multi-storey blockwork structure. Also included was a theoretical approach for analysing such structures and comparisons between experimental and theoretical results which showed reasonable agreement.

The cross-section of the test structure, which had been selected because of availability, consisted of a symmetrical arrangement of rectangles and equal angles so that the warping effect reduced theoretically to zero and, as a result, the influence of the effect of warping could not be determined for the more general case.

In addition the theoretical results calculated with and without the contribution of the floor slabs to the overall stiffness indicated that the slab stiffness was being overestimated so that the measured rotations were larger than the theoretical when the slab effect was included.

In this paper the analysis is applied to an arrangement of masonry channel sections, connected by floor slabs, for which the warping effect does not reduce to zero.

Previous workers[1,2,3] have also used this type of channel core section for the development of their theories and the results of their analyses together with the experimental values obtained by Kalita[2] are available for comparison. An opportunity is therefore provided for the estimation of the influence of both the warping and slab effects on this type of construction.

BASIC EQUATIONS

The basic equations for the rotation at any section is derived by considering the equilibrium of the external twisting moment (M_T) and the internal twisting moments. The internal moments can be conveniently separated into four parts, the first three being contributed by the individual walls and the fourth by the floor slab.

1. The St. Venant Torsion (M_{sv})
2. The Warping Contribution (M_w)
3. The moment due to shear forces acting about the shear centres (M_f)
4. The moments carried by the floor slabs (M_s)

Starting with

$$M_T = M_{sv} + M_w + M_f + M_s \tag{1}$$

and introducing the necessary boundary conditions it can be shown that, for a concentrated torque

$$\theta = [\gamma z - \mathrm{Sinh}(\gamma z) + \mathrm{Tanh}(\gamma H)(\mathrm{Cosh}(\gamma z) - 1)]\,\frac{M_T}{\gamma^3 EI_w^*} \tag{2}$$

and for a distributed torque

$$\theta = \left\{\left[\mathrm{Tanh}(\gamma H) + \frac{\mathrm{Sech}(\gamma H)}{\gamma H}\right](\mathrm{Cosh}\,(\gamma z) - 1) + \frac{\gamma z}{2H}(2H - Z) - \mathrm{Sinh}(\gamma z)\right\}\frac{m_T H}{\gamma^3 EI_w^*} \tag{3}$$

Once the rotation θ has been calculated the deflection of the structure, and the stresses over the cross-section can be calculated.

The St. Venant Torsion

The St. Venant theory was originally applied to circular shafts and assumes that the twisting at any cross section produces only shear stresses. It has been applied to sections which are made up of several rectangles and can be expressed as

$$M_{sv} = GK_T\frac{d\theta}{dz} = G\left(\frac{1}{3}\sum_1^n s_i t_i^3\right)\frac{d\theta}{dz} \tag{4}$$

where s_i and t_i represent the lengths and thickness of the n rectangles forming the section.

The Warping Effect

The warping effect can be expressed as

$$M_W = E\frac{d^3\theta}{dz^3}I_w = E\frac{d^3\theta}{dz^3}\int_0^3 \omega^2(s)t\,ds \tag{5}$$

where ω represents the sectional co-ordinate introduced by Vlasov[4] (Fig. 3).

The Moment M_F

The external moment induces shear forces at the centre of gravity of each of the rectangles and since, in general, the shear centre does not coincide with the centre of gravity there will be a resulting moment of

$$M_f = -E\frac{d^3\theta}{dz^3}I^* = -E\frac{d^3\theta}{dz^3}\left\{\sum_1^n (a_{xi}\cdot e_{yi} + Q_{yi}\cdot e_{xi})\right\} \quad (6)$$

where Q_i represent the shear forces and e_{xi}, e_{yi} the distances to the shear centres.

The Slab Effect

It is assumed that the system of connections formed by the beams or floor slabs can be replaced by a continuous connecting medium of the same stiffness and that the walls deflect equally, with points of contraflexure at the mid-points of the connecting beams.

The resulting moment can be expressed

$$M_s = EK_s\frac{d\theta}{dz} = E(\lambda_1 m_1\bar{m}_1 + \lambda_2 m_2\bar{m}_2)\frac{d\theta}{dz} \quad (7)$$

where m_i, $\bar{m}_i$ are functions of the distances from the centre of the structure to the centre of gravity and shear centre of each element and λ_i are related to the slab properties.

Appplication to Channel Core Section

The stiffness coefficients of equations 4 to 7 for the core wall structure shown in Figs. 1, 2 become

$$\begin{aligned} K_T &= 2(b + 2d)t^3/3 \\ I_w &= b^2t(be^2 + 2d^3 + 6de(d - e))/12 \\ I^* &= 2I_{x1}e_{x1}^2 \end{aligned} \quad (8)$$

$$K_s = \lambda_1 m_1\bar{m}_1 = \lambda_1\frac{b}{2}(b + e_{x1})(b + c_{x1})$$

where λ_1 is given by

$$\frac{1}{\lambda_1} = \left\{\frac{l^3}{24Id} + \frac{1.18}{Ad}\right\}h$$

and the effective area and moment of inertia (Ad, Id) of the connecting media (Fig. 4) are calculated using a depth equal to the floor slab thickness.

CONCLUSIONS

In Fig. 7 the values of the rotation calculated with and without the slab effect are compared with the practical results obtained by Kalita and the theoretical results of others. A comparison is also shown in Table 1. Although the results obtained when the slab effect is neglected are closer to the practical results than those obtained when the slab influence is included this is probably caused by the rotation at the base of the structure. The theory assumes that the base is fully fixed but this condition will not be realized in practice.

The important parameter γ which appears in equations 2 and 3 is defined as

$$\gamma^2 = \frac{K_T/2.2 + K_S}{I^* + I_w} \quad (9)$$

The value of I^* is approximately a hundred times larger than I_w so that the neglect of the warping effect is justified but there is no theoretical justification for neglecting K_s compared to K_T since the ratio of these terms is not so large. The actual rotation θ_a due to torsion could be expressed as

$$\theta_a = \theta - \theta_b \quad (10)$$

where θ is the calculated value from equations 2 or 3 and θ_b is the rotation due to the partial fixity of the base.

It would appear, both for the structure described in this paper and also for the structure reported in Ref. 5 that θ_b is of a similar magnitude to the part of θ due to the slab so that their effects cancel.

The stresses calculated from the rotations are shown in Fig. 8 and these compare favourably with those obtained by Kalita[2].

REFERENCES

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2. Kalita, U.C., *Experimental and Theoretical Studies of the Structural Behaviour of Brickwork Crosswall Systems,* Ph.D. Thesis, University of Edinburgh, May 1970.
3. Tso, W.K. and Biswas, J.K., *General Analysis of Nonplanar Coupled Shear Walls.* Journal of the Structural Division. A.S.C.E. No:St3 March 1973.
4. Vlasov, V.Z., *Thin-Walled Elastic Beams.* Translated from the Russian by the Israel program for Scientific Translations. Jerusalem, 1961.
5. Keskin, O. and Davies, S.R., *The Effect of Torsion on Multistorey Structures.* International Symposium on Loadbearing Brickwork. London, June 1974.
6. Davies, S.R. and Keskin O., *The Effect of Eccentric Loads on Multistorey structures,* 2nd International Symposium on Bearing Walls, Warsaw, September 1975.

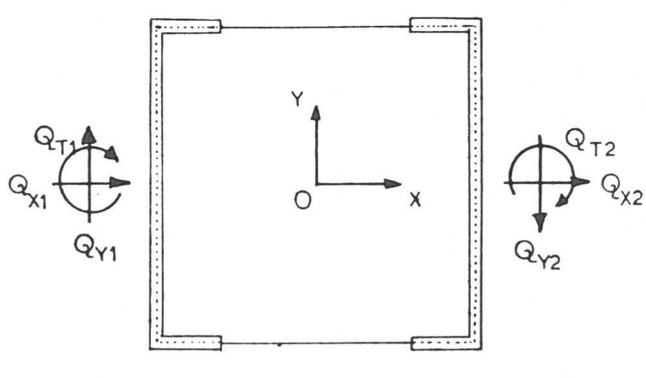


Figure 5. Shear forces and torques from walls

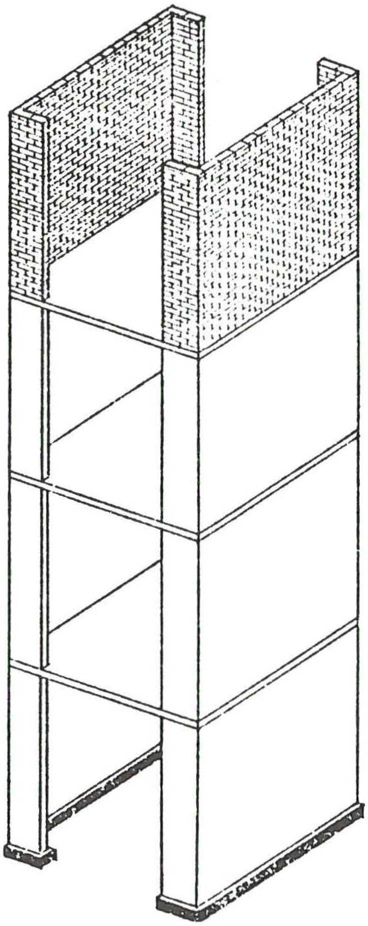


Figure 1. Isometric view of structure

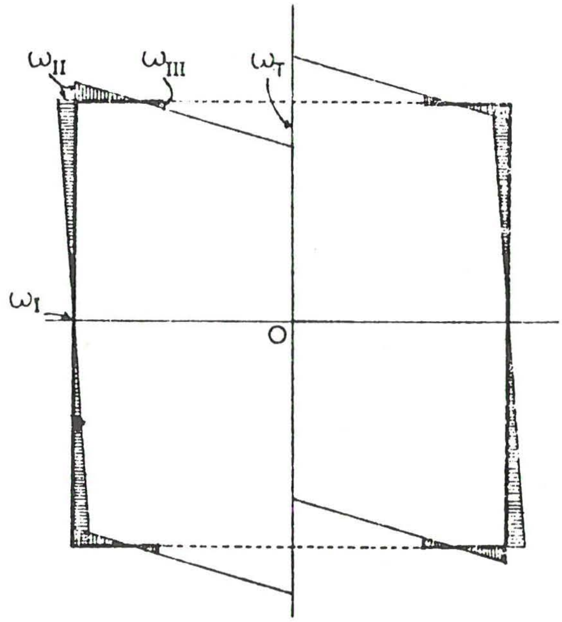


Figure 3. Diagram of the sectorial coordinate

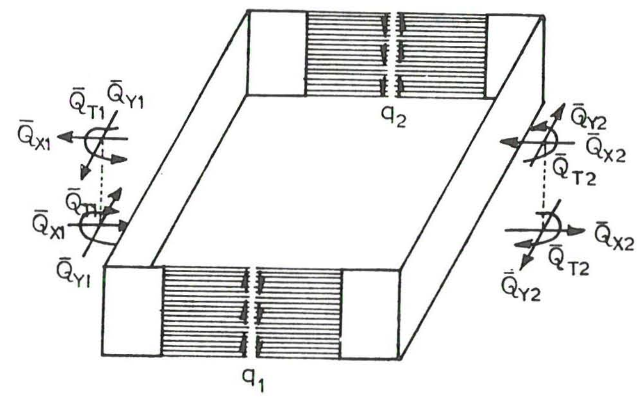


Figure 4. Internal shear forces and torques from individual walls due to distributed shear forces

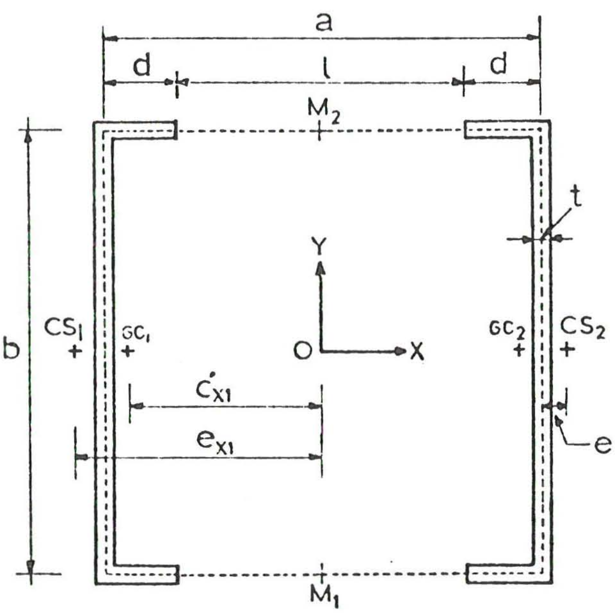


Figure 2. Plan of core wall structure

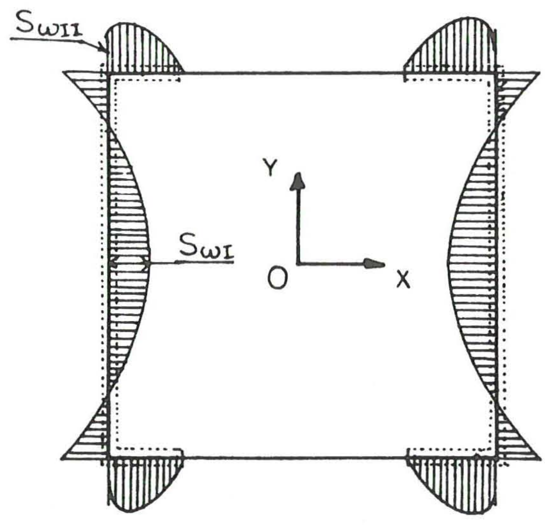


Figure 6. Diagram of the sectorial static moment

TABLE 1—Comparison of theoretical and experimental results, showing the terms of rotation included in the formulae and the percentage difference. The terms are listed below

$E = 0.98 \times 10^6$ lbf/in^2		St. Venan. Torsion	Slab Effect	Shear Forces Effect	Warping Effect	ROTATION, (Rad x 10^{-5}) For EACH STOREY				Percentage difference from experimental
						1st	2nd	3rd	4th	
A. COULL A.W. IRWIN		GK_T	EK_{SC}	EI^*	EI_{wc}	2.2	6.8	11.4	13.6	15 %
W.K. TSO		GK_T	EK_{ST}	EI^*	EI_{wT}	0.4	0.9	1.2	1.5	91 %
Ö. KESKİN	1	GK_T	EK_{SK}	EI^*	EI_{wK}	2.0	6.2	10.3	12.3	22 %
	2	GK_T	—	EI^*	EI_{wK}	2.4	7.6	13.0	15.5	2 %
	3	GK_T	—	EI^*	—	2.4	7.7	13.2	15.7	1 %
	4	GK_T	—	—	EI_{wK}	98.0	263.	386.	431.	—
U.C. KALITA*		FROM J.R. BENJAMIN				3.2	6.8	10.1	13.6	15%
U.C. KALITA'S EXPERIMENTAL DATA						4.1	8.6	12.5	15.9	

$$EI_{wc} = \frac{Eb^2t}{196}\left[b-l-t\right]\left[1+\frac{b^3t}{4I_{xx}}\right]$$

$$EI_{wT} = \frac{Eb^2t}{12}\left[\frac{b^2}{4}\left(b+6d\right)+d^2\left(3b+2d\right)\right]$$

$$EI_{wK} = \frac{Eb^2t}{12}\left[be+2d^3+6de\left(d-e\right)\right]$$

AND

$$EK_{SC} = \frac{24EI_d b^3 e_x}{hl^3}$$

$$EK_{ST} = 2E\left[be+b^2\right]^2\lambda_1$$

$$EK_{SK} = m_{c_1}\bar{m}_{c_1}\lambda_1 E = \frac{Eb^2}{2}\left[b+e_x\right]\left[b+c_x\right]\lambda_1$$

AND

$$GK_T = \frac{2G}{3}\left[b+2d\right]t^3 \quad \text{AND} \quad I^* = \sum_{i=1}^{i=2}\left(I_{x_i}e_{x_i}^2 + I_{y_i}e_{y_i}^2\right)$$

* $E_1, E_2, E_3, E_4 = \left[0.26\times10^6 \text{ to } 0.12\times10^6\right]$ lbf/in^2

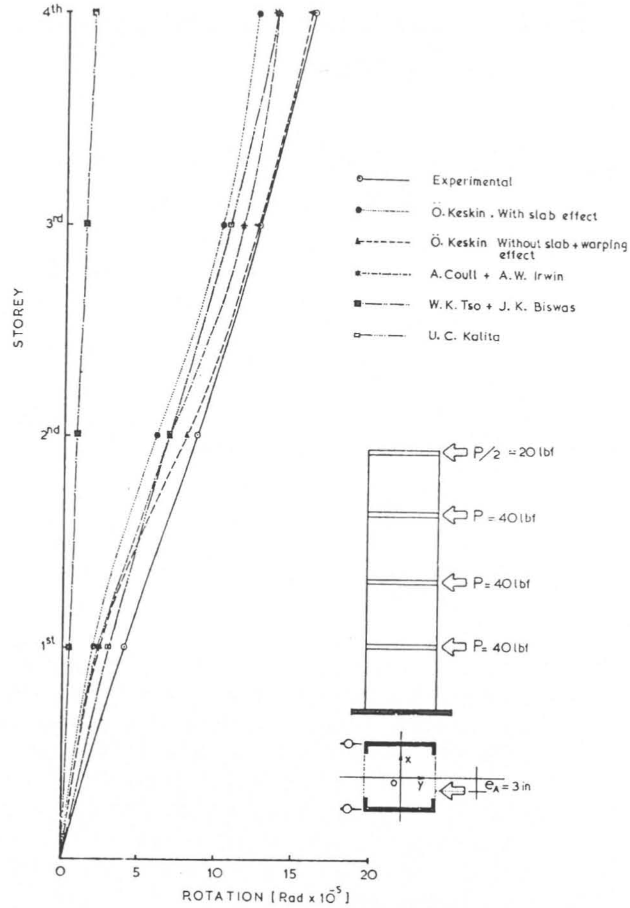
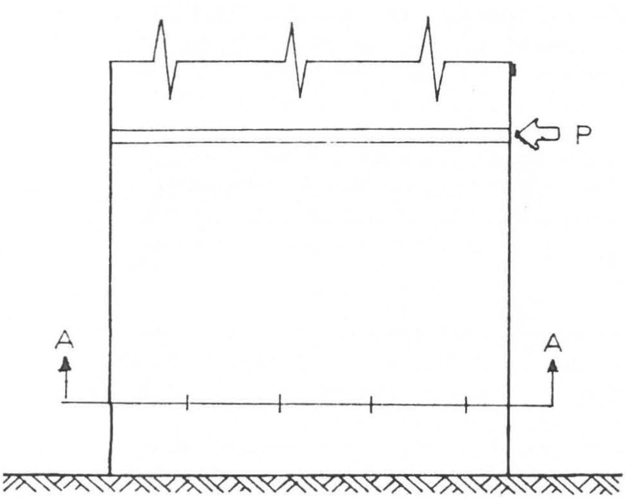


Figure 7. Storey/Rotation curves for structure. Comparison of theoretical and experimental results from "Kalita"



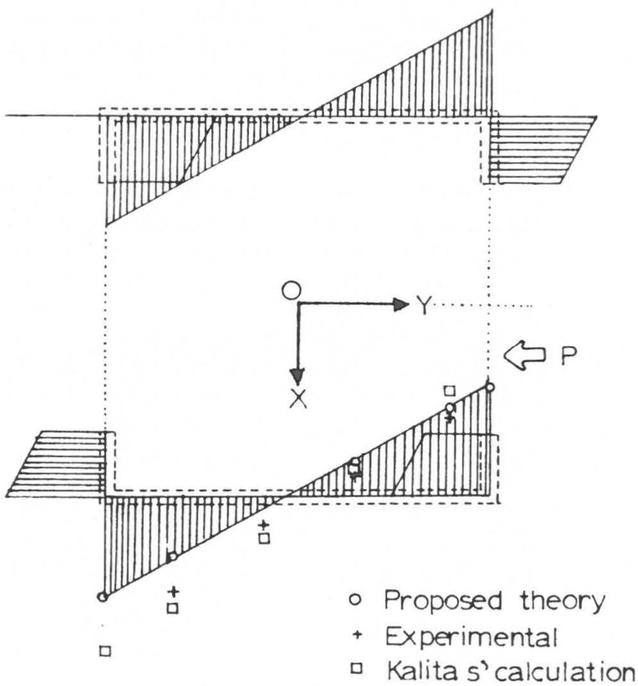


Figure 8. Vertical stress distribution in the web at section A-A