IV–19. Lateral Load Distribution in Cavity Walls

Russell H. Brown and Rudolf E. Elling

ABSTRACT

In the design of laterally-loaded cavity walls, it is generally assumed that each wythe carries lateral load in proportion to its flexural stiffness, EI. Such treatment presumes that the walls are sufficiently tied together that a unit deflection in the loaded wythe will produce a comparable deflection in the unloaded wythe. It does not account for different boundary conditions and connection details in the two wythes. Nor does it consider the size and spacing of wall ties.

The paper presents the results of theoretical analysis of cavity walls connected at discrete intervals by linear springs. Each wythe may have different boundary conditions which represent various methods of wall construction; the inner wythe is considered as either continuous or simply supported, while the outer wythe may have simply-supported, continuous or elastically-supported end conditions. The distribution of lateral load between wythes is seen to depend on end boundary conditions, number and spacing of wall ties, as well as the relative flexural stiffness of the wythes. An evaluation is made of present design requirements of various masonry building codes.

INTRODUCTION

Masonry cavity walls consist of two wythes of masonry separated by an airspace, and tied together at discrete points with metal ties. The large majority of such walls have an outer wythe of brick and an inner wythe of concrete masonry. Such walls have an excellent resistance to rain penetration if properly designed and constructed. The airspace is usually 2 to 4½ in. thick and may be insulated to provide excellent thermal and sound resistance. The fire ratings of cavity walls is also excellent. This paper determines the structural behavior of cavity walls under lateral loads and evaluates the current design procedure.

BACKGROUND

In order to rationally design cavity walls for lateral load, the flexural stiffness of each wythe, including boundary conditions, as well as the stiffness of the wall ties should be considered. It is not unusual for an exterior wythe to be continuous while having an interior wythe which is interrupted at each floor level, thus having different boundary conditions (Fig. 1). If the exterior wythe is supported on shelf angles at each floor level (Fig. 2), or alternate floor levels, then continuity will also be interrupted. The interior wythe may span vertically the entire opening between floor slabs, and therefore be restrained against end rotation at both ends, especially if it is load bearing. If compressible filler is placed between the top of the interior wythe and the bottom of the floor above to accommodate differential movements, then little if any rotational restraint will be provided at that connection.

The manner in which the two wythes are interconnected affects their interaction. The connection may be exclusively wall ties or joint reinforcement, but other connection devices having different stiffness such as shelf angles or dovetail anchors may be present at floor level.

The various building codes which address the lateral load distribution in cavity walls differ in their recommendations. The Brick Institute of America[1] requires for both non-loadbearing and loadbearing cavity walls that "the wind pressure shall be assumed to be distributed to the wythes according to their respective flexural rigidities." The BIA *Recommended Practice*[2] interprets this requirement by assigning wind load to a wythe in proportion to the ratio of EI of that wythe to the total EI of both wythes. At least three authors[3,4,5] use the definition flexural rigidity for the term EI.

The NCMA Code[6] requires "for computing flexural resistance due to horizontal load, the section modulus of a cavity wall shall be assumed to be equal to the sum of the section moduli of each wythe." There are apparently no provisions for differences in elastic moduli for each wythe.

The Uniform Building Code[7] requires "for computing the flexural resistance of cavity walls, the lateral load shall be distributed to the wythes according to their respective flexural rigidities."

BOCA[8] and Standard Building Codes[9] each make reference to BIA[1] and NCMA[6] standards. ANSI A41.1[10] and A41.2[11] are empirical codes and have no specific design procedures pertaining to distribution of lateral load between wythes.

In summary, many of the design standards require lateral load to be distributed to each wythe in a cavity wall in proportion to its flexural rigidity. The flexural rigidity, EI, alone does not adequately reflect boundary conditions or wall tie stiffness on flexural behavior. A rational design procedure which recognizes boundary conditions and wall stiffness is needed.

DEVELOPMENT OF ANALYTICAL MODELS

Analytical models were developed to determine the interaction of masonry wythes connected by wall ties. The problem was approached in two different ways: First as walls connected by a discrete number of springs, and secondly, as walls connected by continuous springs. Each method will be discussed separately.

Discrete Springs

The analysis of two wythes interconnected by discrete springs (Fig. 3) was done using a displacement method. Variables considered were number of interior springs (n), stiffness of interior springs (k), relative stiffness of the two wythes ($EI_1/EI/i_2$), relative stiffness of interior springs to end springs (k/k_e), rotational restraint provided at each floor

level, and sign of wind pressure. The number of interior springs (n) was varied from one to six. The wythes were either completely free to rotate or completely restrained against rotation at their boundaries. Relative displacement at the boundaries was permitted with non-zero values of k/k_e.

The formulation of the problem involved writing the deflection equation in the outer wythe at interior spring location i due to spring forces i, j, k . . . n, to uniformly distributed lateral load, and to end support settlement. The same was done for location i in the inner wythe. The difference in the deflection between the outer and inner wythes was related to the spring force P_i by the spring stiffness constant k. Such equations were written for each interior spring, taking advantage of symmetry, and solved for spring forces. Since all cases considered were symmetric, the largest matrix was 3 × 3.

The formulation of the case having six interior springs follows the notation on the drawings in Fig. 3.

$$\begin{aligned} \Delta_1 &= \Delta_1' + \Delta_e + \delta_{11}P_1 + \delta_{12}P_2 + \delta_{13}P_3 \\ \Delta_a &= \Delta_a' + \delta_{aa}P_1 + \delta_{ab}P_2 + \delta_{ac}P_3 \\ (\Delta_1 - \Delta_a)K &= P_1 \end{aligned} \tag{1}$$

where

Δ_i = lateral deflection of either wythe at location of interior spring i. Numbered subscripts are for outer wythe, lettered for inner wythe.
Δ_i' = deflection Δ_i due to lateral loads without interior springs or end settlement
δ_{ij} = deflection of spring i due to a pair of symmetric unit loads at j.
P_i = interior spring force
P_e = exterior spring force

The deflection of the exterior spring is related to P_e by the end spring stiffness constant k_e. The force P_e in the end spring is related to the interior spring forces by equilibrium resulting in the equation.

$$\Delta_e = \frac{\frac{ql}{2} - P_1 - P_2 - P_3}{k_e} \tag{2}$$

where q is the load on the outer wythe. For the case where the inner wythe is loaded, q is taken as zero in Eq. 2.

A deflection equation similar to Eq. 1 and 2 is written for 3 interior springs (without symmetry 6 equations are required) resulting in the following matrix:

$$\Delta_1' - \Delta_a' + \frac{ql}{2k_e} = \left(\delta_{11} + \delta_{aa} + \frac{1}{k_e} + \frac{1}{k}\right)P_1 + \left(\delta_{12} + \delta_{ab} + \frac{1}{k_e}\right)P_2 + \left(\delta_{13} + \delta_{ac} + \frac{1}{k_e}\right)P_3$$

$$\Delta_2' - \Delta_b' + \frac{ql}{2k_e} = \left(\delta_{21} + \delta_{ba} + \frac{1}{k_e}\right)P_1 + \left(\delta_{22} + \delta_{bb} + \frac{1}{k_e} + \frac{1}{k}\right)P_2 + \left(\delta_{23} + \delta_{bc} + \frac{1}{k_e}\right)P_3$$

$$\Delta_3' - \Delta_c' + \frac{ql}{2k_e} = \left(\delta_{31} + \delta_{ca} + \frac{1}{k_e}\right)P_1 + \left(\delta_{32} + \delta_{cb} + \frac{1}{k_e}\right)P_2 + \left(\delta_{33} + \delta_{cc} + \frac{1}{k_e} + \frac{1}{k}\right)P_3$$

Values of Δ_i' and δ_{ij} are input depending on boundary conditions, and the matrix solved for P_1, and P_2 and P_3. The percentage of load carried by the outer wythe is the ratio of the force carried by the end supports to the total load applied multiplied by 100%.

Continuous Springs

Two wythes interconnected by uniformly distributed springs (n approaches ∞) loaded with a uniformly distributed load is shown in Fig. 3. Either wythe may individually have either simply supported or fixed boundary conditions. However, no end settlement was permitted and each wythe had the same flexural rigidity. The origin of coordinates is taken at the mid length of each beam (so that even and odd loadings may be treated separately) and positive transverse displacement (v) is taken in the direction of the positive y axis.

$$EI_1 \frac{d^4v_1}{dx^4} = -q - k_u(v_1 - v_2) \tag{3}$$

$$EI_2 \frac{d^4v_2}{dx^4} = k_u(v_1 - v_2) \tag{4}$$

Eqs. 3 and 4 can be solved simultaneously for v_2:

$$\left[\frac{d^8}{dx^8} + \frac{2k_u}{EI_1}\frac{d^4}{dx^4} + k_u^2\left(\frac{1}{EI_1^2} - \frac{1}{EI_1EI_2}\right)\right]v_2 = \frac{-qk_u}{EI_1EI_2} \tag{5}$$

Eq. 5 can be solved readily for the special case:

$$EI_1 = EI_2 = EI$$

The even solutions to Eq. 5 for this special case are:

$$v_1 = -C_1 \cos\frac{\alpha x}{\sqrt{2}} \operatorname{Cosh}\frac{\alpha x}{\sqrt{2}} - C_2 \sin\frac{\alpha x}{\sqrt{2}} \operatorname{Sinh}\frac{\alpha x}{\sqrt{2}} + C_3 + C_4x^2 - \frac{q}{2k_u} - \frac{qx^4}{48EI} \tag{6}$$

$$v_2 = C_1 \cos\frac{\alpha}{\sqrt{2}} \operatorname{Cosh}\frac{\alpha x}{\sqrt{2}} + C_2 \sin\frac{\alpha x}{\sqrt{2}} \operatorname{Sinh}\frac{\alpha x}{\sqrt{2}} + C_3 + C_4x^2 - \frac{qx^4}{48EI} \tag{7}$$

where $\alpha = \left[\frac{2k_u}{EI}\right]^{1/4}$

A similar set of solutions exist which are odd with respect to the y axis.

Constants C_1, C_2, C_3, and C_4 can be evaluated when specific boundary conditions are specified for each wythe. Mo-

ments (M) and Shears (V) can be evaluated for each beam through the relationships:

$$M_i = (EI_i)\frac{d^2v_i}{dx^2}$$

and

$$V_i = -(EI_i)\frac{d^3v_i}{dx^3}$$

Results

The mathematical model permits a determination of the force in each spring in the discrete case as a percentage of lateral load on the wall. Bending moments were calculated at supports and midspan of both top and bottom wythes. The total load carried by the outer wythe, determined from the lateral loads and spring forces, was selected as the single most meaningful output parameter for both continuous and discrete springs. It is plotted against spring stiffness or flexibility in Figs. 4 through 10. Each figure includes a drawing of sufficient detail to identify the wall properties it represents.

DISCUSSION OF RESULTS

The analysis reveals that the portion of load carried by each wythe in a cavity wall depends on the size and number of wall ties, the boundary conditions of each wall, and the relative flexural rigidity of the wythes. Each of the effects will be discussed separately.

Wall Tie Flexibility

The wall tie flexibility relative to that of the outer wythe is expressed in dimensionless form by the relative flexibility parameter A which is defined as follows:

$$A = \frac{EI_1}{kl^3} \qquad (9)$$

where

EI_i = flexural stiffness of the *outer* wythe
l = vertical wall span
k = axial stiffness of a single wall tie

The percentage of lateral load carried by the outer wythe plotted against the flexibility parameter A shows that for a given set of boundary conditions, the load carrying participation of the outer wythe is relatively insensitive to wall tie flexibility. The BIA Standard[1] requires a 3/16 in. diameter tie for each 4½ sq. ft. of wall area spaced not more than 24 in. vertically and 36 in. horizontally. This minimum requirement is applicable for cavities from 2 to 3½ in. in thickness. For cavities from 3½ in. to 4½ in. in thickness, one metal tie is required for each 3 sq. ft. Using these minimum requirements with a 3½ in. thick outer wythe of masonry having a modulus of elasticity, E_m = 3,000,000 psi, and a 2 in. thick cavity, the resulting value of A is about 0.002. A 5½ in. thick outer wythe which is otherwise similar has an A value of about 0.0054. This range of A values is plotted on Fig. 6 representing current practice. The effect of doubling the area of each tie, which halves the A value, can be seen to have little effect on participation of the outer wythe in resisting lateral load.

Relative Wall Stiffness

If the term "relative flexural rigidity" in the various codes is interpreted to mean the ratio of EI of a wythe to the total EI of the wall, then for $EI_1/EI_2 = 1$, the design load on the outer wythe by most design standards would be 50 percent of the total load. Inspection of all of the figures having $EI_1/EI_2 = 1$ indicates that for reasonable values of A, the percentage of load carried by the outer wythe ranges from about 20 percent (Fig. 10), to about 90 percent (Fig. 7) for positive wind pressures. If EI were used as the sole criterion for lateral load distribution, the error may be unacceptably large.

Wall Tie Forces

The axial forces in the wall ties depend on the relative flexibility parameter A, the tie stiffness ratio k/k_c, the intensity of wind load, and the direction of the wind pressure. Interior tie forces may be either tension or compression for a positive wind pressure. Among the interior ties, the ones nearest the end typically carry more axial force. In some cases, as much as 25% of the total lateral force on a strip of wall is transmitted through the first interior wall tie. If a wall with a 10 ft. vertical span with ties spaced horizontally at 36 in. were subjected to a 30 psf wind load, the first interior tie would be required to carry about 225 lbs. of axial compression. Data published by NCMA[12] indicates that forces of this magnitude can easily be handled even with No. 9 gage wire. The ties at floor level are subjected to even greater axial forces, up to about 95% of the total load on a span, especially if EI_1/EI_2 is large. Using the same span, wind pressure, and tie spacing as in the previous example, the end tie might have a compressive force on the order of 900 lbs. According to procedures suggested by NCMA[12], a 3/16 in. diameter tie would buckle at 900 lbs. compression if the cavity were 6.3 in. in width. If the cavity were 4½ in. thick, the factor of safety against buckling would be less than 2. If the ties were bent to provide for a water drip, the factor of safety would further reduce. It appears that walls with wide cavities should have heavier or more closely spaced ties at the floor levels.

Effect of Boundary Conditions

The boundary conditions of a wythe affect its flexural stiffness. Wythes which are continuous tend to attract more of the lateral load since they are stiffer than discontinuous spans. Comparison of Fig. 6 with Fig. 7 shows the effect of a continuous outer wythe compared to a simply supported one. Clearly the continuous outer wythe attracts considerably more of the lateral load than the simply supported one. The same trend may be observed if the interior wythe is continuous. Again, the use of EI as the sole criterion for lateral load distribution may result in extreme error.

Number of Ties

The number of interior-wall ties used to connect two wythes was varied from one to six. In order to study the limiting case, walls connected by continuous springs were also evaluated. The percentage of load carried by the outer wythe is plotted against a new stiffness parameter Z (Fig. 4 and 5) where

$$Z = \frac{k_u l^4}{32\ EI_1} = \frac{nkl^3}{32\ EI_1}$$

where k_u = stiffness of continuous spring per foot of beam length. The other terms in the equation are the same as those defined previously. The curve is plotted in terms of Z rather than A in order to prevent the number of ties from increasing the total area of tie reinforcing. When interpreting Fig. 5, for a given value of Z, the single interior tie which corresponds to $n = 1$ would have the same area as the total area of all six interior ties for $n = 6$. In this way, it is the distribution of ties rather than their area that is being varied. It appears that using $n = 6$ gives virtually the same result as continuous springs for reasonably small values of Z. The use of $n = 1$ gives errors on the order of 0–20% depending upon the value of Z. Based on this evaluation, there appears to be no justification for suggesting closer tie spacing, since so little benefit results.

Effect of k/k_e

The effect of stiffness of interior ties to stiffness of end ties (k/k_e) can be determined from Figs. 6 through 10. In every case, a large ratio of k/k_e causes a reduction in load carried by the outer wythe. A large value of k/k_e implies a flexible end connection which permits rigid body displacement of the outer wythe relative to the inner wythe. The effect is to reduce the stiffness of the outer wythe and, consequently, reduce its tendency to attract the load. In the case where the exterior wythe is continuous and the interior wythe simply supported (Fig. 7), with non-settling supports ($k/k_e = 0$) and equal relative flexural rigidities, the outer wythe carries about 90% of the load. When k/k_e is 2, the outer wythe carries about 50–60% depending upon the value of A. If the method of distributing lateral loads to each wythe in proportion to its relative EI value is used, flexible end supports ($k/k_e > 0$) result is a more accurate prediction. On the other hand, if the outer wythe is not continuous and the inner wythe is, flexible end supports increase the error.

Negative Wind Pressure

In the analysis for negative wind pressure, it was assumed that the cavity was sufficiently vented to the outer atmosphere that a pressure differential across the wall would result in positive pressure applied to the inside of the interior wythe (Fig. 3c). Since most codes require the design for both positive and negative pressure, both loading cases must be considered. If both cases do not result in exactly the same distribution of lateral load between wythes, then the sum of the required design load for each wythe will exceed 100% of the lateral load on the wall. For example, in Fig. 7 for $A = .002$, $EI_1/EI_2 = 1$, $k/k_e = 1$, the outer wythe to inner wythe load ratio is about 80/20. For negative pressure, this ratio is 65/35 (Fig. 9). Therefore the outer wythe must be designed for 80% of positive wind pressure, and the inner wythe for 35% of the negative wind pressure. For equal positive and negative wind pressures, 115% of the load is designed for.

Steel Studs

Steel stud backup systems for brick veneer walls can be analyzed with the mathematic model developed herein. BIA Tech Note 28-B[13] suggests one tie for each 2⅔ sq. ft., with maximum horizontal and vertical spacing of 24 in. For a 3⅝ in. brick veneer and a metal stud with strong axis moment of inertia of 0.477 in.^{4} and 3⅝ in. thickness, 1 in. airspace and minimum requirements of BIA for tie spacing and size, the value of A is about 0.0004. The ratio EI_1/EI_2 is around 20. Fig. 6 indicates that regardless of k/k_e, almost all of the load is carried by the outer wythe. The only way the steel stud can properly participate in resisting the lateral load would be for the brick wythe to undergo flexural failure. This would result in leaky walls if not structural failure.

Wall Bending Moments

The percentage of load carried by the outer wythe is the main criterion used in this paper to evaluate lateral load distribution. The discrete mathematical model developed also calculated bending moments treating the spring forces as discrete loads, rather than a uniformly distributed load. Comparison of the "exact" moments with those computed from the net load acting on a wythe treated as though it were uniformly distributed shows that in some cases considerable error can result. The trend is not generally to overestimate or underestimate the moments, but rather, appears to be random. It may be an over-simplification to calculate moments from a net uniformly distributed load. However, any reasonable design procedure would preclude calculating moments from discrete spring forces.

CONCLUSIONS

An analytical model was developed to determine the interaction of wythes in a cavity wall connected by linear springs. The degree of participation of each wythe in resisting lateral load was found to be a function of relative stiffness of the wythes, number and stiffness of interior springs, end support conditions, and sign of wind pressure. The result of systematic variation of these parameters led to the following conclusions:

1) Distribution of lateral load to each wythe of the cavity wall in proportion to its relative flexural rigidity without consideration of end support conditions can result in unacceptably large errors.
2) Size and spacing requirements for wall ties in most design standards for masonry are adequate for interior ties.
3) Ties at floor level, unlike interior ties, have a low factor of safety against buckling. Increased tie area requirements at floor levels should be considered.

4) Continuity of either an exterior or interior wythe significantly increases the portion of lateral load that wythe must carry. Substantial errors can result from neglecting the continuity effect.
5) Exterior wythes connected at floor levels by flexible ties carry substantially less lateral load than those supported by unsettling supports at floor levels, such as shelf angles.
6) Designing for positive and negative wind pressures results in design capacities totaling in excess of 100% of the lateral load. For equal positive and negative wind pressure, the sum of the required design load for each wythe was often about 120% of the lateral load.
7) Steel stud backup systems do not carry an appreciable portion of the lateral load until the outer masonry wythe experiences flexural failure.

REFERENCES

1. *Building Code Requirements for Engineered Brick Masonry,* August 1969, Structural Clay Products Institute, McLean, Virginia
2. *Recommended Practices for Engineered Brick Masonry,* November 1969, Brick Institute of America, McLean, Virginia
3. Popov, E.P., *Mechanics of Materials,* Second Edition, 1976, Prentice-Hall, Inc., Englewood Cliffs, New Jersey, p. 359
4. Singer, Ferdinand L., *Strength of Materials,* Second Edition, 1962, Harper and Brothers, New York, p. 185
5. Timoshenko, Stephen, and Young, D. H., *Elements of Strength of Materials,* Fourth Edition, 1962, D. Van Nostrand Company, Inc., Princeton, New Jersey, p. 114
6. *Specifications for the Design and Construction of Load-Bearing Concrete Masonry,* May, 1976, National Concrete Masonry Association, Arlington, Virginia
7. *Uniform Building Code,* 1976 Edition, International Conference of Building Officials, Whittier, California
8. *The BOCA Basic Building Code/1975,* Sixth Edition, Building Officials and Code Administrators International, Inc., Chicago, Illinois
9. *Southern Standard Building Code,* 1973 Edition, Southern Building Code Congress, Birmingham, Alabama
10. *American Standard Building Code Requirements for Masonry,* ANSI A41.1-1953, American National Standards Association, Inc., New York, New York
11. *American National Standard Building Code Requirements for Reinforced Masonry,* ANSI A 41.2-1960 (R1970), American National Standards Institute, Inc., New York, New York
12. "The Structural Role of Joint Reinforcement in Concrete Masonry," *NCMA TEK 99,* National Concrete Masonry Association, McLean, VA, 1978
13. "Brick Veneer Panel and Curtain Walls," *BIA Technical Notes on Brick Construction, 28B,* Brick Institute of America, McLean, VA, Jan. 1979

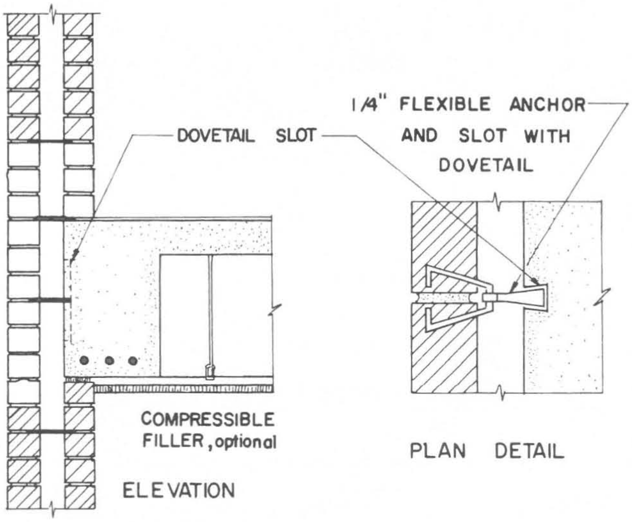


Figure 1. Flexible Support of Exterior Wythe at Floor Level

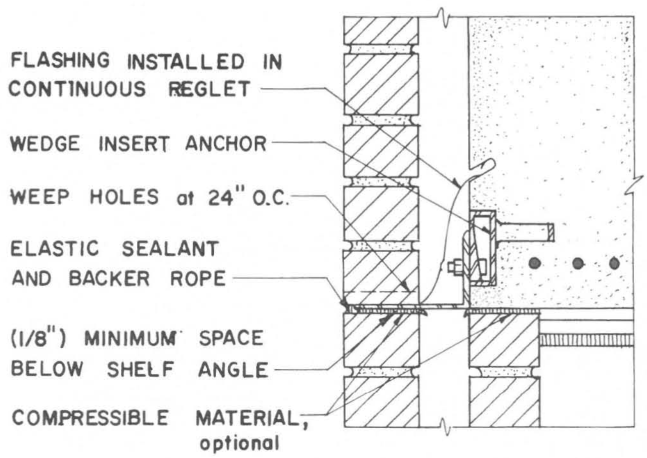


Figure 2. Shelf Angle Support Detail

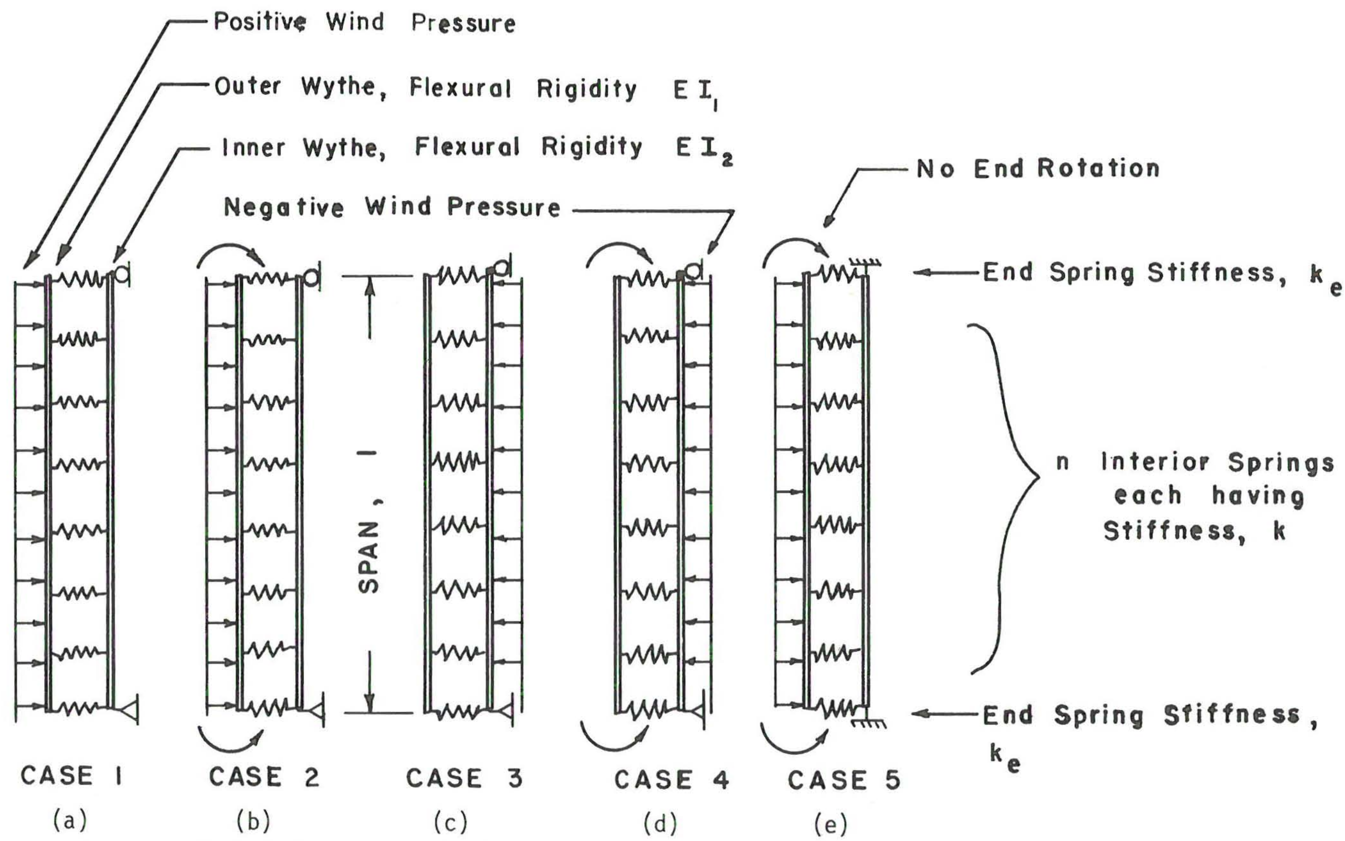


Figure 3. Cavity Wall Configurations with Various Loadings and Boundary Conditions

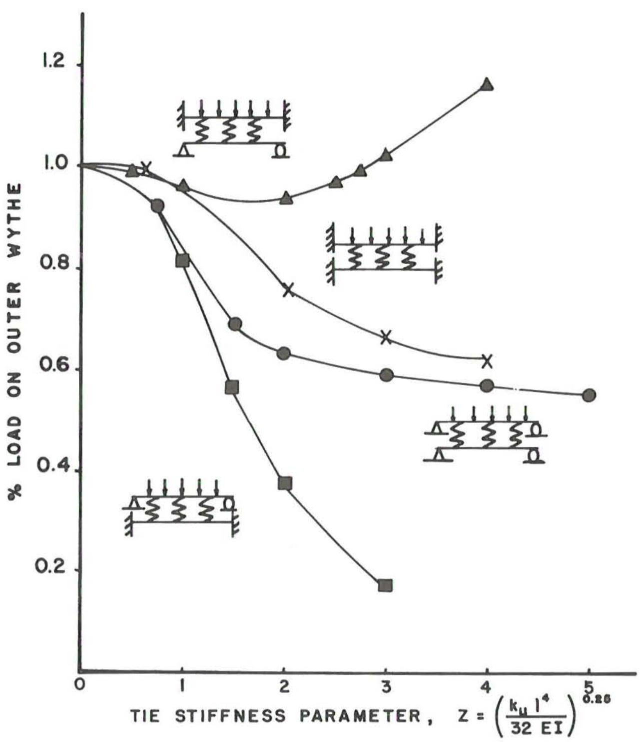


Figure 4. Percentage of Load on Outer Wythe vs. Wall Tie Stiffness Parameter for Continuous Springs

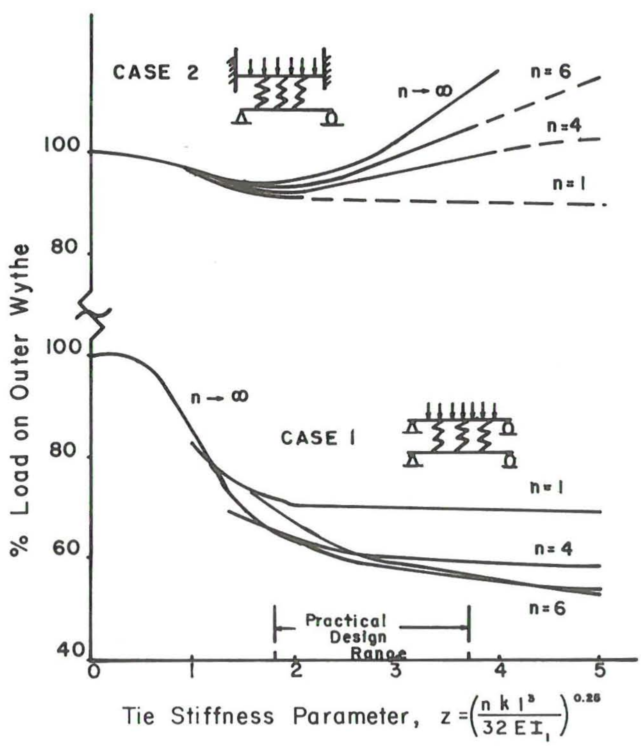
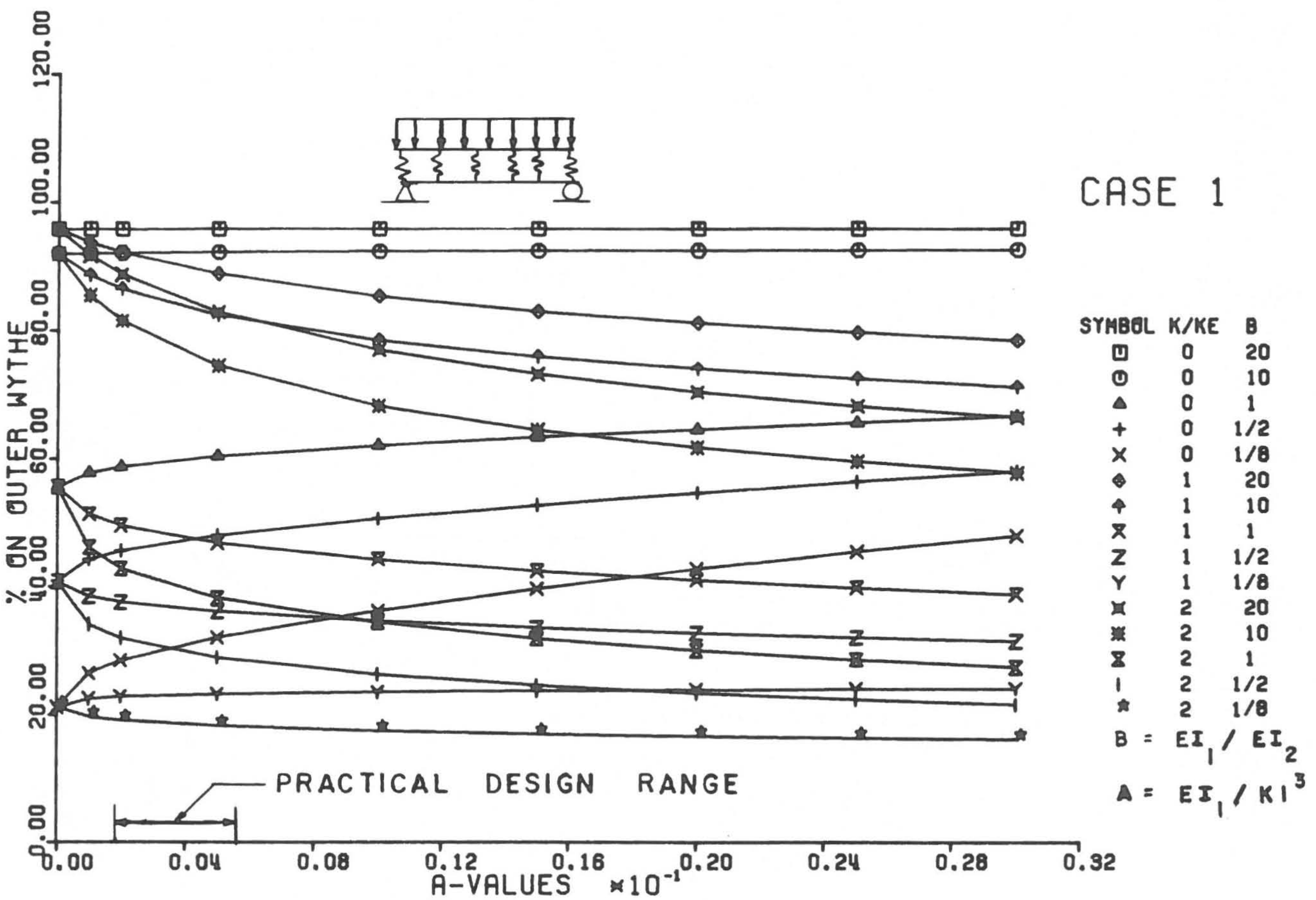


Figure 5. Percentage of Load on Outer Wythe vs. Wall Tie Stiffness Parameter for Differrent Numbers of Interior Springs

Figure 6. Percentage of Load on Outer Wythe vs. Wall Tie Flexibility for Case 1 configuration



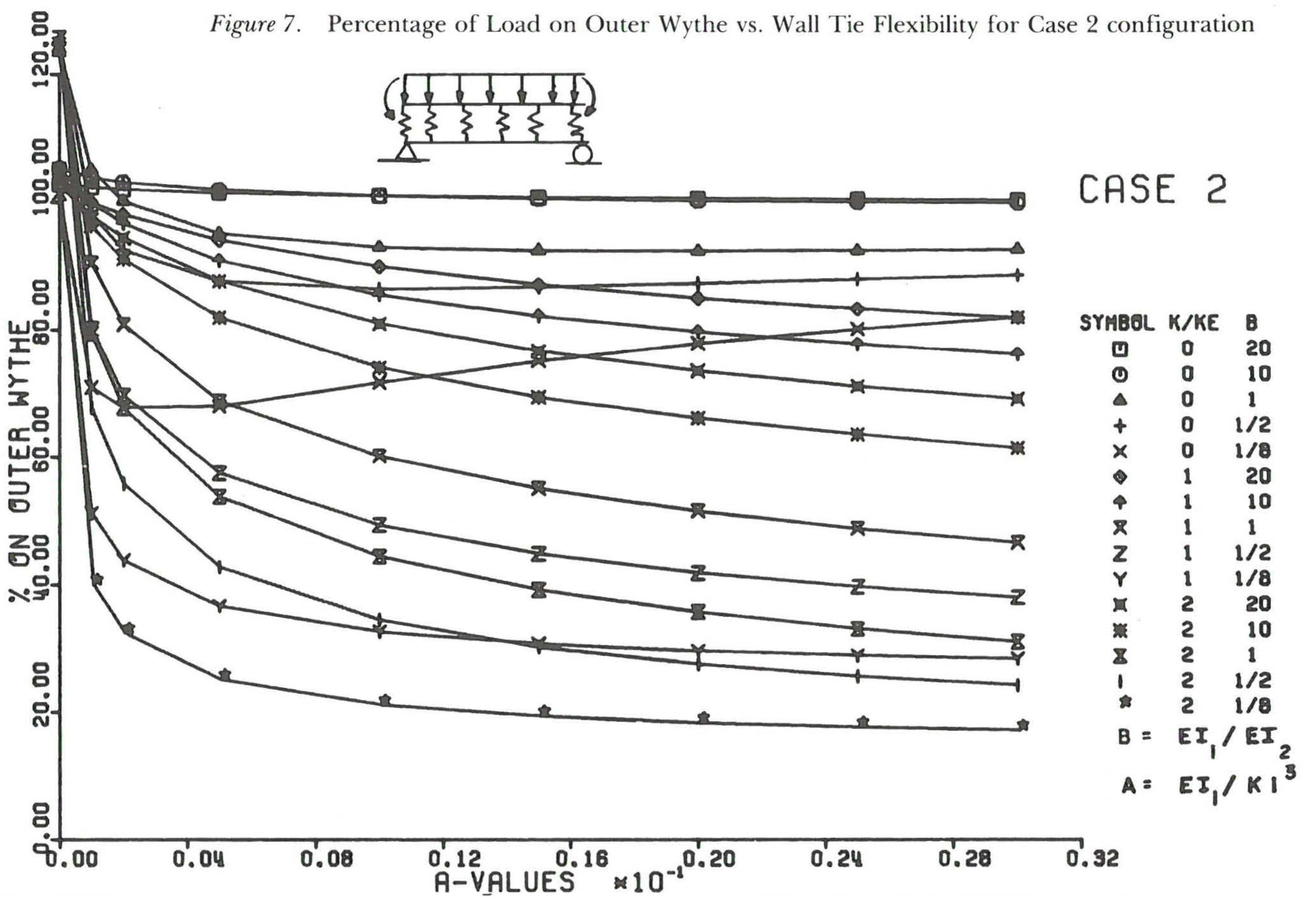


Figure 7. Percentage of Load on Outer Wythe vs. Wall Tie Flexibility for Case 2 configuration

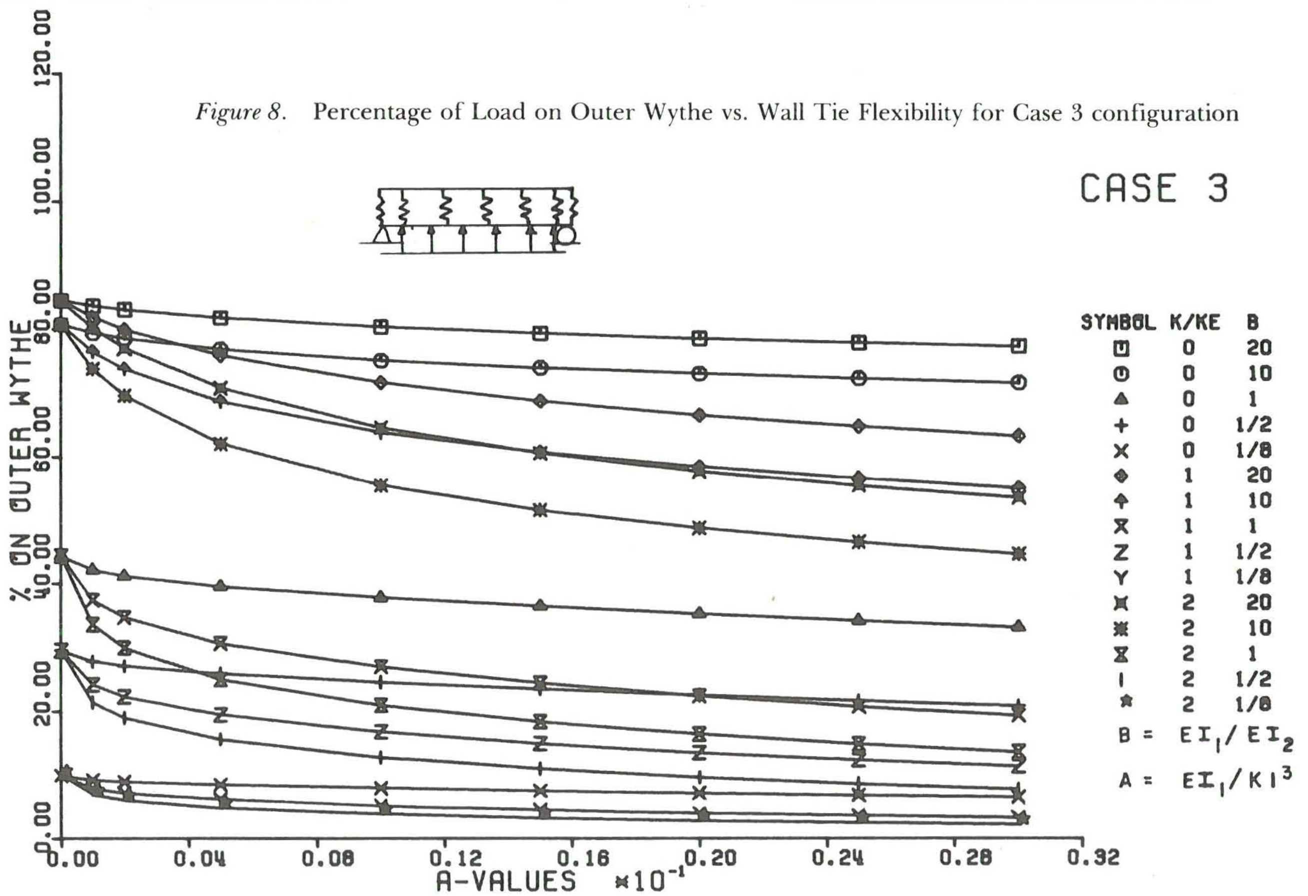


Figure 8. Percentage of Load on Outer Wythe vs. Wall Tie Flexibility for Case 3 configuration

Figure 9. Percentage of Load on Outer Wythe vs. Wall Tie Flexibility for Case 4 configuration

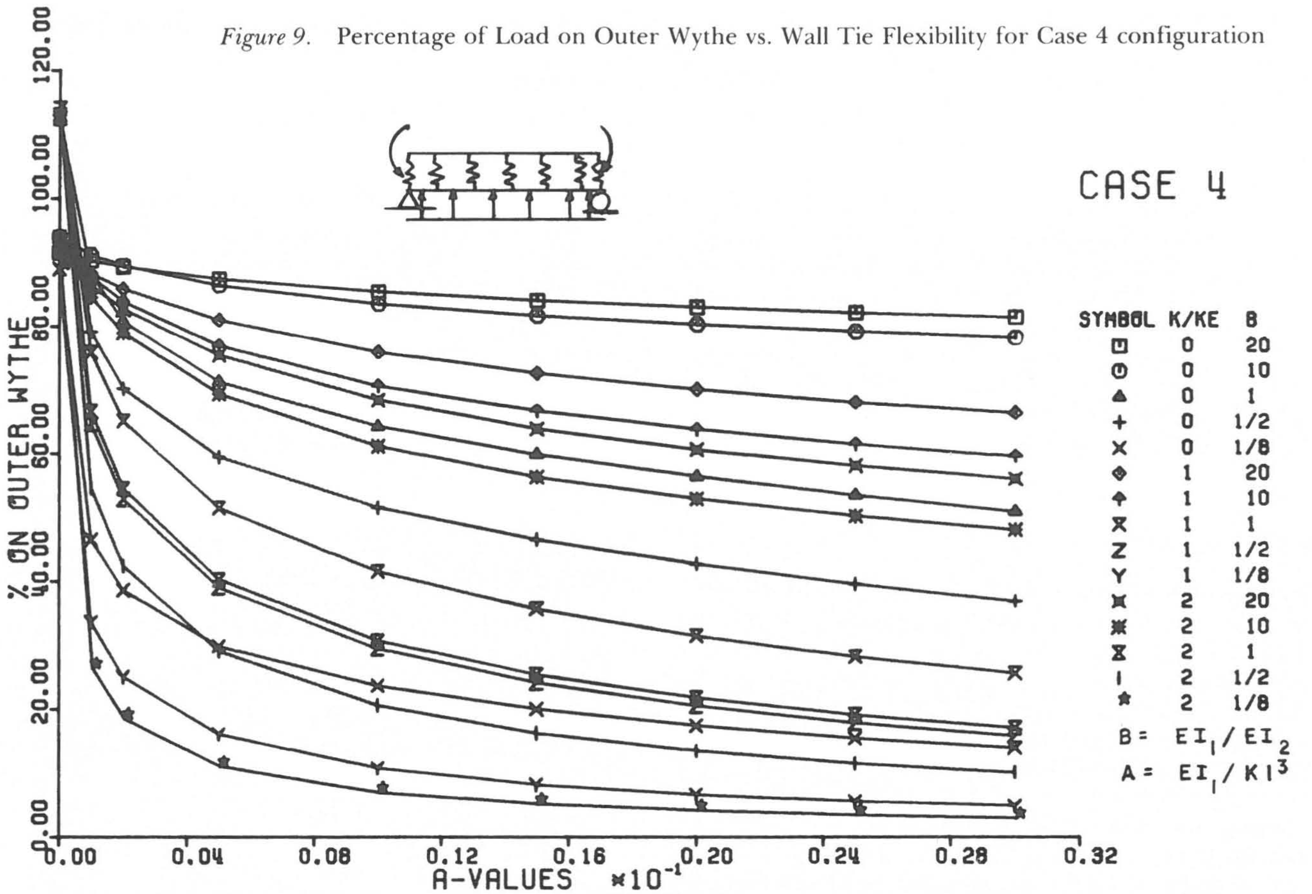


Figure 10. Percentage of Load on Outer Wythe vs. Wall Tie Flexibility for Case 5 configuration