IV–36. The Numerical Simulation for the Prevision of the Load Carrying Capacity of Masonry Structures"

M. Contaldo, Research Assistant
Institute of Technique of Constructions, Engineering Faculty—University of Naples—Italy

C. Faella, Assistant Prof.
Institute of Technique of Constructions, Engineering Faculty—University of Naples—Italy

F.M. Mazzolani, Head of the Institut
Institute of Technique of Constructions, Engineering Faculty—University of Naples—Italy

ABSTRACT

A simulation method for the analysis of the buckling behaviour of brickwork walls is presented here. This method allows to take into account the true stress-strain law for the material, which can be based on test results, together with the actual shapes and the presence of mechanical and geometrical imperfections. Some numerical results show the flexibility of such method and a comparison with theoretical and experimental results is also given.

INTRODUCTION

The degree of confidence of a theoretical prevision of the load carrying capacity of masonry structures, based on classical bifurcation methods, is rather unsatisfactory, due to various difficulties to interpret the actual behaviour of the structures by means of simple mathematical models.

A more refined approach must take into account both the true constitutive material law, which generally differs in tension and in compression, and the presence of imperfections, as initial out-of-plumb of the structural axis and load eccentricities, together with typical aspects of the constructional system, as shape cavity, stiffeners, edge condition.

The non-linearity of such problem requires to perform an incremental analysis, which simulates the step by step behaviour of the structure under different loading conditions, by providing the simultaneous solution of the set of equilibrium, compatibility and plasticity equations.

If the in-put data of the simulation procedure are coming from the statistical exploitation of several experimental results, its output may be considered equivalent to a laboratory test.

The aim of the present research is, first of all, to check the reliability of a simulation method of calculating masonry structures, by comparing its results to the testing ones, given by the up-to-date literature.

MODELS OF STRESS-STRAIN RELATIONSHIP FOR BRICKWORK

The knowledge of the true stress-strain relationship is the first important step in the analysis of the load carrying capacity of slender brickwork walls.

The influence of the stress-strain relationship model is rather important, as it has been observed in a previous paper[1]. Fig. 1 shows that the plasticity increases the collapse load of the wall especially in the range of low slenderness, whereas the tensile strength of the material gives effects in the range of large slendernesses ($L/\xi > 20$ to 30).

The elastic-perfectly plastic models with limited strains for interpreting cracking and crushing, as assumed by many Authors[1,2,3], have been substituted by more refined approaches which consider continuous $\sigma - \epsilon$ curves with a post-collapse decreasing branch.

Neglecting any tensile strength, Kirtsching, Corder and Schöner[4] proposed the following general formula:

$$\sigma/\sigma_c = a\epsilon - b\epsilon^2 \qquad (1)$$

where the parameters a and b depend upon the initial Young modulus E and the compression strength σ_c.

By assuming

$$a = E/\sigma_c \text{ and } b = a^2/4$$

the formula (1) becomes

$$\sigma = \epsilon\, E\left(1 - \frac{\epsilon}{2\,\epsilon_c}\right) \qquad (2)$$

where

$$\epsilon_c = 2\sigma_c/E.$$

These parabolic curves give a good agreement with the test results of Powel and Hodgkinson[5], as shown in Fig. 2, where the formula (2) is presented in non-dimensional form (case A):

$$\sigma/\sigma_c = 4\alpha\epsilon - 4\alpha^2\epsilon^2 \qquad (3)$$

with $\alpha = E/4\,\sigma_c$.

Formula (1) may be generalized by introducing the exponent n in the form:

$$\sigma = E\epsilon - k\epsilon^n \qquad (4)$$

The parameters k and n can be derived by imposing the following conditions (for $\epsilon = \epsilon_c$):

$$\sigma = \sigma_c$$

$$\frac{d\sigma}{d\epsilon} = 0$$

which give:

$$k = \frac{E}{\eta\, \epsilon_c^{n-1}}$$

$$n = \frac{E\, \epsilon_c}{E\, \epsilon_c - \sigma_c}$$

The use of the formula (4) gives the results of fig. 2 (case B), which are also very close to the testing ones. We observe that the value of the exponent n is approximatively equal to 2 as in the formula (1), what explains the practical coincidence of both curves A and B.

THE SIMULATION METHOD

The incremental procedure considers the displacements as independent incremental parameters and gives the step-by-step solution of the inelastic behaviour of the brickwork wall in the pre and post-buckling range. Some of the salient features of the theory are given below (Fig. 3):

a) the wall is considered as a prismatic beam-column simply supported at the edges;
b) the wall is submitted to end axial loads (N) with unequal end eccentricities (e_A and e_B);
c) the cross-section is arbitrary in shape;
d) an initial curvature may be considered as sinusoidal or parabolic curve characterized by the middle span deflection v_0;
e) the $\sigma - \epsilon$ relationship of the material may be arbitrary;
f) the mechanical characteristics of the material may vary over the cross-section.

The method supposes the wall to be subdivided into finite longitudinal (m) and transversal parts (q). The number of the longitudinal subdivisions (m) depends upon the required accuracy of the solution, whereas the transversal subdivisions (q) allow to take into account the shape effects.

At each stage of the incremental process the solution of the equation system, which governs the problem (equilibrium, compatibility, plasticity), is given by an iterative procedure.

The out-put of the computer gives the "load versus deflection curve" beyond the maximum carrying load for finite values of the deflection parameter v both in stable and in unstable range.

SOME NUMERICAL RESULTS

The stress-strain relationship given by the formula (4) has been assumed for the simulation calculations, but neglecting the decreasing branch. The parameters E, k and n have been evaluated by comparing the analytical parabola with the testing results given in Reference 6. They correspond to:

$$n = 2 \text{ and } \sigma_c/E = 0{,}0038$$

The influence of the longitudinal subdivision (m) has been preliminarily analysed. Some results are given in Tab. I, for $e = 0$ and $v_o/L = 1/1000$.

They show that the ratio N/N_u for different values of the slenderness parameter L/t is practically independent from the number of longitudinal elements for $m \geq 8$.

The load versus deflection curves are given for different slenderness ratios L/t and initial curvatures v_o/L (Fig. 4). The post-buckling range is negligible for the considered cases, due to the absence of plasticity in the assumed $\sigma - \epsilon$ curve, what corresponds to a brittle behaviour of the brickwork wall.

The simulation method allows to follow the evolution of the transversal deflection during the loading process. Some corresponding out-puts are plotted in Fig. 5, for $e = 0$, $L/t = 25$ and $v_o/L = 1/2000$. It is also possible to evaluate the spreading over the wall of the crushed and cracked zones at each incremental stage.

The large influence of the eccentricity on the load carrying capacity is analysed in Fig. 6 for different values of inside and outside of the Kern zone.

At the contrary the influence of the initial curvature seems to be rather limited in the considered range ($v_o/L = 1/500, 1/1000, 1/2000$), as it is shown in Figs. 4 and 7.

Some average test results taken from Hasan[6] are also plotted in Fig. 7. They have been normalized by means of the theoretical values for $L/t = 6$ which are always less than one, making the comparison between numerical and testing results more significant.

The agreement in-between is satisfactory for high eccentricities ($e = t/3$), whereas it becomes worst by reducing e. It can be interpreted as an effect due to the difficulties to obtain a physical model strictly coincident with the theoretical one. This fact is confirmed by the comparison with different theories, which is quite good independently from the eccentricities (Fig. 8).

The Turkstra's and Angervo's theories have been applied together with the present simulation method. The Turkstra's theory utilizes a parabolic $\sigma - \epsilon$ curve, while the Angervo's theory is based on a linear elastic curve with limited strain.

As it can be expected from these hypotheses, the agreement with simulation and Turkstra's results is quite good, being the small differences due to the initial curvature assumed in the simulation calculations ($v_o/L = 1000$). The discrepances with Angervo's results tend to disappear when the eccentricity increases, what can be explained by considering the predominance of geometrical effects due to bending.

LITERATURE

1. Mazzolani, F.M., Faella C, "Inelastic stable and unstable behaviour of eccentric loaded walls." Proceedings of II Int. Symp. on Bearing Walls, Warsaw, September 1975.
2. Yokel, F.Y., "Stability and load capacity of members with no tensile strength." Journal of Structural Division, Proceedings ASCE, ST.7.
3. Chen, W.F., Atsuta, T., "Strength of eccentrically loaded walls." Int. J. Solids Structures, vol. 9, 1973.
4. Kirtschig, K., Cordes, R., Schoner, W., "Computation of the load bearing capacity of masonry by means of stress-strain curves." Proceedings of Third International Brick Masonry Conference, 1975.

5. Powell, B., Hodgkinson, H.R., "The determination of stress-strain relationship of brickwork." Proceedings of the Fourth International Brick-Masonry Conference. Brugge, April 1976.
6. Hasan, S.S., "Investigation of the effect of slenderness ratio on the compressive strength of masonry wall panels." Department of Civil Engineering and Building Science. University of Edimburgh, England, November 1975.
7. Page, A.W., "A model for the in-plane deformation and failure of brickwork." University of Newcastle, Faculty of Engineering, Australia. Engineering bulletin CE 8 March 1978.

TABLE I—N/N$_u$ ratios for different values of m

L/t \ m	4	8	12	16	20
5	0,917	0,882	0,880	0,897	0,894
15	0,519	0,506	0,508	0,506	0,506
25	0,244	0,250	0,255	0,241	0,256
35	0,123	0,131	0,131	0,132	0,132
45	0,073	0,076	0,076	0,077	0,077

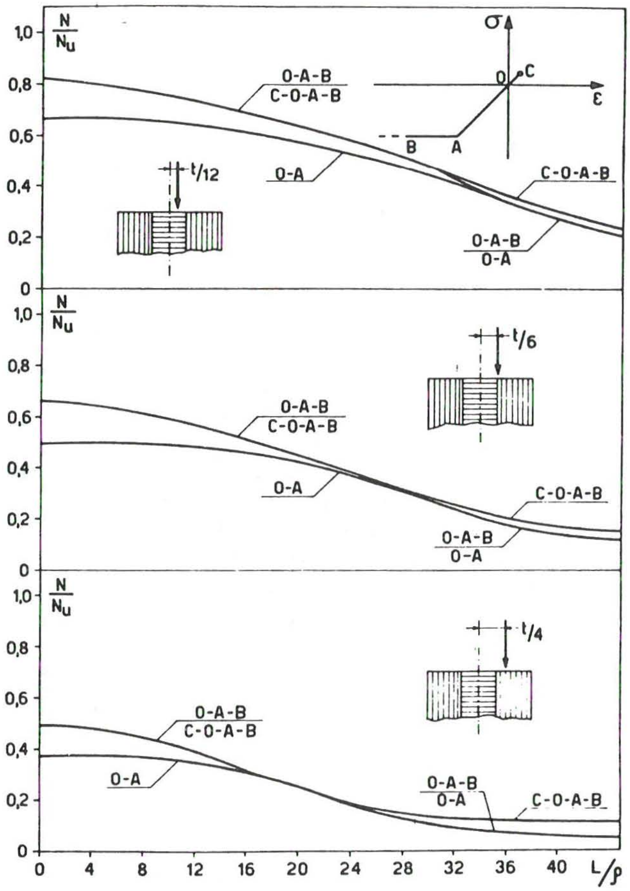


Figure 1.

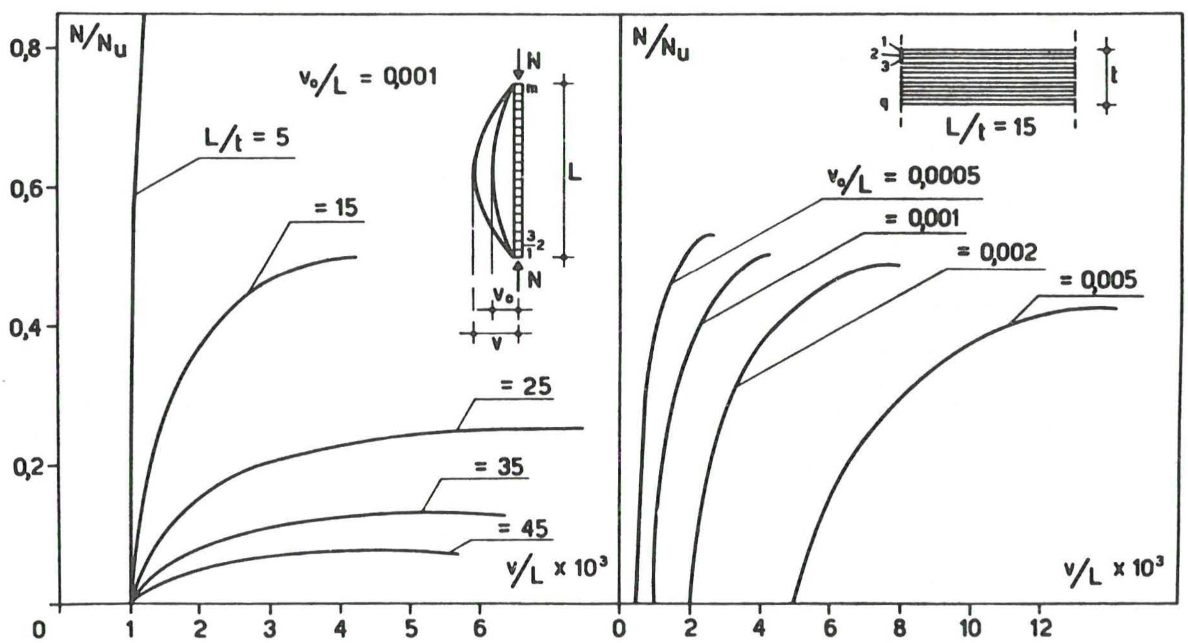
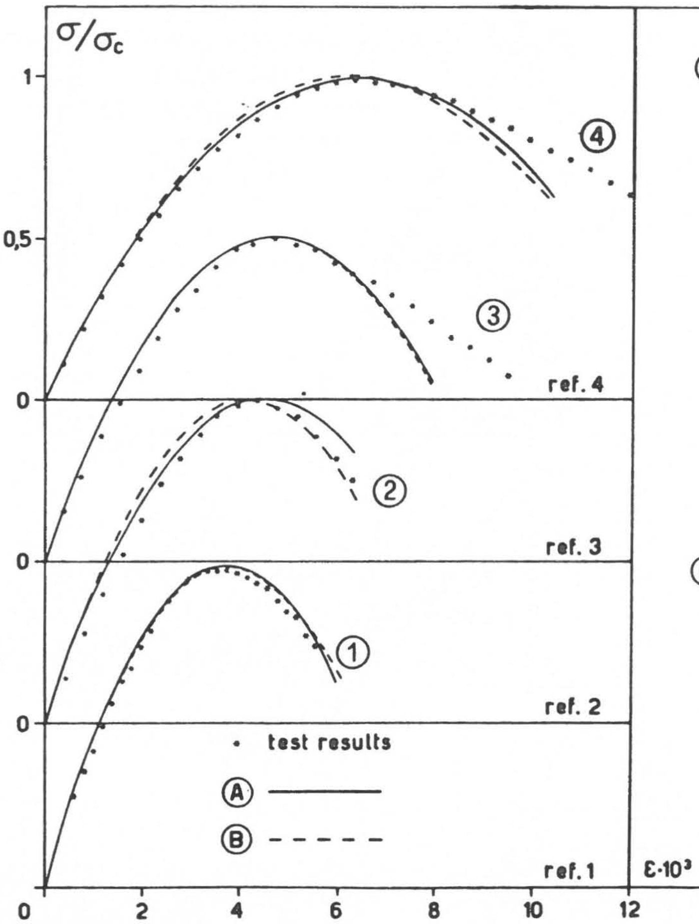


Figure 4.



Ⓐ $\sigma/\sigma_c = 4\alpha\varepsilon - 4\alpha^2\varepsilon^2$

CURVE	E (N/mm²)	σ_c (N/mm²)	$\alpha = \frac{E}{4\sigma_c}$
1	4960	9,33	132,90
2	18203	18,98	239,67
3	16830	20,10	209,33
4	17370	27,70	156,77

Ⓑ $\sigma/\sigma_c = (E\varepsilon - K\varepsilon^n)/\sigma_c$

CURVE	n	K
1	1,9802	590280
2	1,8512	1755642
3	1,9905	3327471
4	1,9520	2067732

Figure 2 The parameters k and n can be derived by imposing the following conditions.

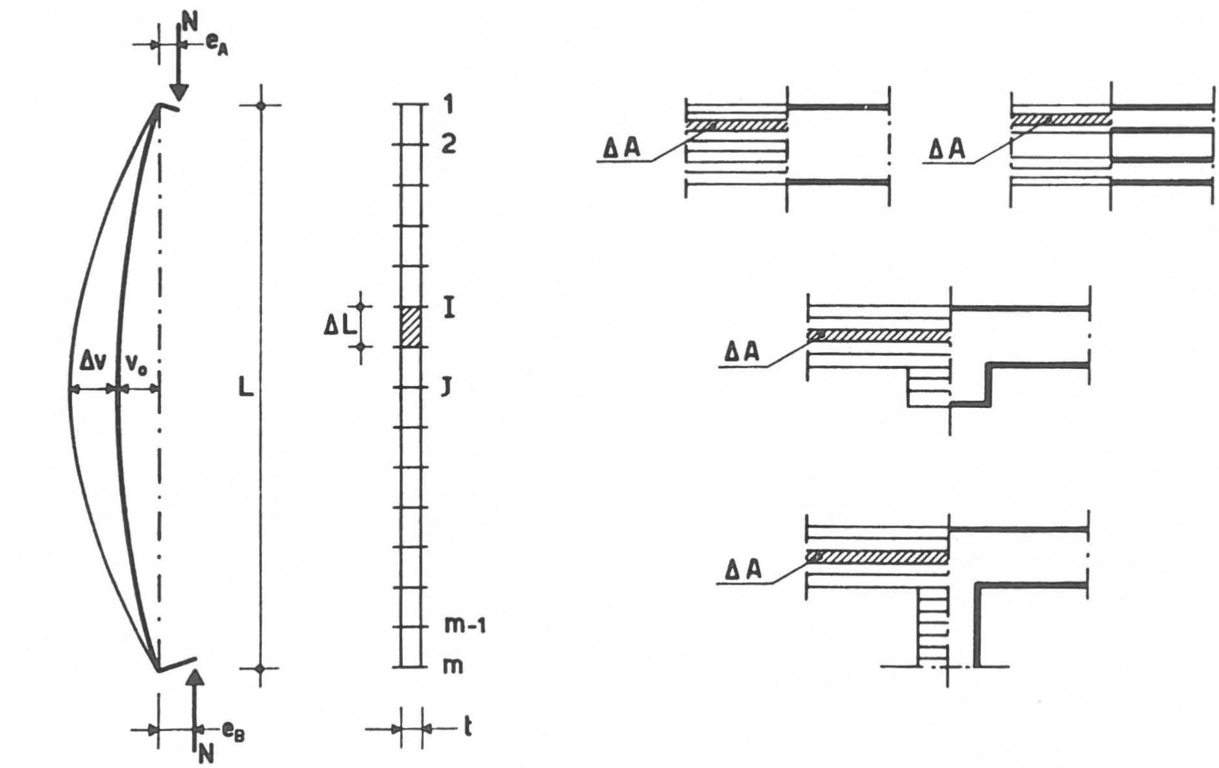


Figure 3. The out-put of the computer gives the "load versus deflection curve" beyond the maximum carrying load for finite values of the deflection parameter v both in stable and in unstable range.

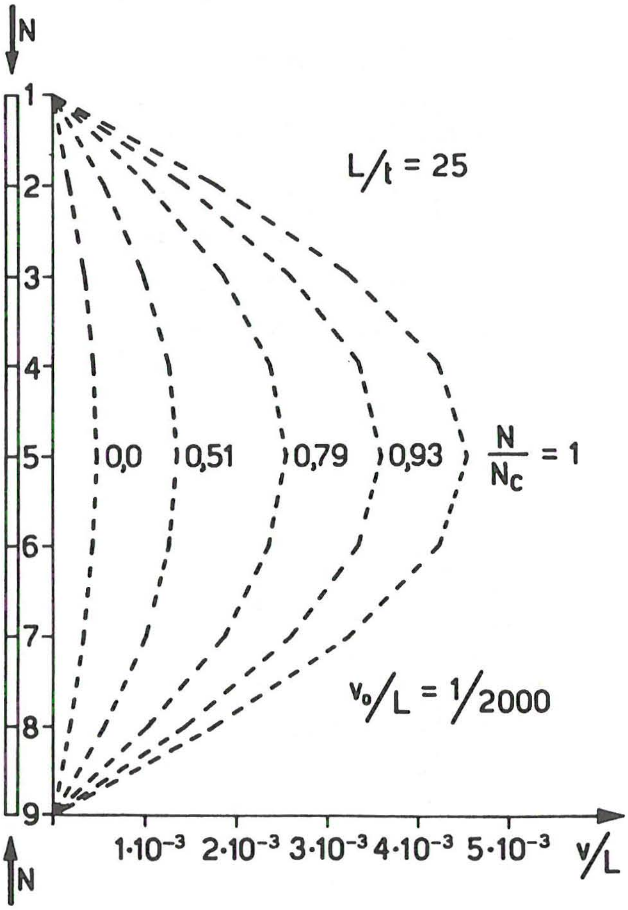


Figure 5.

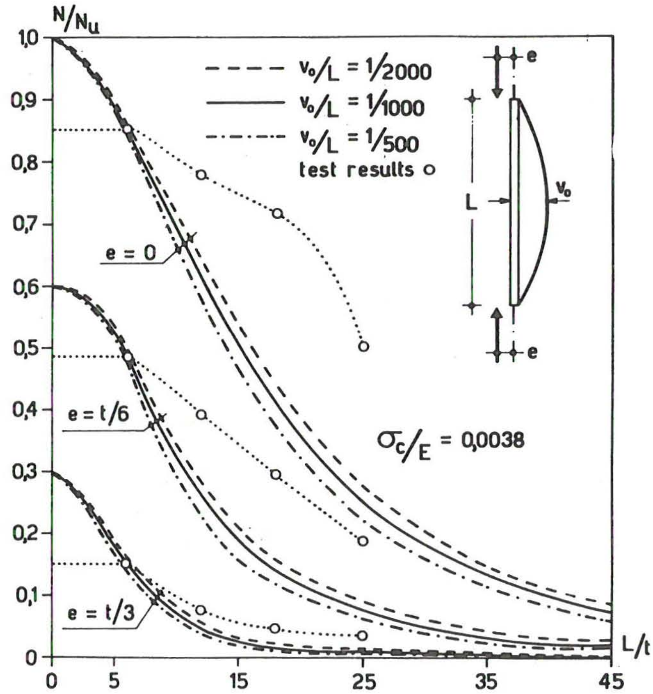


Figure 7.

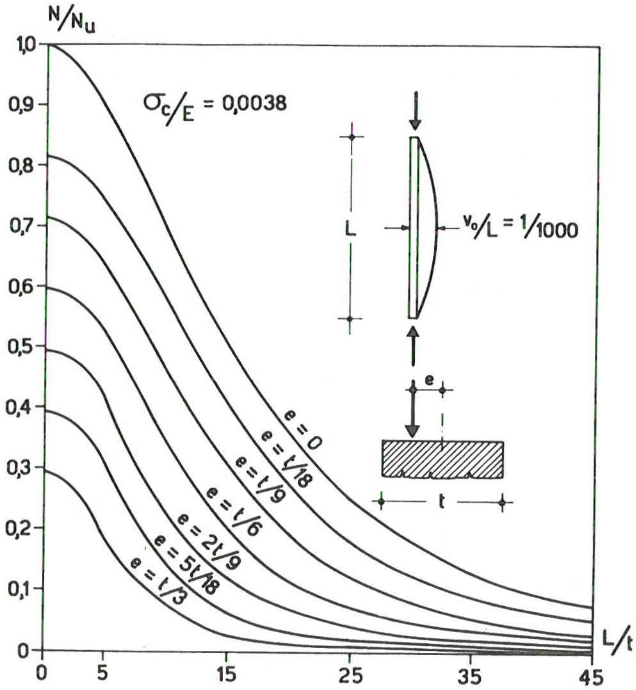


Figure 6.

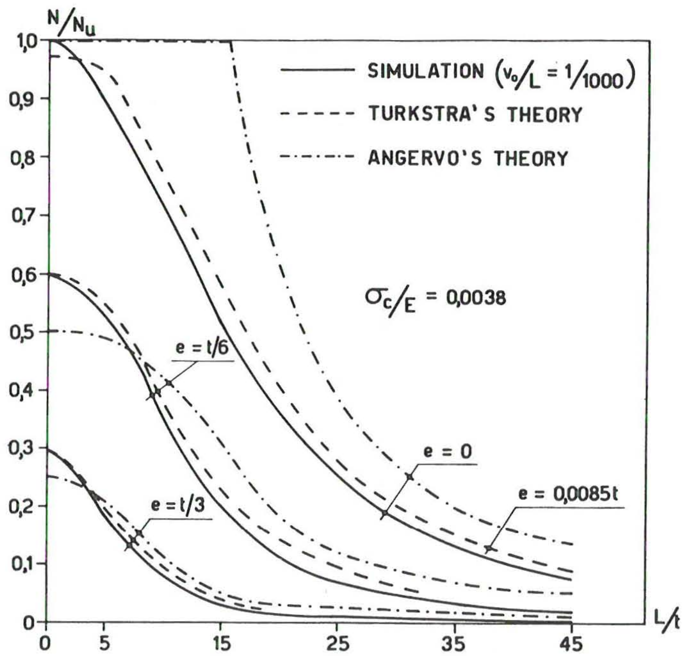


Figure 8.