IV–38. Masonry Structures: Masonry as Continuum Medium with Generalized Planes of Weakness

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ABSTRACT

This paper concerns formulation of hypothesis of masonry behaviour as continuum medium—brick—in which joints—mortar-brick interface—are treated as planes of weakness. This formulation gives available to numerical treatment the classic criteria of failure, as Mohr-Coulomb criterion, particularly efficacious in the treatment of media with low tensile strength, how widely verified in the analysis of rock boulders.

Furthermore the geometrical regularity of masonry joints imposed by obvious technological reasons, agrees to linear conditions of failure and gives them available to algorithms of linear programming.

Indeed the theoretical model is a discretization by finite elements, turned to determine with load factor, the failure mechanism more expressively referable to experimental results.

Dans cette mémoire on formule l'hypothèse selon la quelle le comportement de la maçonnerie en brique est assimilable a ce d'un milieu continu—la brique—dans le quel les manchons—c'est à dire l'interface mortier-brique—peuvent être regardés comme des plans de faiblesse. Cette formulation permet d'utiliser, au fin du traitement numerique des ces problèmes, les principes classiques de rupture, comme celui de Mohr-Coulomb; en effet ce dernier ressort particulièrement efficace pour l'étude des milieux avec faible résistence aux efforts de traction comme il est bien été demonstré dans l'analyse des roches.

De plus la régularité géométrique des manchons est perfaitement convenable aux conditions linéaires de rupture et pourtant permet de traiter ces dernières avec les algorithmes de la programmation linéaires.

Le modèle théorique est constitué par une discrétization en éléments finis capable d'individuer soit le charge critique que le méchanisme de rupture le plus proche aux essais éxperimentaux.

In dieser Abhandlung wird die Hypothese über das Verhalten des Mauerwerks als Kontinuum—Ziegeln—in denen die Verbindungen zwischen Zement—Ziegeln als schwache Flächen—behandelt werden. Diese Formulierung ist für die numerische Behandlung der klassischen Bruchkriterien von Nutzen, wie das Kriterium Mohr-Coulomb, das besonders für die Behandlung der Mittel mit niedriger Zugwiderstandskraft wirksam ist, wie es weitgehend in der Analyse der Felsmassen festgestellt wurde.

Weterhin ist die geometrische Regelmässigkeit der Verbindungen, die aus eindeutig technologischen Gründen auferlegt wird mit den linearen Bruch nedingungen in Enklang zu bringen und lässt sich mit den Algorithmen der linearen Programmierung anwenden. In der Tat ist das teorethische Modell eine Discretisierung mit finite-Elements und dient dazu mit der kritieschen Belastunng den Bruchmechanismus so nah wie möglich an die Versuchsergebnisse zu bringen.

La presente memoria concerne la formulazione di una ipotesi del comportamento della muratura come mezzo continuo—il mattone—nel quale i giunti—l'interfaccia malta-mattone—sono trattati come piani di debolezza. Una tale formulazione rende utilizzabile per il trattamento numerico i criteri classici di rottura, come il criterio di Mohr-Coulomb, particolarmente efficace nel trattamento di mezzi a bassa resistenza a trazione, come è ampiamente verificato nell'analisi delle masse rocciose.

Inoltre la regolarità geometrica dei giunti, imposta da ovvie ragioni tecnologiche, si accorda con le condizioni lineari di rottura e le rende utilizzabili con gli algoritmi della programmazione lineare.

Infatti il modello teorico è una discretizzazione ad elementi finiti, volto a determinare con il carico critico, il meccanismo di rottura più espressivamente riferibile ai risultati sperimentali.

INTRODUCTION

In the past many studies have considered masonry as an assemblage of brick and mortar with average properties. Isotropic elastic behaviour has usually been assumed to semplify the problem, and the influence of mortar joints acting as planes of weakness has been ignored. Assumptions of this nature may be satisfactory in predicting deformations at low stress levels. Really in this condition we can observe extensive stress redistribution caused by non-lin-

ear material behaviour—principally in the mortar joints—and failure in localized areas due to loss of bond between mortar and brick. Consequently, to develope rational design methods for masonry structures it is essential to know the response, in terms of failure behaviour, to the load charging. Therefore, on one side experimental results and on other side the set up of methods and failure criteria, which allow to work out acceptable hypothesis on ultimate strength, assume significance.

It is very difficult to formulate theoretical hypothesis, describing satisfactorily masonry structure failure behaviour. Anyhow, experience and experimental tests demonstrate it is possible to individualize two collapse configurations types:

i) failure on interface mortar-brick
ii) failure in basic material.

Consequently, the aim of our study is to establish an unitarian theoretical formulation that can describe masonry structure behaviour in terms of limit analysis. This consents to individualize failure mechanism acquainting us about (i) or (ii) collapse configuration occurred and coupled load factor, in the same time.

In despite of evidence, numerous experimental data give for masonry structures an aspect as "continuity"—all one—. This suggested us the opportunity of approaching the problem from basic hypothesis formulation of masonry as continuum medium, but not in continuum meaning for acquired ways of literature—granular continuum, equivalent continuum, and so on—, but affected by continuity characteristics of the "fired clay" material. Therefore brick characteristics take signification not as such as one but as "fired clay" material, so that mortar-brick joints are easily assimilable to planes of weakness, limiting continuum properties into the body of basic medium.

This approach gives immediateness to the choice, as limit criterion, of Mohr-Coulomb criterion for effectiveness that describes the failure in the basic material—let us remind, assumed with no tensile strength—and on the planes of weakness. Moreover it consents treatment of cases multiplicity, varying significant parameters—cohesion and angle of internal friction—substantially maintaining same formulation of limit conditions.

On the other side, geometrical regularity of joints, imposed by obvious technological reasons, consents turning the limit analysis problem from non-linear programming problem to linear programming problem, by suitable adaptations.

Furthermore, finite element analysis method for a masonry panel subjected to in-plane loading is developed.

The necessary material properties to define this model have been experimentally determined by tests on masonry panels and on single brick.

DEVELOPMENT OF GOVERNING EQUATIONS

Numerical implications and operative effectiveness of assumed hypothesis about masonry structures behaviour are carried out by analysis of discretized triangular finite element and constant stress tensor patterns.

Ultimate loading problem, as problem of searching maximum and/or minimum of a function of variables subjected to constraint conditions, is formulated using finite element method basic concepts and, secondarily, plasticity classic tools.

According to the lower bound theorem of theory of plasticity, stress components σ_{ij} must be in equilibrium with external loads and satisfy limit conditions everywhere into the continuum.

Therefore, problem is reduced to establishing maximum load factor, complying with equilibrium and limit conditions, that, in general assumption, are carried out as stress non-linear conditions. General governing equations have following aspect:

$$
\begin{array}{lll}
1) & \lambda \longrightarrow \text{maximum} & \\
2) & [\mathcal{A}]\{\sigma\} + \lambda\{q_o\} + \{P\} = 0 & \\
3) & \Phi_h(\sigma) \leq 0 & h = 1, \ldots, m \\
4) & \Psi_k(\sigma) \leq 0 & k = 1, \ldots, n
\end{array}
$$

where:

1) maximum condition
2) equilibrium equations
3) limit conditions on planes of weakness
4) limit conditions in basic material

Limit load can be predicted, in general form, by non-linear programming tools. The method proposed in this paper gives a more substantial choice of analysis by reducing computational charge and semplifying numerical treatment of the problem.

From this point of view, the significant assumptions are two:

i) linear limit conditions on planes of weakness
ii) linear limit conditions in basic material.

Observation can be carried that mortar-brick joints are regular in the most cases and, above all, orthogonal between them. Thus choice of joints directions as stress tensor reference directions is suggested. Internal stresses assume so the clear meaning: normal stress on joint, shear stress on joint. In this manner Mohr-Coulomb criterion is expressed in linear terms.

Besides, we formulate hypothesis that, into basic material body, joints directions, assumed as stress tensor reference, are principal. Thus, limit conditions assume a similar linear formulation.

The so set out problem solution allows to determine load factor and to establish which of the two limit conditions has been acting: either the one on plane of weakness or the other in brick.

In the first case, the carried out error, by assuming joints directios as principal in basic material body, results, after all, of no great entity and a little affecting final value of load factor.

In the other case, when brick limit condition is acting, non negligible error suggests a second writing of equilibrium equations in the real system of principal reference and a second iteration, by leaving, nevertheless, non modified limit conditions on plane of weakness.

So approximation, and together error, carry from (ii) to (i) conditions, now not acting or less significative. Thus we are able to develope a successive iterations method in terms of the linear programming.

Problem governing equations now assume the following formulation:

1) $\lambda \longrightarrow$ maximum

2) $[A]\{\sigma\} + \lambda\ \{\sigma_o\} + \{P\} = 0$

3) $[B]\{\sigma\} + [I]\{V\} + \{P\} = 0$

4) $[C]\{V\} - f[D]\{V\} \leq 0$

5) $[E]\{\sigma\} - \{\gamma\} - \varphi[F]\{\sigma\} \leq 0$

6) $[M]\{\sigma\} - [N]\{S_0\} \leq 0$

where:

1) maximum condition
2) equilibrium equations
3) boundary equilibrium equations
4) horizontal reactions limitations
5) limit conditions on planes of weakness
6) limit conditions in basic material

This mathematical formulation agrees with the two failure configurations of masonry structures for which collapse can verify or in basic material or on joints, related to load conditions.

EXPERIMENTAL DETERMINATION OF MASONRY PROPERTIES REQUIRED FOR FINITE ELEMENT MODEL

Experimental tests have been performed to assign values to material characteristics to employ in finite element model.

The dimensions of each brick were typically 24 cm × 12 cm × 6 cm. Bricks were assumed isotropic, elastic and with no tensile strength and tested to shear strength, frictional and compression tests. Tests have been performed on five series of fifteen bricks, at various imbibition degree.

The mortar used for cementing the bricks together was in the following proportion:

cement	=	450 gms
sand	=	1300 gms
water	=	225 cc

The following average values are experimentally tests results:

frictional support coefficient	f	=	0,5
vertical joints cohesion	γ_v	=	8 kg/sc
horizontal joints cohesion	γ_o	=	16 kg/sc
angle of internal friction on planes of weakness	Φ'	=	35°
basic material cohesion	S_0	=	70 kg/sc
angle of internal friction in "fired clay" material	Φ	=	20°

NUMERICAL TESTS

Numerical experimentations have been performed on masonry panel according to the finite element model of scheme in Fig. 1. Panel is subjected to various in-plane loading conditions, which are function of the only δ-parameter, deflection angle from vertical (Fig 2) Results are represented iu the diagrams of Fig. 3 and 4, which relate respectively per cent error e% and load factor λ versus angle δ.

Analysis of results allows emphasizing:

—at first, a load factor extreme variableness related to collapse configuration (ii) in brick and (i) on planes of weakness;

—secondarily, a very much oriented and, therefore, very much variable bearing capacity related to loading condition, which one or other collapse configuration can prime;

—at last, the initial error, introduced in the simplified form of proposed method, is, even though of no great entity, significant only relating to (ii) collapse configuration (see region I) and unconsiderable in joints failure case (see region III).

EXPERIMENTAL TESTS

Experimental tests were performed on same dimensions panels than finite element model. The amount of performed tests has not permitted an exact evaluation of limit loads in all load conditions.

However, for load conditions, with no exact load factor evaluation, tests confirmed goodness of numerically obtained results. Experimental verifications, examined thoroughly, on adequate number of tests, are in our future programs.

CONCLUSIONS

Proposed method in this paper, by finite element analysis, and in a simplified formulation using linear programming, allows determining limit load for masonry panels with in-plane acting load.

Accuracy of analysis is certainly influenced by variability degrees inherent to material properties—brick, mortar—but it is not so greatly to prevent any significative estimation. Masonry, as continuum medium, develops at end a strength capacity largely influenced by joints network. The whole joints network can or cannot make possible collapse configuration (i), that hardly reduces potential bearing of masonry; e.g., the influence in our model of horizontal joints creating continuous planes of weakness—bed joints—, that conditions strength of whole structure ($\delta = \pi/2$).

Thus, problem is carried to be able using masonry structure in its full capacities by a correct correspondence between loads and joints network: for example, to prevent hammering phenomena of masonry for horizontal lines, against lateral structures during earthquake. Applications of proposed method in this paper can be carried for a systematical treatment of ultimate strength problem and for every masonry structure patterns, by describing collapse modalities and characteristics.

Further developments of this procedure are envisaged involving accuracy of material laws by testing specimens.

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APPENDIX I—REFERENCES

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APPENDIX II—NOTATION

The following symbols are used in this paper

λ = load factor

$[\mathcal{A}]$ = global equilibrium matrix

$\{\sigma\}$ = stress parameter vector

$\{q_o\}$ = load vector

$\{P\}$ = body forces

$\Phi(\sigma)$ = planes of weakness limit function

$\Psi(\sigma)$ = basic material limit function

$[A]$ = equilibrium matrix

$[B]$ = boundary equilibrium matrix

$[I]$ = identity matrix

$\{V\}$ = support reactions

$[C]$ = Boolean coefficient matrix of horizontal support reactions

f = coefficient of friction in the constraints

$[D]$ = Boolean coefficient matrix of vertical support reactions

$[E]$ = Boolean coefficient matrix of shear stress on planes of weakness

$\{\gamma\}$ = joint cohesion

φ = coefficient of friction

$[F]$ = Boolean coefficient matrix of normal stress on planes of weakness

$[M]$ = Boolean coefficient matrix of normal sress in basic material

$[N]$ = Boolean coefficient matrix of cohesion in basic material

$[S_o]$ = basic material cohesion

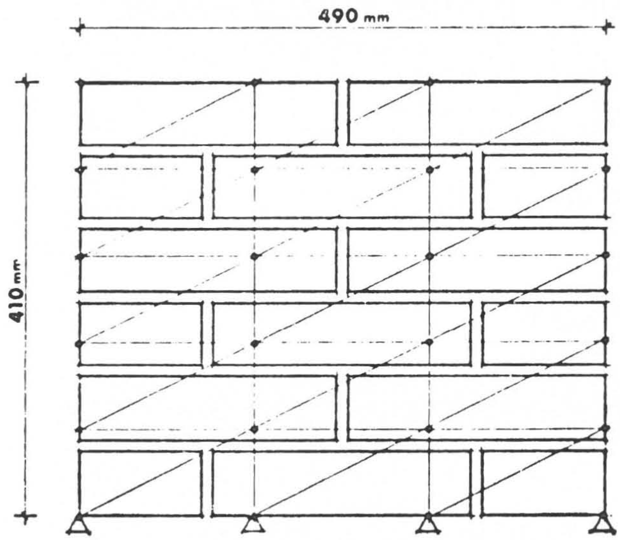

Figure 1. Specimen tested with typical finite element subdivision

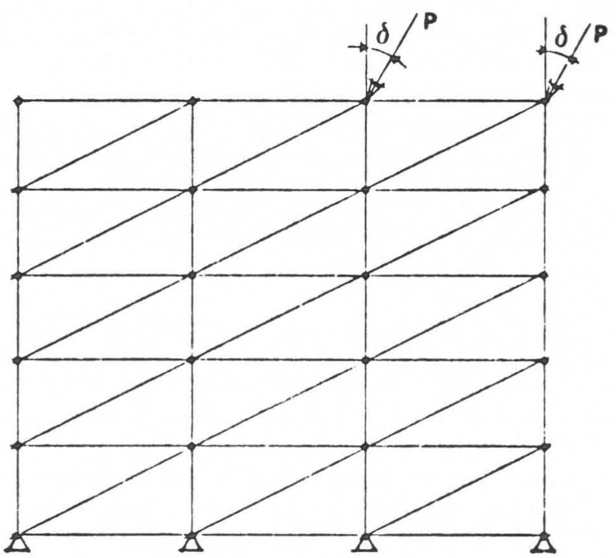

Figure 2. Specimen load types tested ($b=0, \pi/12, \pi/6, \pi/4, \pi/3, 5\pi/12, \pi/2$)

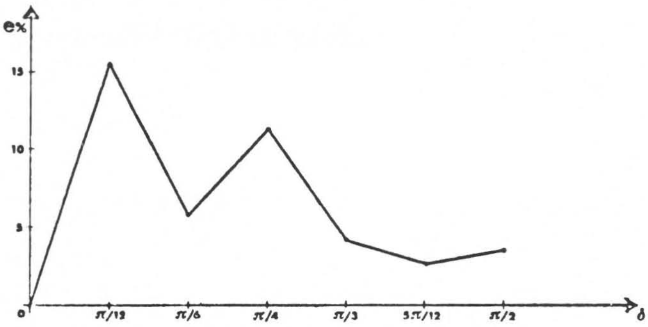

Figure 3. Percent error v/s load angle diagram

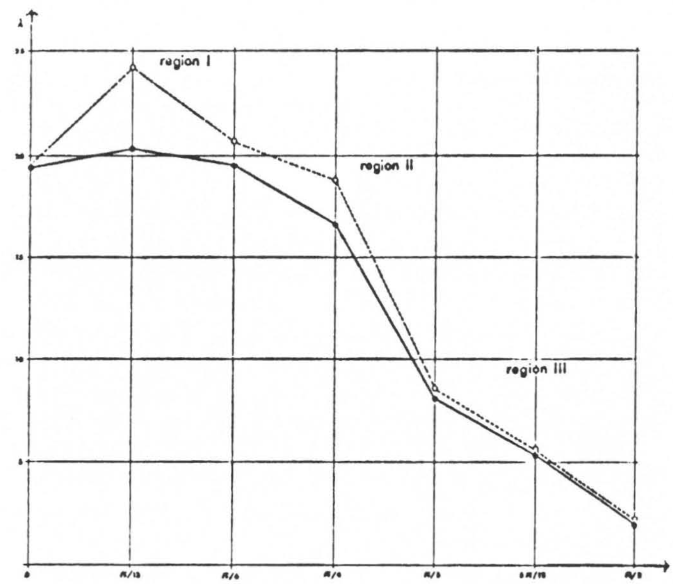

Figure 4. Load factor v/s load angle diagram
*first order approximation values
*corrected values