V-4. Plates and Shells of Hollow Blocks

K. Nag and S.R. Davies

ABSTRACT

In this paper the authors discuss in general the use of hollow clay blocks as structural materials in the form of flat or shell roofs. Although clay blocks have been in use for the last 50 years there seems to have been no reasonable design methods available for such construction. The authors therefore develop general theories and sets of displacement equations in terms of U, V and W for a doubly curved shell, taking membrane and bending actions into consideration, and then show how these equations can be used for flat plates and cylindrical shells. The method of solving the sets of displacement equations by using finite difference approximation is included. Finally experimental results for a fixed plate and a cylindrical shell fixed at edges built of clay blocks are compared with values obtained by the analytical solutions.

INTRODUCTION

Hollow clay blocks have been used in many European countries for over fifty years for the construction of walls, beams, slabs, cylindrical shells and doubly curved shells. The use of these blocks result in a number of advantages such as simple and mobile shuttering, savings in dead weight, use of less reinforcement, better thermal insulation and a financial economy resulting from the use of a cheaper material.

However, approximate methods for the design of hollow clay block construction are still being used and slabs are designed as beam strips spanning in one direction, and other curved surfaces by simplified methods based on the membrane theory.

In previous papers[1,2] the authors have outlined methods for the design of plates and cylindrical shells for this form of construction allowing for bending effects and in this paper a similar approach is developed for doubly curved shells. In fact the displacement equations for the doubly curved shell, taking account of both membrane and bending actions, represents a general case from which other solutions for plates and cylindrical shells can be readily derived.

Results obtained from tests on model plates and cylindrical shells have been fully described in Ref. 3 but a selection of the results obtained are included in this paper for comparison with the theory.

DEVELOPMENT OF THEORY

In practice hollow clay blocks may be of different cross sectional form and to simplify the analysis the cross section of the shell is assumed to be represented by the sections shown in Figs. 1 and 2. The method can be modified to allow for other types of cross section.

Other assumptions made are:

a. The shell is thin so that the ratio of the radii to overall thickness is large.
b. The shell is shallow and dz/dx and dz/dy are small and can be neglected compared to unity.
c. Plane sections normal to the middle surface remain plane and normal during deformation.
d. Normal stresses on planes parallel to the middle surface are neglected.

The development of the basic equation follows the usual methods used for slabs except that additional extensional and bending rigidities have to be defined to allow for the cross-sectional shape in directions x and y. These rigidities are as follows.

Extensional Rigidities

$$D = (E/(1-\mu^2))\int_s dz$$

$$D_x = D + E\int_r \left(\frac{b''}{b_2}\right) dz$$

$$D_y = D + E\int_r \left(\frac{b'}{b_1}\right) dz$$

Bending Rigidities

$$K = (E/(1-\mu^2))\int_s z^2\, dz$$

$$K_x = K + E\int_r \left(\frac{b''}{b_2}\right) z^2\, dz$$

$$K_y = K + E\int_r \left(\frac{b'}{b_1}\right) z^2\, dz$$

where

$\int_s$ = Integration over slab

$\int_r$ = Integration over rib.

Using the above and successive substitution of the five relationships shown below the partial differential equations connecting the middle surface displacements and the external forces are obtained (Eqns. 1, 2 & 3 and Figures 3 & 4).

1. Middle surface strains and changes of curvature with middle surface displacements.
2. Parallel surface strains and middle surface strains.
3. Normal and shear stresses with parallel surface strains.
4. Forces and moments with normal and shear stresses.
5. Forces and moments in the form of the equilibrium equations.

The resulting partial differential equations are—

$$D_x(\partial^2u/\partial x^2) - (D_x/R_x + \mu D/R_y)(\partial W/\partial x) + (D/2)(1+\mu)(\partial^2 v/\partial x\partial y) - (D/R_{xy})(1-\mu)(\partial W/\partial y) + (D/2)(1-\mu)(\partial^2 u/\partial y^2) + P_x = 0 \quad (1)$$

$$(D/2)(1-\mu)(\partial^2 v/\partial x^2) - (D/R_{xy})(1-\mu)(\partial W/\partial x) + (D/2)(1+\mu)(\partial^2 u/\partial x\partial y) - (D_y/R_y + \mu D/R_x)(\partial W/\partial y) + Dy(\partial^2 v/\partial y^2) + P_y = 0 \quad (2)$$

$$(D_x/R_x + \mu D/R_y)(\partial u/\partial x) + (D/R_{xy})(1-\mu)(\partial v/\partial x) + (D_y/R_y + \mu D/R_x)(\partial v/\partial y) + (D/R_{xy})(1-\mu)(\partial u/\partial y) - (D_x/R^2{}_x + 2\mu D/R_xR_y + (2D/R^2{}_{xy})(1-\mu) + D_y/R^2{}_y)W - K_x(\partial^4 W/\partial x^4) - 2K(\partial^4 W/\partial x^2\partial y^2) - K_y(\partial^4 W/\partial y^4) + P_z = 0 \quad (3)$$

APPLICATION OF BASIC EQUATIONS TO PARTICULAR CASES

(1) Flat slab under Pure Banding

For this case

$$1/R_x = 1/R_y = 1/R_{xy} = 0$$

and the equations reduce to a single equation.

$$-(K_x(\partial^4 W/\partial x^4) + 2K(\partial^4 W/\partial x^2\partial y^2) + K_y(\partial^4 W/\partial y^4)) + P_z = 0 \quad (4)$$

(2) Cylindrical Shell

For this case

$$\frac{1}{R_x} = \frac{1}{R_{xy}} = 0 \quad \text{and} \quad 1/R_y = 1/R$$

and the equations reduce to

$$D_x(\partial^2 u/\partial x^2) - (\mu D/R)(\partial W/\partial x) + (D/2)(1+\mu)(\partial^2 v/\partial x\partial y) + (D/2)(1-\mu)(\partial^2 u/\partial y^2) + P_x = 0 \quad (5)$$

$$(D/2)(1-\mu)(\partial^2 v/\partial x^2) + (D/2)(1+\mu)(\partial^2 u/\partial x\partial y) - (D_y/R)(\partial W/\partial y) + D_y(\partial^2 v/\partial y^2) + P_y = 0 \quad (6)$$

$$(\mu D/R)(\partial u/\partial x) + (D_y/R)(\partial v/\partial y) - (D_y/R^2)W - K_x(\partial^4 W/\partial x^4) - 2K(\partial^4 W/\partial x^2\partial y^2) - K_y(\partial^4 W/\partial y^4) + P_z = 0 \quad (7)$$

METHOD OF SOLUTION

The surface of the plate or shell is divided into a convenient number of rectangles and simultaneous equations derived at each node of the rectangles by application of finite difference approximations to the displacement equations.[4]

These simultaneous equations are linear and contain, as unknowns, the values of U, V and W at each node together with fictitious values which have to be eliminated by using appropriate boundary conditions.

APPLICATION TO PLATES

The fourth order displacement equation in W is reduced to a set of linear simultaneous equations in the form

$$AW = P_z \quad (8)$$

where A is a matrix of coefficients and W is the vector of displacement at each node.

The equation is readily solved by computer and, once the values of W are known, bending moments determined from the equations.

$$M_x = -[K_x(\partial^2 W/\partial x^2) + K\mu(\partial^2 W/\partial y^2)] \quad (9)$$

$$M_y = -[K_y(\partial^2 W/\partial y^2) + K\mu(\partial^2 W/\partial x^2)] \quad (10)$$

$$M_{xy} = M_{yx} = -[K(1-\mu)(\partial^2 W/\partial x\partial y)] \quad (11)$$

APPLICATION TO CYLINDRICAL SHELL

The three governing equations reduce to a set of linear simultaneous equations in the form

$$\begin{aligned} AU + BV + CW &= P_x \\ DU + EV + FW &= P_y \\ GU + HV + KW &= P_z \end{aligned} \quad (12)$$

where A,B, . . . K are matrices of coefficient and U, V and W are vectors of displacements at each node.

These equations can be solved by computer for U, V and W and forces and moments determined from the following

$$\begin{aligned} N_x &= D_x(\partial u/\partial x) + \mu D(\partial v/\partial y - W/R) \\ N_y &= D_y(\partial v/\partial y - W/R) + \mu D(\partial u/\partial x) \\ N_{xy} &= N_{yx} = (D/2)(1-\mu)(\partial u/\partial y + \partial v/\partial x) \\ M_x &= -K_x(\partial^2 W/\partial x^2) - \mu K(\partial^2 W/\partial y^2) \\ M_y &= -K_y(\partial^2 W/\partial y^2) - \mu K(\partial^2 W/\partial x^2) \\ M_{xy} &= M_{yx} = -K(1-\mu)(\partial^2 W/\partial x\partial y) \end{aligned} \quad (13)$$

EXPERIMENTAL WORK

Ideally results obtained from the method outlined above should be compared with those obtained from full scale testing but unfortunately such results are not available. However, in order to make some comparison between theoretical and practical values the authors have constructed models representing a slab and a cylindrical shell.

The slab, constructed of 1/6th scale hollow clay blocks (Fig. 7) measured 762mm × 762mm and was fixed at all edges. Top and bottom reinforcement was placed, along grooves cast into the blocks, in one direction only and the slab was loaded incrementally with a uniformly distributed load.

The cylindrical shell constructed with similar blocks measured 762mm (length) 762mm (width) and 762mm (radius). The shell was fixed at the edges and free at the arch ends. Top and bottom reinforcement was placed in the transverse direction and the shell was loaded with a uniformly distributed load.

Further details of the test procedure and results obtained both for deflection and strain measurements are given in Ref. 3.

Table 1 and 2 show a comparison between the theoretical and experimental values for the slab and cylindrical shell respectively.

The location of the dial gauges are shown in Figs. 5 and 6.

CONCLUSIONS

Comparison of the values of the vertical deflections obtained by the use of the proposed theory and the measured values are in fairly close agreement both for the plate and the cylindrical shell.

No results are available for doubly curved shells and additional experimental work, preferably from full scale tests is required in this field.

The largest variation in results for the cylindrical shell occur at locations 1, 4 and 8 and since these are the locations nearest to the fixed boundaries there may be some influence due to the size of the mesh near the boundary and the accuracy of the fictitious values introduced at these boundaries. Greater accuracy could be obtained at the expense of greater computation by introducing a graded mesh which is finer at the boundary.

ACKNOWLEDGMENTS

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REFERENCES

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NOTATION

x,y,z	co-ordinate system.
U,V,W	Displacement components in the x,y,z directions respectively.
P_x,P_y,P_z	External loads.
N_x,N_y	Normal Forces.
N_{xy},N_{yz}	Shear Forces.
M_x,M_y	Bending Moments.
M_{xy},M_{yz}	Twisting Moments.
μ	Poisson's Ratio.
R_x,R_y	Radii of Curvature.
D,D_x,D_y	Extensional Rigidities.
K,K_x,K_y	Bending Rigidities.
E	Young's Modulus

TABLE 1—Comparison of Experimental and Analytical Results for Slab Built of Hollow Blocks (Load Intensity — 460 N/m^2)

Dial Gauge No.	Vertical Deflections (W) mm × 10^{-4} Experimental	Analytical
1	238	263
2	171	161
3	170	160
4	89	99

TABLE 2—Comparison of Experimental and Analytical Results for Cylindrical Shell (Load Intensity — 460 N/m^2)

Dial Gauge No.	Vertical Deflection (W) mm × 10^{-4} Experimental	Analytical
1	95	71
2	114	108
3	127	122
4	76	71
5	102	108
6	102	108
7	127	122
8	79	71
9	102	108
10	124	122

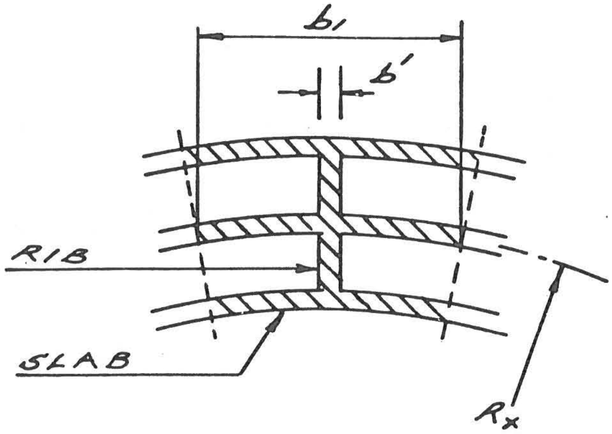

Figure 1.

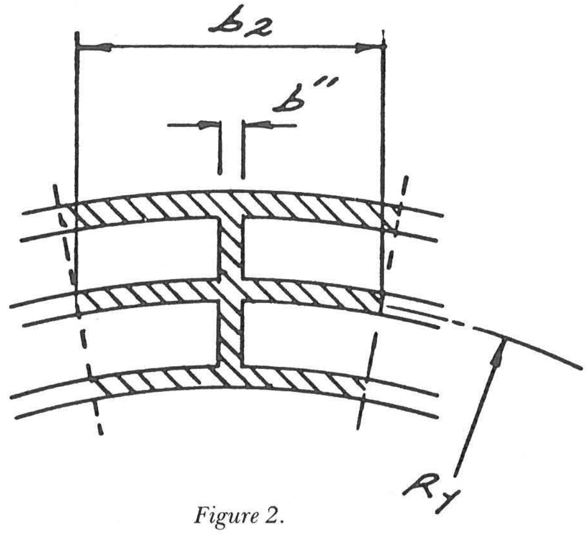

Figure 2.

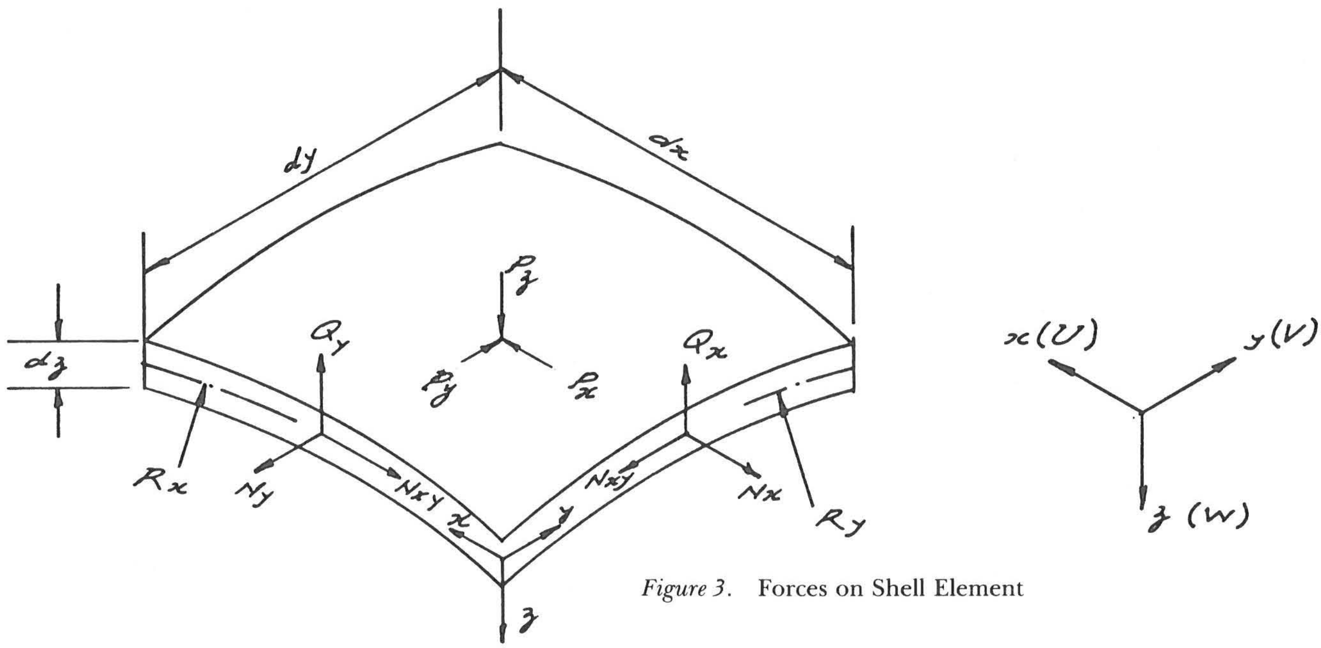

Figure 3. Forces on Shell Element

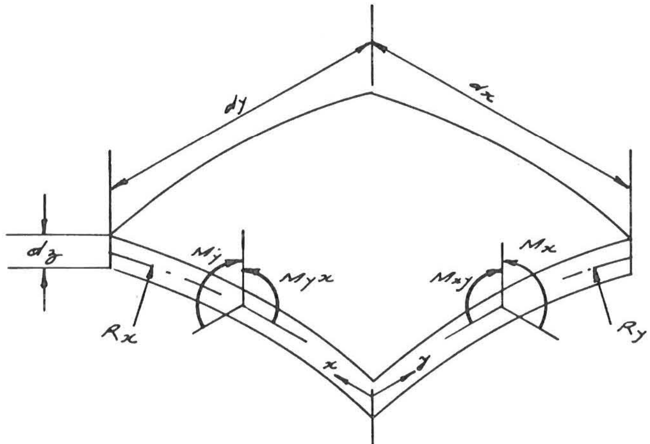

Figure 4. Moments on Shell Element

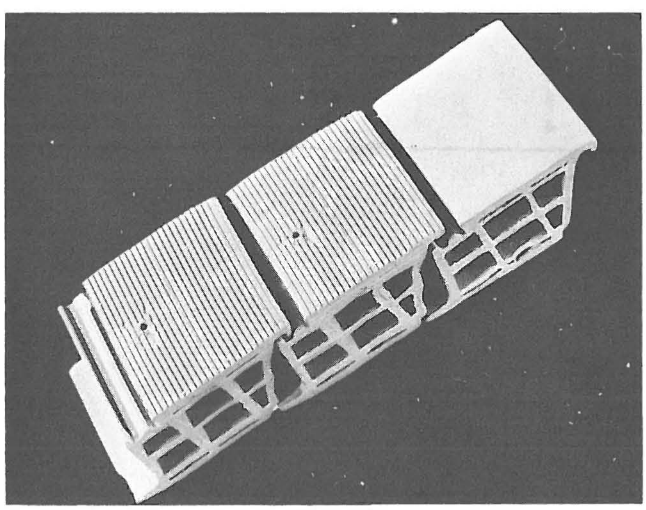

Figure 7.

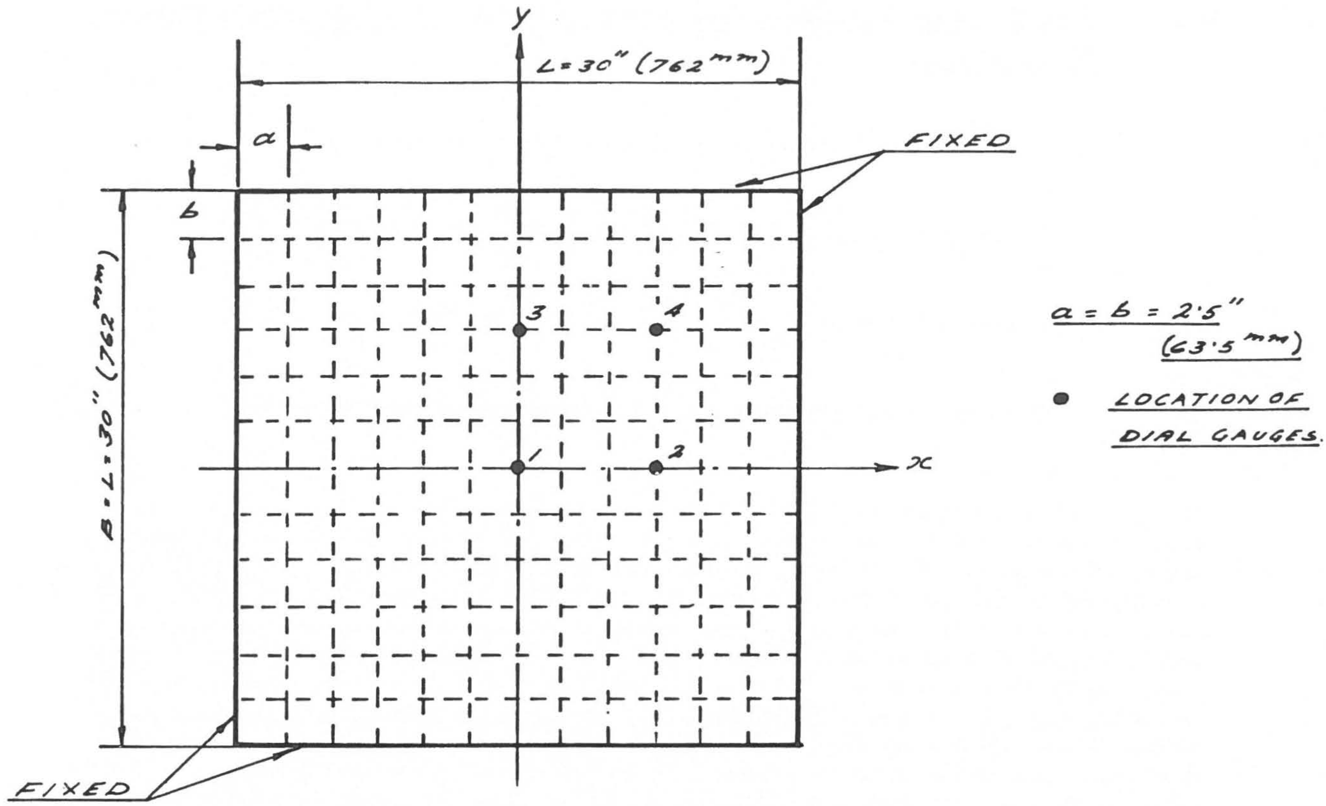

Figure 5. Plate Built of Hollow Blocks
Dimensions & Location of Dial Gauges.

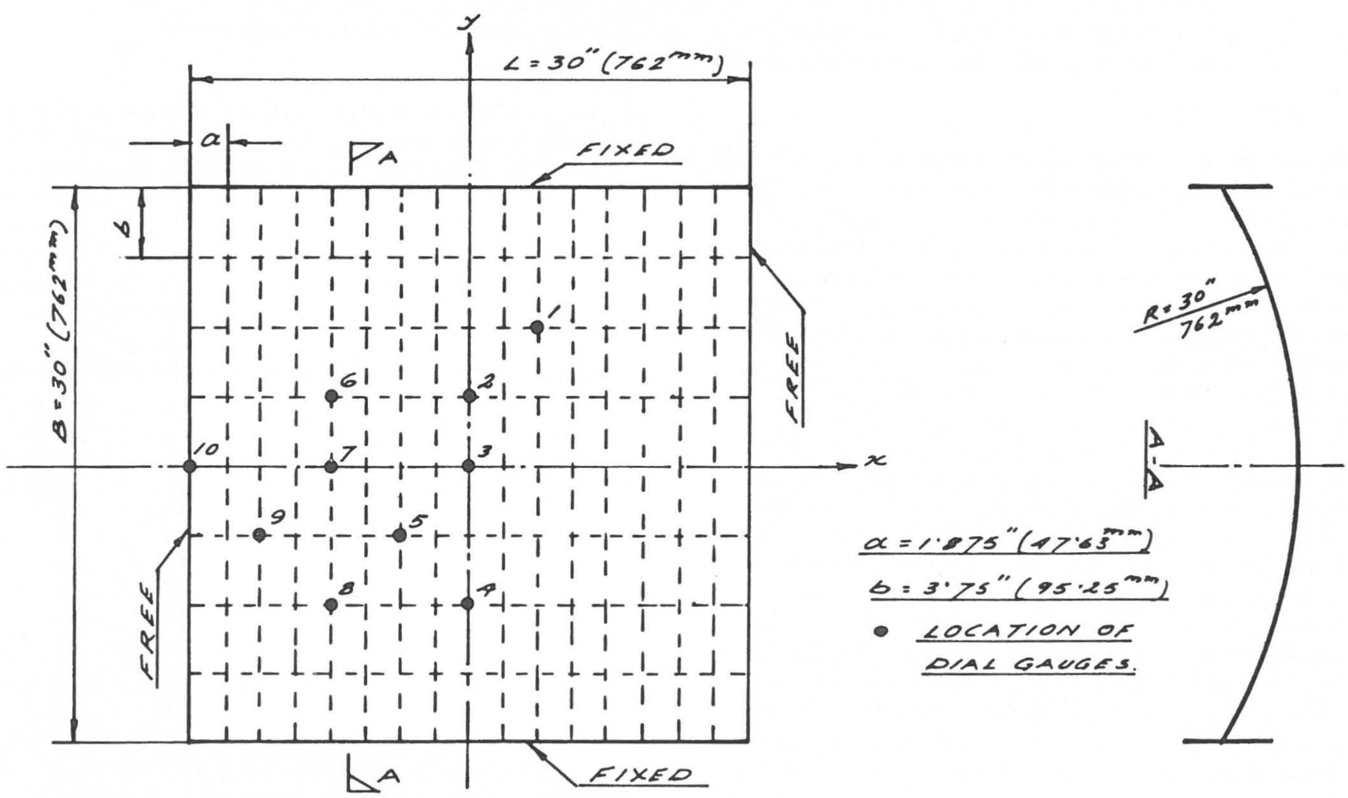

Figure 6. Cylindrical Shell Built of Hollow Blocks
Dimensions & Location of Dial Gauges.