V-15. Stress Reduction Factor Design for Unreinforced Masonry Walls

James Colville
Assoc. Dean, College of Engrg., Univ. Md., College Park, Md.
Wynn Chyn
Grad. Res. Asst., Dept. Civ. Engrg., Univ. Md., College Park, Md.

ABSTRACT

A simplified design procedure for solid load bearing brick masonry walls which considers eccentricity of wall loading and wall slenderness has recently been developed.

The purpose of this paper is to extend the simplified procedure to include hollow brick masonry wall construction. Firstly simplified wall bearing capacity equations and moment rotation relations which reflect the effect of wall cracking are developed. Following this, a procedure for estimating the eccentricity of axial load in multistory structures is presented. Finally relations relating wall load eccentricity, wall slenderness and appropriate design stress reduction factors are given.

In order to provide a check on the validity of the simplified design procedure, a comparison of results with values predicted using a modified version of a finite element model originally developed for consideration of slenderness effects in concrete columns is included. The numerical procedure considers the effects of tension cracking, and secondary bending on the displacements and moments in eccentrically loaded walls.

The simplified design procedure is illustrated with a design example.

INTRODUCTION

A stress reduction factor method of design of masonry walls has recently been proposed[1] based on a simplified theoretical analysis of the response of the wall to combined axial compressive load and weak axis bending. Details of the development of generalized stress failure equations and moment-rotation equations necessary for computation of appropriate stress factors are given in Refs. (1, 2, 3).

Although there is some evidence to suggest that the proposed method yields reasonable values for the stress reduction factors (1, 2, 4), the procedure is valid only for solid walls.

The purpose of this paper is as follows:

(1) To provide additional information related to the validity of the stress reduction factors previously presented in Refs. (1, 2, 3, 4) and
(2) To provide information on stress reduction factors for hollow walls.

NUMERICAL METHOD OF ANALYSIS

In a paper published in 1975, (5) a numerical procedure capable of considering the following complexities in the behavior of slender, reinforced beam-column members was presented:

(a) low tensile strength of the beam-column materials;
(b) post-elastic behavior of the longitudinal reinforcing;
(c) nonlinear material properties in compression; and
(d) the effects of axial load on the member bending stiffness

Details of the finite element procedure, which considers the effects of both material and geometric nonlinearities, are given in Ref (5) and are based on research carried out by Abbasi (6, 7).

The aforementioned procedure has been extended herein to include consideration of hollow members composed of two equal wythes of brickwork. It is assumed that the two brick wythes are tied together such that they may be assumed to act as a composite section. Although the procedure is also capable of considering both the effects of a nonlinear constitutive relationship in compression, and the influence of longitudinal reinforcement, these latter complexities have been suppressed and results presented herein are for unreinforced walls with an assumed linearly elastic compressive stress-strain relationship.

Failure of a beam-column is assumed to occure when the maximum compression stress reaches 1.5 times the nominal axial compressive strength of the wall; σ_{br}. This criterion is based on information presented in Refs (8, 9). The stress reduction factor, ν, used in this paper equals the wall failure load divided by the axial compression failure load, thus

$$\nu = \frac{P_{failure}}{\sigma_{br} A_{wall}}$$

NUMERICAL RESULTS

Solid Walls

Using the computer program a number of solid walls were analyzed. The major variables considered were the wall slenderness ratio, and eccentricity of the axial loading. Certain data required in the computer analysis were maintained at constant values.

A summary of this information is given below:

Modulus of elasticity of masonry = 1333.3 ksi
Axial compressive strength of masonry = 2 ksi
Number of elements = 10

A comparison of theoretical values of ν for walls in single curvature obtained following procedures outlined in Ref (2) with values obtained using the numerical procedure is given in Table 1. From these results, it is evident

that the theoretical results give good agreement with the program. Although this is itself does not confirm the correctness of the theoretical values, since both theory and program use the same failure criterion, it does support the observation that the simplified theoretical procedure compares well with a more rigorous analysis procedure.

As a further check on the simplified theory, a few cases of walls in double curvature with equal but opposite end eccentricities have been studied. The results are given in Table 2. Again the agreement between theory and program is excellent although the following comments should be noted. First of all, for those walls with slenderness ratios of 20 or less, the stress reduction factors are constant. This is due to the fact that the section of maximum eccentricity is at the wall ends. Thus failure occurs at the wall end and the wall slenderness does not reduce the failure load. In the recommended simplified theoretical approach for computing v, it is recognized that stress conditions at the wall ends are more complex that simple theory would predict and a correction is applied to the evaluation of v. For the wall with a slenderness ratio of 40, failure occurs away from the wall end and in this case the theoretical value is only 1% less than the value obtained from the program.

Hollow walls

Considerable difficulty results in developing simplified equations for v for cracked hollow walls in which the cracking is limited to partial cracking of one wythe. As a result theoretical values of v for hollow walls have been developed only for the following conditions:

(1) both wythes uncracked
(2) one wythe fully or completely cracked

In this paper, stress reduction factors for unreinforced hollow walls bent in single curvature, computed using the aforementioned numerical procedure are presented. The width of the hollow core is expressed as a fraction, k, of the wall thickness, t. Thus the core width = kt. Numerical values of v for walls with k = 0.225, and k = 0.5 are given in Table 3.

An examination of the results indicates the following:

(1) The stress reduction factors increase in value as the core size is increased. Note, however, that these stress reduction factors are based on the actual areas of the walls. Thus, for example, the wall area with k = 0.5 is one half of the wall area of a solid wall.
(2) The variation in v is very nearly linearly related to the core width, k. This is illustrated in Fig. 1 which shows the relationship between v and k for several end eccentricities for a wall with a slenderness ratio of 5 bent in single curvature.

Based on these observations it would appear possible to estimate the stress reduction factor, v, for any symmetrical composite hollow wall (i.e. equal wythes of masonry) in single curvature by linear interpolation of the values given in Table 3.

CONCLUSIONS

The following conclusions and observations are based on the information presented herein.

(1) the simplified stress reduction factor design method presented in Refs. (1,2,3) gives reasonable agreement with numerical results for solid walls bent in single curvature.
(2) stress reduction factors for hollow walls tend to be greater than those for corresponding solid walls.
(3) approximate values for stress reduction factors for hollow walls in single curvature may be obtained by linear interpolation of Table 3.

Work is continuing towards developing stress reduction factor values for hollow walls in double curvature and reinforced masonry walls using the numerical analysis procedure referenced herein.

REFERENCES

1. Colville, James, "Stress Reduction Design Factors for Masonry Walls," to be published in the Structures Division Journal, ASCE, October 1979.
2. Colville, James, "Analysis and Design of Brick Masonry Walls," The University of Edinburgh, Scotland, June 1977.
3. Colville, James, "Simplified Design of Load Bearing Brick Masonry Walls," British Ceramic Society, Proceedings 27, Load-Bearing Brickwork (6), December, 1978.
4. Colville, J. and Lears, M. "A Comparison of Masonry Design Parameters," Proceedings, Fifth International Brick Masonry Conference, Washington, D.C., October, 1979.
5. Colville, James, "Slenderness Effects in Reinforced Concrete Square Columns," Publication SP50–, Reinforced Concrete Columns, American Concrete Institute, 1975.
6. Abbasi, Jamil, "Reinforced Concrete—A Finite Element Formulation," thesis presented to the University of Maryland, College Park, Maryland, in December, 1972, in partial fulfillment of the requirements of the degree of Doctor of Philosophy.
7. Colville, James, and Abbasi, Jamil, "Plane Stress Reinforced Concrete Finite Elements," Journal of the Structural Division, ASCE, Vol. 100, No. ST5, Proc. Paper 10554, May 1974, pp 1067–1083.
8. Yokel, F.Y., and Dikkers, R.D. "Strength of Load-Bearing Masonry Walls, Journal of the Structural Division ASCE, Vol. 97, No. ST 5, Proc. Paper 8143, May 1971, pp. 1593–1609.
9. Yokel, F.Y., and Dikkers, R.D., Closure to "Strength of Load-Bearing Masonry Walls," Journal of the Structural Division, ASCE, Vol. 99, No. ST5, Proc. Paper 9693, May, 1973, pp 948–950.

TABLE 1—Comparison of Stress Reduction Factors-Walls in Single Curvature

Wall Slenderness Ratio H/t	End eccentricity Wall thickness (ϵ = e/t)	Stress Reduction Factor, ν Theory	Stress Reduction Factor, ν Program	Difference in percent
5	.10	.923	.921	+0.22
5	.15	.775	.774	+0.13
5	.20	.660	.656	+0.61
5	.25	.545	.546	−0.18
10	.10	.855	.865	−1.16
10	.15	.735	.728	+0.96
10	.20	.612	.600	+2.00
10	.25	.483	.483	0
15	.10	.790	.773	+2.20
15	.15	.656	.654	+0.31
15	.20	.526	.483	+8.90
15	.25	.304	.306	−0.65
20	.10	.670	.642	+4.36
20	.15	.427	.452	−5.53
20	.20	.296	.290	+2.07
20	.25	.180	.175	+2.86

TABLE 2—Comparison of Stress Reduction Factors-Walls in Double Curvature

Wall Slenderness Ratio H/t	ϵ=e/t	ν theory	ν program	Difference in percent
10	.2	.675	.699	−3.43
15	.2	.675	.696	−3.02
20	.2	.675	.694	−2.74
40	.2	.591	.597	−1.00

TABLE 3—Comparison of Stress Reduction Factors (numerical results)-Walls in Single Curvature

Wall Slenderness Ratio	End eccentricity Wall Thickness (ϵ= e/t)	v Solid	v hollow k=.22 5	v hollow k=.5
5	.10	.921	1.00	1.00
5	.15	.774	.865	.976
5	.20	.656	.762	.878
5	.25	.546	.670	.796
10	.10	.865	.961	1.00
10	.15	.728	.825	.950
10	.20	.600	.722	.850
10	.25	.483	.628	.770
15	.10	.773	.887	1.00
15	.15	.654	.761	.900
15	.20	.483	.656	.800
15	.25	.308	.527	.728
20	.10	.642	.775	.972
20	.15	.452	.657	.820
20	.20	.290	.494	.730
20	.25	.180	.293	.646

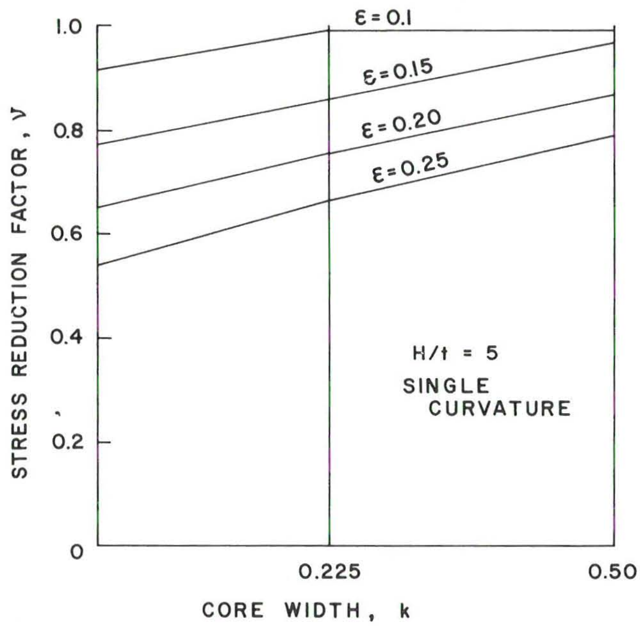

Figure 1. Stress Reduction Factor vs. Core Width