V–16. An Empirical Formula for the Estimation of Lateral Strength

H.W.H. West and D.G. Beech

ABSTRACT

The British Ceramic Research Association has, for a number of years, been engaged upon an investigation of the performance of storey-height half-brick thick walls; the results have been previously reported at various conferences. The work has latterly been extended to cover walls up to 5.2 m high and in various thicknesses, both bonded and unbonded, up to 340 mm thick. The results have been subjected to multiple regression analysis and a relationship established.

La British Ceramic Research Association a été, depuis un certain nombre d'années, engagée dans l'étude de la résistance aux charges latérales des murs de hauteur d'étage et d'épaisseur en briqueton; les résultats ont été préalablement rapportés dans plusieurs conférences. Le travail a été récemment étendu pour couvrir les murs de jusqu'à 5.2 m de hauteur et d'épaisseurs diverses de jusqu'à 340 mm d'épaisseur. Les résultats ont été soumis à l'analyse de régression multiple et un rapport a été établi.

Seit etlichen Jahren hat sich die Britische Keramische Forschungsgesellschaft mit dem Seitenlast-Aufnahmevermögen stockwerkhoher halb-Ziegel-dicker Mauern befaßt. Über die Ergebnisse wurde schon mehrmals anläßlich verschiedener Tagungen berichtet. In letzter Zeit hat sich das Forschungsprogram auf 5,2m.-hohe Mauern mit verschiedenen Dicken bis 340mm ausgedehnt. Die Ergebnisse sind einer Mehrfach-Regressions-Analyse unterzogen worden, wobei eine Beziehung (bzw. Gsetzmäßigkeit) festgestellt worden ist.

La British Ceramic Research Association si ha occupato da molti anni di una indagine sulla resistenza ai carici laterali dei muri ad un piano aventi un spessore d'un mezzo mattone; i resultati sono stati communicati anteriormente ai parecchi conferenze. Di recente il lavoro e stato estenduto per includere muri fino alla altezza di 5.2 m e di parecchi spessori fina alla spessore di 340 mm. I resultati sono stati sogetti alla analisi di regressione multipla e una relazione e stata confermata.

INTRODUCTION

The very considerable programme of wallette testing[1] has enabled the effect of the properties of bricks to be determined and specifically it has been possible to establish with some statistical rigour that the flexural strength of the wallettes in the two orthogonal directions is related to the water absorption of the bricks and the grade of mortar. Sufficient results were available for the characteristic strengths of clay bricks to be established for three ranges of water absorption <7%, 7–12% and 12–30%, and these values of f_{kx} have been written into the new Code of Practice BS 5628: Part 1: 1978[2].

Haseltine[3] has been able to devise a design method which, although less than perfect, provides more guidance than was hitherto available. This was a modified yield line method of calculation and relies upon the Code values of f_{kx} though provision is made for the value determined from wallette tests on the particular brick and mortar to be used if this is, as is usually the case, more advantageous. The method of calculation is, however, perhaps more intricate than might be desirable and it must be conceded, as indeed Haseltine does, that yield line ought not to apply to masonry.

West[4] has shown that the experimental results for full-sized walls conform to an equation of the general form

$$p \propto \frac{k}{L^n}$$

where p is the lateral wall strength; L is the length of the wall; and k varies with the brick and mortar.

This was, however, based only on results for storey-height walls of one thickness (single-wythe 102.5 mm) but since that time walls up to two storeys high of lengths from 2.74 to 5.5 and in thickness up to 307.5 mm have been tested. All these walls have been tested with 3 sided support in steel frames with the upper edge free and dpc at the bottom.

The seductive attraction of a simple empirical relationship can therefore now be explored. Intuitively it might be expected that the characteristic flexural strength should enter into this and a positive disadvantage of the broad relation in Eqn. 1 is that this does not appear directly. Similarly the other geometrical parameters of height and thickness need also to be entered. Sufficient data are available to test this statistically and the results at least bear further examination.

METHOD

The results for some seventy walls were used comprising both 1:¼:3 and 1:1:6 mortar and it was decided to take the variables as f_x, flexural strength of wallettes normal to bed joint, t, thickness in mm, L, length in m, and h, height in m. An equation of the form

$$y = k\,(fx)^{n_1}\, t^{n_2}\, (L - c)^{n_3}\, h^{n_4} \qquad (2)$$

was used and fitted in logarithmic form by multiple regression. This means that the least squares technique is applied to the logarithms, so that the relative errors are minimised rather than the absolute ones, which is not unreasonable where a power law is concerned.

The constant c was introduced because in a previous investigation it had seemed better to take the reciprocal of L − 0.79 as a parameter rather than the reciprocal of L. Here two values of c were first tried: C = 0.79 and c = 0. Almost identical values of the multiple correlation coefficient (0.964) were obtained, the residual standard deviation being 0.07514 for c = 0.79 and 0.07552 for c = 0. The index for the L term and the constant k were different, but the other indices were little changed.

It seemed worth while taking additional values for c to see whether an optimum value of the correlation coefficient would be obtained; values of 0.2, 0.5 and 1.0 were taken. The correlation coefficient changed very little, the best value being 0.965 when c = 0.5. The residual standard deviations were

c	0.0	0.2	0.5	0.79	1.0
R.S.D.	0.07552	0.07513	0.07472	0.07514	0.07661

It is clear that the behaviour of the correlation is not very sensitive to variation in c. An additional advantage of c = 0.5 is that $n_3 = -1.0073$, which being close to -1 is intuitively attractive.

The equation in terms of the original variables is:

$$y = \frac{0.01359\ fx^{0.4661}\ t^{1.5310}}{(L - 0.5)^{1.0073}\ h^{0.5864}} \tag{3}$$

the indices being rounded to 4 decimal places.

The residual standard deviation of 0.07472 for the logarithms implies a 95% confidence limit for the observed values of about 29% below the calculated value and 41% above it.

The graph of observed against calculated is shown in Figure 1. The different bricks and mortars are distinguished by symbol. They seem to be reasonably well "mixed up". There is a tendency at high strengths for the calculated value to be an underestimate but this is a consequence of fitting in terms of logarithms. When logs are plotted (Fig. 2) the fit looks better. The constant could be corrected empirically to give a better fit at high values, but this would increase the tendency to overestimate low strengths. It is safer to accept a tendency to underestimate high strengths.

A plot of the residuals log (observed) − log (calculated) against L/h in Fig. 3 shows no marked trend.

Power laws are troublesome dimensionally. Here the dimensions of the size factors L, h, t roughly cancel, leaving approximately

$$y \propto fx^{1/2} \tag{4}$$

This is no more objectionable than the type of equation used for compressive strength with wall strength proportional to the square root of the brick strength and the fourth root of the mortar strength.

MODIFIED RELATIONSHIP

The precision implied by Eqn. 3 may be justified though all experience with masonry and its constituent products suggests that mean values derived from proper replicated experiments and an adequate though not excessive safety factor are more reliable guides to long term performance. With that in view it is proposed to simplify Eqn. 3 to

$$p = \frac{k\ \sqrt{fx} \cdot \sqrt{t^3}}{(L - 0.5)\ \sqrt{h}} \tag{5}$$

This Eqn. is applied to all the wall results and gives Fig. 4.

The gradient k now becomes 0.0146. The 95% confidence limits are also shown in the figure. The gradient of this line is 0.0104. Thus we can approach the single digit number so beloved of engineers by defining the safe empirical relationship as

$$p = 0.01\ \frac{f_{kx}^{0.5}\ t^{1.5}}{(L-0.5)\ h^{0.5}} \tag{6}$$

CONCLUSIONS

An empirical relationship has been established to enable the lateral load resistance of walls to be calculated from their length, height and thickness and the flexural strength for the same bricks and mortar derived from wallette tests.

In the precise form this came out as

$$p = \frac{0.01359\ f_x^{0.4661}\ t^{1.5310}}{(L-0.5)^{1.0073}\ h^{0.5864}}$$

but the preferred version is the simplified form

$$p = \frac{0.01\ f_{kx}^{0.5}\ t^{1.5}}{(L-0.5)\ h^{0.5}}$$

which has a slope less than that of the 95% confidence limit shown in Fig. 4 and thus provides a safe and simple working formula.

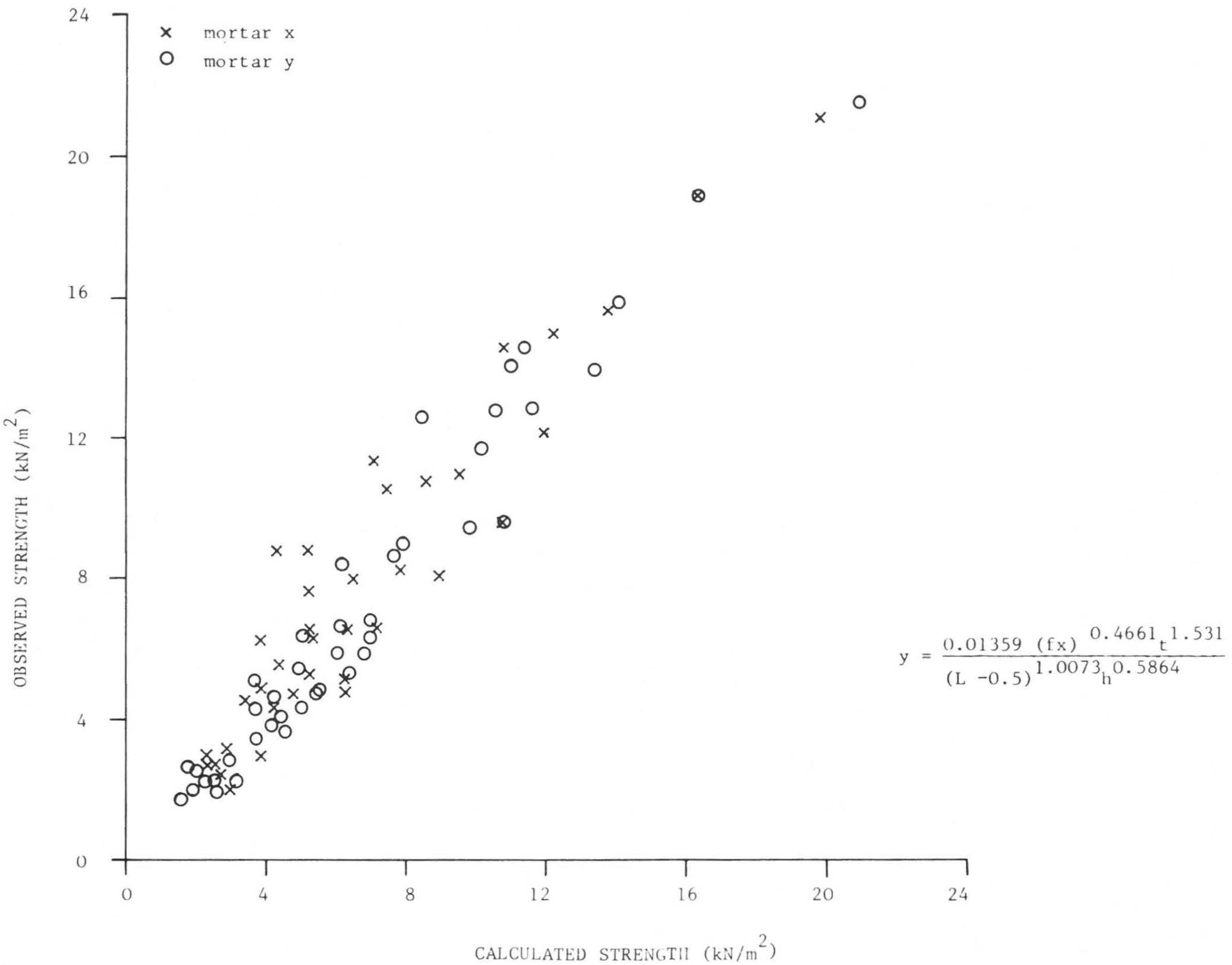
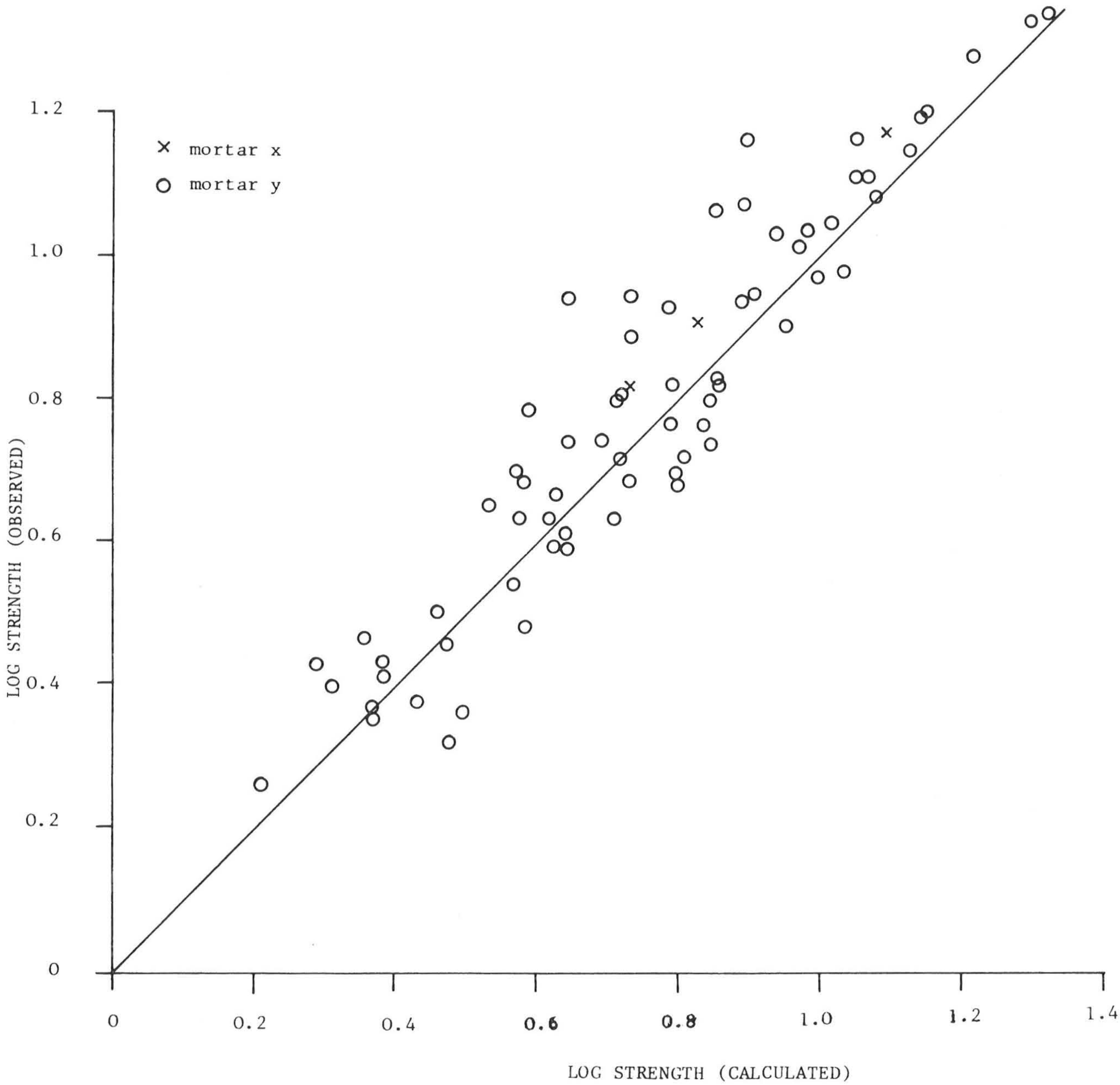


Figure 1. Calculated Strength (kN/m²)



× mortar x
○ mortar y
1.2
1.0
0.8
0.6
0.4
0.2
0
LOG STRENGTH (OBSERVED)
0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
LOG STRENGTH (CALCULATED)

Figure 2.

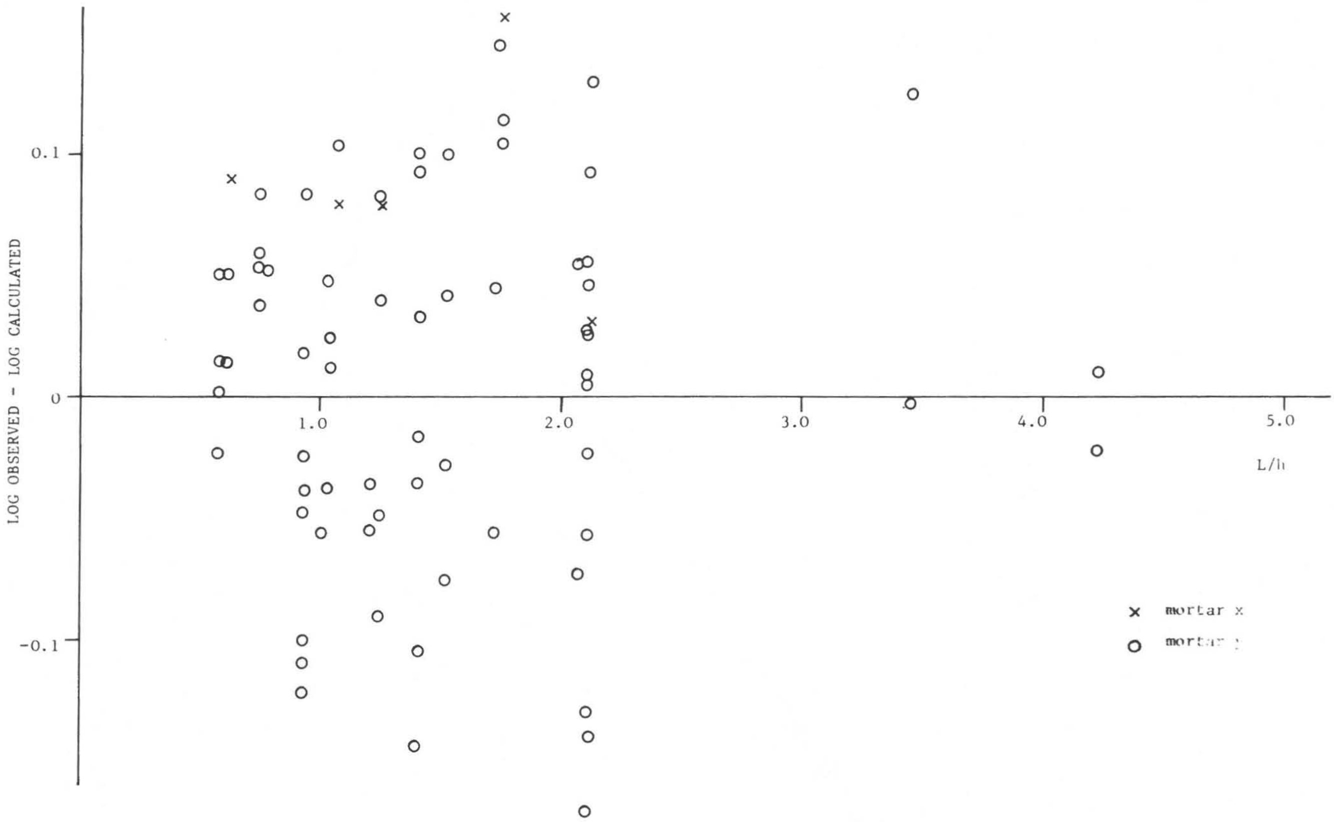

LOG OBSERVED - LOG CALCULATED
0.1
0
-0.1
1.0
2.0
3.0
4.0
5.0
L/h
mortar x
mortar

Figure 3.

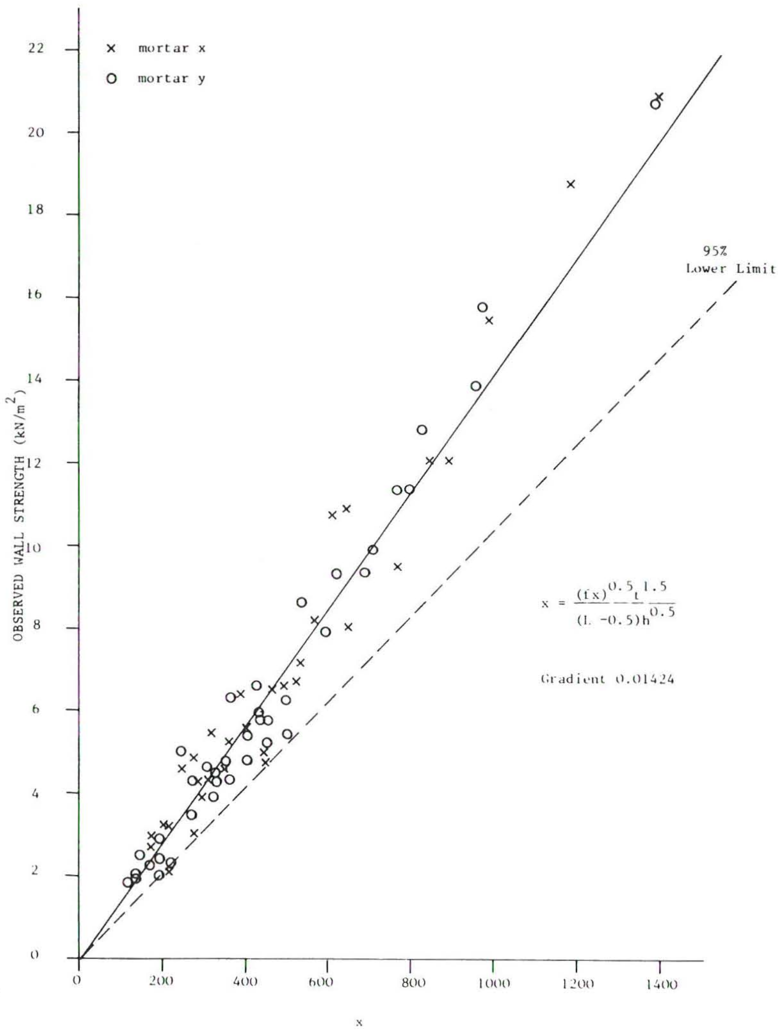


Figure 4. Observed Wall Strength (kN/m²)