II–7. A Failure Criterion for Brickwork in Bi-Axial Bending

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ABSTRACT

When brickwork walls supported on three or four sides are submitted to lateral wind pressures, they bend about both the vertical and horizontal axes. Various authorities advocate either the plastic or elastic method of analysis to assess the lateral flexural resistance of these walls. In both cases, the failure criteria adopted in the past assume that the vertical flexural strength of brickwork is unaffected by the application of a horizontal moment and vice-versa. In short, it has been assumed that there is no interaction between vertical and horizontal moments in assessing the strength of brickwork subjected to bi-axial bending. Experimental evidence in the paper indicates that this is an incorrect and unconservative assumption.

Another common assumption made in analysis is that an applied vertical compressive stress increases the effective vertical moment of resistance but not the horizontal moment of resistance. Experimental evidence in the paper indicates that an applied vertical stress significantly increases both the vertical and horizontal moments of resistance.

INTRODUCTION

Practical brickwork panels usually resist wind pressures by a combined bending in the vertical and horizontal directions. In addition, the brickwork is subjected to an axial compressive load due to its own weight. At a particular location in a brickwork panel the material is subjected to a combination of vertical moment, horizontal moment and compressive force. This combination changes from point to point in the panel. Whatever method of analysis is used to calculate these moments and forces, the assessment of the panel strength is dependent upon the criterion adopted for the failure of the material.

It may be possible to match a method of analysis and a failure criterion to closely predict panel strengths and yet neither may correspond very well with actual behaviour. While this may be an acceptable state of affairs if one confines its application within the range of cases that have been experimentally verified, it indicates a lack of full understanding of real behaviour. One would have far greater confidence in the applicability of strength predictions outside the immediate range of experimentally verified instances if the method of analysis and the failure criterion used corresponded approximately to actual behaviour. It is with this in mind that the following considerations of the failure criterion in bi-axial bending is presented.

Consideration is first given to the ratio of bending strengths when moments are applied to produce bending in the horizontal and vertical directions respectively. Next, the behaviour of brickwork subjected to various combinations of vertical and horizontal moments is presented, followed by a consideration of the effect of a compressive load.

Lawrence^ fitted a curve to experimental values obtained from several sources and found that the modulus of rupture (MPa) for horizontal bending was approximately 2.17 times the square root of the modulus of rupture for vertical bending. In later work using a consistent method to establish all points, Lawrence^ found that a better fitting curve was obtained if the above constant was reduced to 1.75.

The author has gathered values from many sources and these are shown in Fig. 1. It can be seen that Lawrence’s original empirical curve is a reasonable fit to this larger set of data. Also, it can be seen that a large scatter occurs about this curve. With such a large scatter of points it seems pointless to get a better fitting curve. It would be far more desirable to take other parameters into account in a theoretical treatment to produce better predictions.

Scatter of the individual points in Fig. 1 is partly caused by:

a) Different configurations of specimens and loads used to determine the flexural strengths. This factor has been considered previously by the author^ and can be quite significant.

b) Variability of the materials and workmanship. Ideally, each point should be established by averaging results for at least 10 specimens for each orientation of moment.

c) Different bending strengths of the bricks used. Since failure in horizontal bending often occurs by bricks breaking, it is likely that this parameter is important. This is not distinguished by the above empirical curve.

A theoretical relationship for the orthogonal strength ratio is developed below that takes account of the modulus of rupture of the brick. Consider the section of brickwork in Fig. 11 subjected to horizontal bending moments, Mv.

Suppose that, at failure, the extreme fibre stresses in the perpend joints are uniform and equal to fsc. The moment resisted by each perpend joint is therefore Msc = fsc Zsc. Similarly, if the extreme fibre stress at mid-length of the brick is fsc, then the moment resisted by the brick is:

\[ M_{sc} = f_{sc} Z_{sc} \]

ORTHOGONAL STRENGTH RATIO

Brickwork spanning in the horizontal direction has a greater flexural strength than when spanning in the vertical direction. This is because, when spanning vertically, failure occurs by tensile bond in the bed joints but when spanning horizontally, the overlapping bricks in alternate courses cause the bricks to break or the mortar bed joints to fail in torsional shear. Most codes fix this strength ratio at 2, but values ranging from 1½ to 8 have been reported.
Each bed joint also resists a moment, \( M_s \), due to torsional shear between the overlapping bricks. For moment equilibrium of the section shown in Fig. IIIb:

\[
M_m + 2M_s = M_b
\]

or

\[
M_s = \frac{1}{2} (M_b - M_m)
\]

or \( M_s = \frac{1}{2} (f_b - f_m)Z_n \) (1)

The modulus of rupture of brickwork in horizontal bending, \( F'_v \), will be the average extreme fibre stress on length \( BCD \) in Fig. IIIa. That is:

\[
F'_v = \frac{1}{2} (f_v + f_m)
\]

Let \( f_m = k F'_v \) (2)

where \( F'_v \) is the modulus of rupture of brickwork in vertical bending. The maximum value of \( k \) will be unity because tensile bond of the mortar to the end face of the brick is not likely to be greater than to the rougher bed face.

Also, workmanship is likely to be better for the bed joint than the perpend joint. The minimum value of \( k \) will be nearly zero, corresponding to the perpend joint having cracked prior to failure of the brickwork. Combining equations 2 and 3 gives:

\[
F'_v = \frac{1}{2} (f_v + k F'_v)
\]

Finally the orthogonal strength ratio \( R \) is defined as:

\[
R = \frac{F'_v}{F'_v}
\]

Substituting 4 in 5 gives:

\[
R = \frac{1}{2} \left( \frac{f_v}{F'_v} + k \right)
\]

Substituting 3 in 1 gives:

\[
f_v = \frac{2M_s}{Z_n} + k F'_v
\]

and substituting this in 6 gives:

\[
R = \frac{M_s}{F'_v} + k
\]

The orthogonal ratio \( R \) may then be calculated from equation 6 if failure is through a brick \( (f_v \) critical) or from equation 8 if failure is through the mortar joints \( (M_m \) critical). Where the bricks break, \( f_v \) is equal to the modulus of rupture of the bricks, \( F'_v \) and

\[
R = \frac{1}{2} \left( \frac{F'_v}{F'_v} + k \right)
\]

The modulus of rupture of the brick should be measured by applying end moments to a single brick or by testing a beam of bricks glued end to end. Testing a single brick as a centrally loaded beam over a short span will yield values that are too high.

The value of \( k \) may be approximated as follows:

If the modulus of rupture of mortar and brick are the same \( \left( i.e. \frac{F'_v}{F'_v} = 1 \right) \) then there will be no premature cracking in the perpend joint \( (k = 1) \) and from equation 7, \( R = 1 \). At the other extreme, for large values of \( R \) where the horizontal bending strength is much greater than the vertical flexural strength, it is likely that the perpend will have cracked before ultimate load and hence \( k = 0 \). It is considered reasonable to assume that at \( R = 1, k = 1 \) and at \( R > 5, k = 0 \) with \( k \) varying proportionally between these limits. These assumptions lead to the following formulae for calculating the orthogonal strength ratio when failure occurs through the bricks:

For \( R < 5 \)

\[
R = \frac{1}{9} \left( \frac{4 F'_v}{F'_v} + 5 \right)
\]

For \( R > 5 \)

\[
R = \frac{F'_v}{Z_n F'_v}
\]

A family of these curves is plotted for various values of \( F'_v \) in Fig. II. It can be seen that the average of experimental results (represented by Lawrence’s empirical curve \( R = 2.17 \sqrt{F'_v} \)) lie to the left of the family of curves at high values of \( R \) indicating that brick strength is not critical in this region. For lower values of \( R \), the empirical curve enters the family of theoretical curves indicating generally that brick strength is critical there.

As mentioned previously, equation 8 holds when the brickwork fails through the mortar joints. The present state of knowledge does not allow the evaluation of \( M_m \) in terms of the stress \( F'_v \). An empirical curve, however, has been drawn through some of the experimental points established by Satti and Hendry and the author where failure was known to be through the mortar joints. This is shown in Fig. II.

In summary, the full lines in Fig. II represent, in the author’s opinion, an improved means of estimating the orthogonal strength ratio from measurements of the moduli of rupture of the brick and of joints in vertical flexure. For the practical range of brickwork \( (F'_v \geq 0.3 \text{ MPa}) \) the orthogonal strength ratio, \( R \) may be taken as the lesser of:

\[
R = \frac{1}{9} \left( \frac{4 F'_v}{F'_v} + 5 \right) \quad \text{and} \quad R = \frac{2.75}{\sqrt{F'_v}}
\]

**INTERACTION OF VERTICAL AND HORIZONTAL MOMENTS**

The analysis of the previous section is applicable to one-way bending, that is, when the brickwork is subjected to either a vertical or horizontal moment. This section deals with the case of two-way bending, that is, when the brickwork is subjected to both vertical and horizontal bending moments simultaneously. Most codes of practice specify maximum permissible bending stresses without distinction between one-way or two-way bending. In other words, they imply that the horizontal and vertical bending strengths respectively are unaffected by the application of a moment in the other direction. It was suggested by the author that this may be an unconservative approach and experimental work now confirms this.

Because it is very difficult to apply simultaneous vertical and horizontal moments to specimens of brickwork such as that shown in Fig. IIIa, tests have been carried out on “single joint” specimens, Fig. IV. This “joint” approximately
reproduces the behaviour of brickwork when subjected to both vertical and horizontal moments. Clearly the failure mode when subjected to vertical moment alone is the same as in a wall and the modulus of rupture, $F'$, is readily calculated. Consider the "joint" subjected to horizontal moment, $M_h$, alone. Provided one assumes that the moment transmitted through the perpend joint is the same as that transmitted in Fig. IIIa, the stress conditions on the section ABCD in Fig. IVb correspond to those in the wall. Since the "joint" approximates the behaviour of a brickwork wall in both vertical and horizontal bending, it is assumed that it will do so when subjected to combined moments.

A number of joints were tested under various combinations of vertical and horizontal moments. The lower brick of each specimen was clamped to a rigid frame and the moments $M_v$ and $M_h$, applied via clamps on each of the other three bricks. These moments were applied by hydraulic jacks acting on levers. The lengths of the lever arms determined the ratio of the applied moments. Outputs from load cells on each lever gave continuous plots on an X-Y recorder, allowing vertical and horizontal moments at failure to be read. So that the loading system did not impose unwanted constraints on the specimens, each moment was applied through a shaft having a universal joint at each end.

Two series of tests were performed:

Series I —used bricks 150 mm long $\times$ 50 mm high $\times$ 67.5 mm wide with mortar joints 6 mm thick.

Series II—used bricks 225 mm long $\times$ 75 mm high $\times$ 70 mm wide with mortar joints 9.5 mm thick.

In both series, mortar consisted of Portland cement, hydrated lime and sand in the proportions 1:1:6 by volume. Average flow of the mortar was 85%. Results are summarised in the first four columns of Table 1. The extreme fibre stress due to vertical moment, $M_v$, was calculated from

$$ F_v = \frac{M_v}{Z_v} $$

The extreme fibre stress due to horizontal moment, $M_h$, was calculated from equation 4 substituting that

$$ f_h = \frac{M_h}{Z_h} $$

That is

$$ F_h = \frac{M_h}{Z_h} + \frac{k}{2} F'_v $$

Conservatively, $k$ was taken as zero. It can be seen from Fig. V that the failure criterion for combined vertical and horizontal moments is given approximately by the elliptical expression:

$$ \left( \frac{F_v}{F'_v} \right)^2 + \left( \frac{F_h}{F'_h} \right)^2 = 1 $$

The usual assumption of no interaction between these stresses can be seen in Fig. V to be unconservative.

**EFFECT OF AXIAL STRESS**

When an axial compressive stress is applied to a vertically spanning brick wall it may suppress flexural tensile stresses from developing when wind pressure acts. In this case the bond strength between brick and mortar is unimportant. The most critical cases are those panels that have no applied compressive force. These have a compressive stress due to self weight only of about 0.02 MPa per metre below the top of the wall.

The effect of a compressive stress of 0.10 MPa on the failure criterion of the joint specimens in Series I was investigated. Compressive stress was applied by a weight on a hanger bearing on the centre of a steel plate bedded on the top brick. Results are shown in the last four columns of Table 1 and also in Fig. V.

It can be seen that the applied compressive stress increased both the vertical and horizontal bending strength and that the combined action point was on the elliptical failure criterion. The increase in modulus of rupture in vertical bending was greater than the applied compressive stress of 0.1 MPa. This suggests that the neutral axis for brickwork in bending is between the centre of the wall and the compression face.

Interestingly, the increase in horizontal modulus of rupture was such that the orthogonal strength ratio $R$ was the same with or without the applied compressive stress. In this instance, the usual failure criterion assumed for brickwork subjected to vertical compression underestimates that actually measured.

Where failure occurs in the mortar joints, one could expect an increase in horizontal strength because the increased shear strength of the mortar due to the applied compressive stress leads to a greater moment of resistance, $M_r$, of the bed joint. Nearly all the tests reported here were through the mortar joints. Where horizontal bending strength is determined by brick strength, no increase could be expected since a small compressive stress would probably have negligible effect on the bending strength of the brick. Guidance as to whether brick or mortar strength is likely to control for a particular case can be found in Fig. II.

**CONCLUSIONS**

A method of estimating the orthogonal strength ratio for brickwork from the moduli of rupture of the brick and of the joint in vertical flexure has been presented. The value is determined by a theoretical equation where failure in horizontal bending is through the bricks or by an empirical equation when failure is through the mortar joints.

Experimental evidence presented indicates that for brickwork in bi-axial bending, the interaction diagram between the failure stresses in the vertical and horizontal directions is approximately elliptical.

For the specimens tested with an applied vertical compressive stress, the failure criterion in bi-axial bending remained elliptical with both vertical and horizontal moduli of rupture increasing by the same percentage. This result is unlikely to apply to cases where the horizontal bending strength is determined by brick strength rather than mortar strength.

**ACKNOWLEDGMENT**

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REFERENCES


NOTATION

- $M_h$, $M_v$: Applied horizontal and vertical moments
- $f_{h}$, $f_{v}$: Extreme fibre horizontal and vertical stresses
- $F_h$, $F_v$: Ultimate extreme fibre stresses in bi-axial bending
- $F'_h$, $F'_v$: Moduli of rupture of brickwork in horizontal and vertical flexure, respectively
- $Z_h$, $Z_v$: Section Moduli $\frac{ht^2}{6}$, $\frac{ht^2}{6}$
- $b$, $h$, $t$: Length, height and thickness of brick
- $R$: Orthogonal strength ratio $\frac{F_h}{F_v}$
- $M_{b}$, $M_{v}$, $M_{r}$: Moments resisted by brick, perpend joint and bed joint in horizontal bending
- $f_{h}$, $f_{v}$: Extreme fibre stresses in brick and perpend joint in horizontal bending
- $F'_b$: Modulus of rupture of brick
Figure 1. Experimental values

\[ R = 2.17 \frac{F}{{F_v}} \]
Modulus of Rupture of Brick is not critical where lines are dotted

Empirical curve indicates where mortar strength controls in horizontal flexure

Family of theoretical curves for various moduli of rupture of brick. Full lines indicate where this quantity is critical in horizontal flexure.

\[ R = \frac{2 \cdot 17}{F'_{v}} \]

Approximate average experimental results

**Figure II.** Theoretical curves

**Figure III.** Wallette
### TABLE 1—Experimental Results

<table>
<thead>
<tr>
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<th>Axial Stress = Zero</th>
<th>Axial Stress = .10 MPa</th>
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<tbody>
<tr>
<td></td>
<td>$M_1$ Nm</td>
<td>$M_h$ Nm</td>
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<tr>
<td>Mean</td>
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**SERIES I**

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<td>C.V.</td>
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<tr>
<td>Mean</td>
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<td>C.V.</td>
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<tr>
<td>Mean</td>
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<tr>
<td>C.V.</td>
<td>.26</td>
</tr>
<tr>
<td>Mean</td>
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</tr>
<tr>
<td>C.V.</td>
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<tr>
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<td>C.V.</td>
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**SERIES II**

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</tr>
<tr>
<td>C.V.</td>
<td>.20</td>
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<td>138</td>
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<td>C.V.</td>
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<td>Mean</td>
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</tr>
<tr>
<td>C.V.</td>
<td>0</td>
</tr>
</tbody>
</table>

Each mean value was determined from 20 results for Series I and 30 results for Series II.

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**Figure IV.** Single joint

![Diagram of single joint](image)

$M_h$ acting only
Failure criteria in bi-axial bending

Figure V.