Un metodo per l'analisi di edifici multipiano in muratura antisismici
(A method for the analysis of antiseismic masonry multistory buildings)

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Sommario - La progettazione delle strutture murarie soggette ad azioni sismiche richiede la disponibilità di un efficiente strumento per la valutazione delle sollecitazioni. Fino ad oggi tale strumento è stato identificato, almeno in Italia e Jugoslavia, nel metodo di analisi denominato POR e nel relativo programma di calcolo. Il POR d'altronde presenta numerosi punti deboli che possono condurre a valutazioni della sicurezza a collasso della struttura muraria errate per eccesso. Nell'articolo viene presentato un procedimento ed il relativo programma di calcolo PORFLX che elimina i principali difetti del POR. I risultati ottenuti con il PORFLX evidenziano il carattere fortemente non conservativo di quelli del POR.

Summary - Designing masonry structures subjected to seismic actions requires an efficient tool to evaluate stresses. Till today such tool has been identified, at least in Italy and Yugoslavia, in the method of analysis named POR and the relevant computer program. On the other hand POR has many weak points which can lead to overestimations of the safety against collapse of the masonry structure. Herein a procedure of analysis and the relevant computer program PORFLX are presented, which eliminate the main faults of POR. The results obtained through PORFLX show clearly the strongly non conservative character of the POR method.

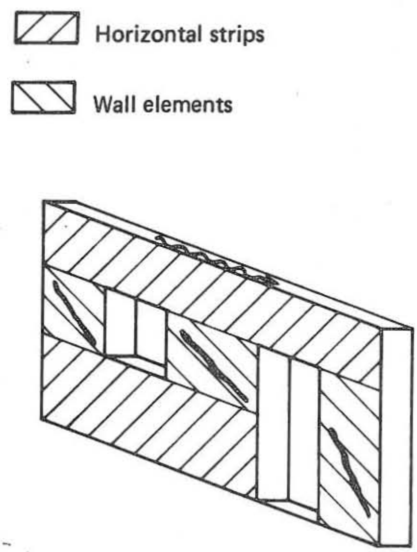
1. INTRODUCTION

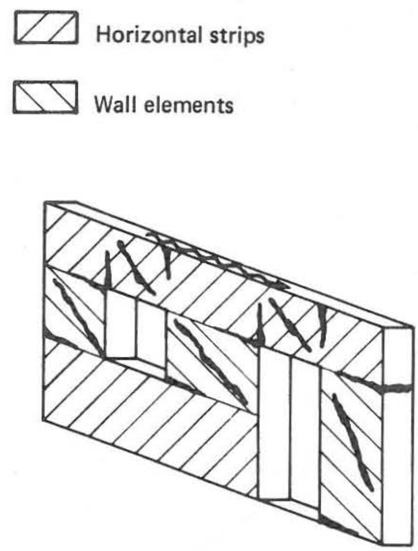
The violent earthquakes which recently occurred in Europe, particularly in Yugoslavia and in Italy, have emphasized the existence of a great problem, which is social and economical as well as one of seismic engineering; this is the problem of the antiseismic strengthening of existing buildings, in particular of "poor" masonry buildings.

It is indeed sufficient to have a look at the damage which has been surveyed in recent earthquakes [1] to see that the most damaged constructive typologies are the "old" masonry constructions, i.e. the buildings with stone masonry walls and wooden or steel horizontal structures (floors and roofs).

Besides being the most vulnerable, these typologies are often the most numerous, especially in the oldest and most historically significant settlements. For this reason a large number of research workers have recently given their attention to masonry buildings, and, as a consequence, some procedures of analysis particularly oriented to the problem have been developed.

Besides the most complicated procedures based upon the finite element method [2,3], which are too expensive to be used in analyses of entire buildings, there are approximate methods, some of which





1) Seismic action parallel or orthogonal to the walls	1) Seismic action parallel or orthogonal to the walls
2) Slabs infinitely rigid in their plan	2) Slabs infinitely rigid in their plan
3) Zero displacement of the floor's centroid orthogonal to the seismic action	3) Zero displacement of the floor's centroid orthogonal to the seismic action.
4) Pierced wall idealised as an assemblage of strips and wall elements.	4) Pierced wall idealised as an assemblage of strips and wall elements.
5) Strips and wall elements idealised as one-dimensional elements.	5) Strips and wall elements idealised as one-dimensional elements.
6) Strips infinitely stiff and infinitely resistant.	6) Strips infinitely stiff but not infinitely resistant in shear and flexure.
7) Elastic - plastic wall elements with shear failure only.	7) Elastic - plastic wall elements with tension, compression and shear failure.
8) Axial force in the wall elements kept always constant.	8) Axial force in the wall elements varying with the horizontal load.
a) POR	b) Proposed method (PORFLX)

Fig. 1 - Assumptions of POR method and of the proposed method.

(Vet, POR) have been extensively used for repair and strengthening work after the 1976 Friuli earthquake [4]. These assume an elastic - perfectly plastic constitutive law for the material, with unlimited (Vet) or limited (POR) plastic deformations, a static idealization for the action, a sheartype behaviour for the structure. Under these hypotheses, they evaluate the safety against collapse.

The procedure of analysis to be presented herein follows such a line of operation and is in fact proposed as an improvement on the POR method. Such a choice aims at providing an operational tool which is more accurate than the POR method but, nevertheless, has the same simplicity of use and capability of making inexpensive analyses of even large — both in plan and in height — buildings.

2. FORMULATION OF THE METHOD

Before presenting the proposed method, it seems opportune to briefly recall the hypotheses on which the POR method is based, in order to make evident the weak points that the proposed method aims at removing. These hypotheses are synthetically shown in fig. 1a.

The assumption of infinite stiffness and strength of the horizontal strips determines the shear type behaviour of the wall elements, so that each floor can be analysed separately. Every wall element gives a contribution to the overall horizontal force which is a function of its stiffness, of its distance from the centre of stiffness and of the distance between the centroid and the centre of stiffness of the floor. The ultimate horizontal load is calculated by a step - by - step procedure which, at every step, increases the displacement of the centre of stiffness and updates the stiffness of the wall elements which are yielding or collapsing and, consequently, the position of the centre of stiffness.

Among the listed assumptions, the most questionable seems to be that of the infinite strength of the strips (6) and that of only one kind of failure for the wall elements (7). Indeed, the strength of the strips is often insufficient (see figs. 2a, 2b) especially because of the thinning of the wall over and under the openings of the windows, due to the presence of blind boxes, radiators or to particular constructive arrangements.

As for the hypothesis of taking into consideration only shear failures for wall elements, even if it is less rough than it might appear at first sight, it is however questionable, especially in the case of slender wall elements; several examples of failures in flexure, indeed, have been observed after strong earthquakes, sometimes accompanied by shear cracks (see figs. 3a, 3b).

Finally it must be observed that the POR method neglects the changes of axial forces in wall elements, which are related to the shear - type idealization and which cause changes of shear strength (see eq. (1)).

The proposed method of analysis is based upon the hypotheses listed in fig. 1b. As can be see, only hipotheses 6, 7, 8, differ from the corresponding ones of POR. Keeping hypotheses 1 to 5, even though some of them (3,4) are questionable, is justified by the

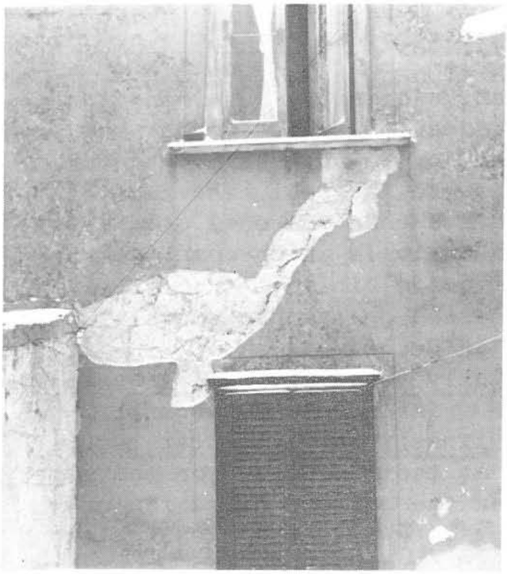


Fig. 2a - Failure of strip in shear.

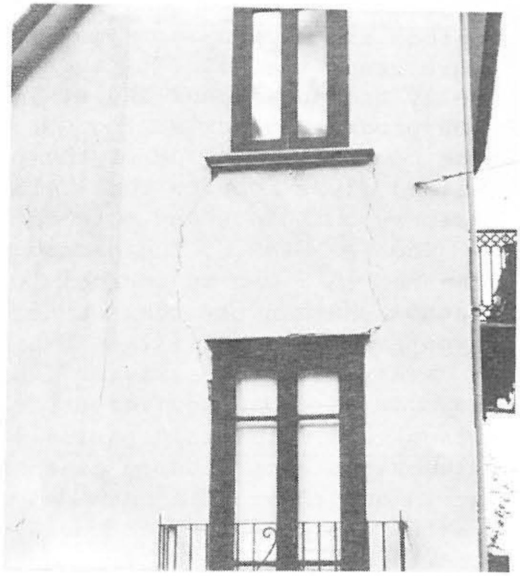


Fig. 2b -Failure of strip in flexure.

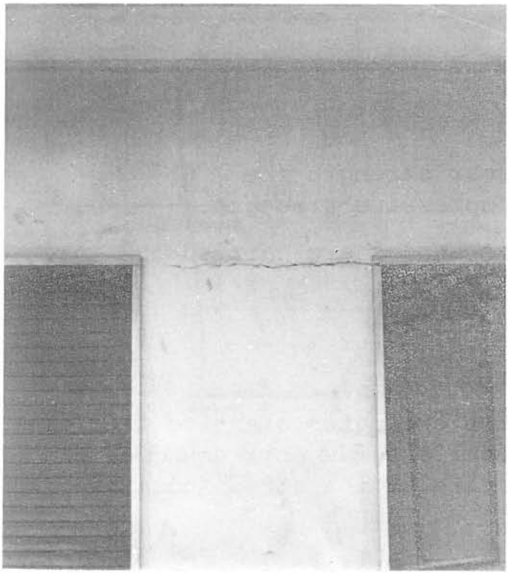


Fig. 3a -Failure of wall element in flexure.

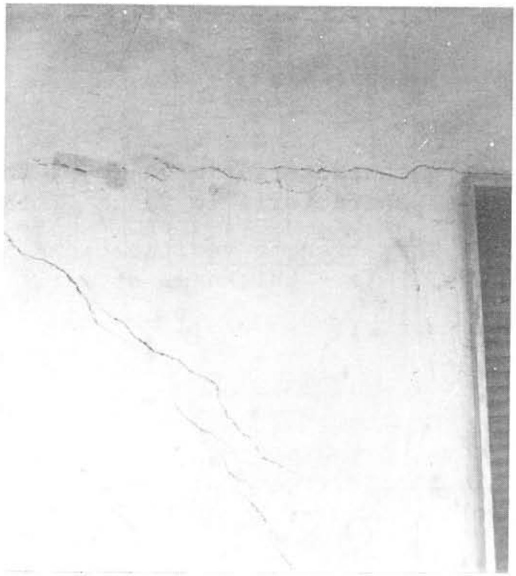


Fig. 3b -Failure of wall element in shear and flexure.

desire of the writers, which is to obtain a method and a computer program whose framework is as close as possible to that of the POR method and the POR program. Thus operating, in fact, the new procedure keeps the velocity an the efficiency of the POR method practically unchanged, and the new program is directly obtained from the POR program by performing some modifications which do not require the complete remaking of the program. On the other hand the proposed method gives results which are closer to the real behaviour of masonry buildings and more conservative than the ones given by POR.

The features of the procedure of analysis which come from hypotheses 1 to 5 can be found in the specific bibliografy of the POR method. Herein the consequences produced on the procedure by hypotheses 6 to 8 will be examined in detail.

Every floor is analysed individually and every wall (see fig. 4) is modeled as an equivalent frame with infinitely rigid parts (heavy lines) and deformable parts (light lines). In particular the wall elements are considered fixed both at their base and on the upper strip and their stiffness depends on the current situation of cracking during the displacement history; as soon as the flexural or the shear strength of the upper strip is attained, the upper constrain of the wall element becomes a hinge.

The following constitutive laws are adopted: the rigid - brittle law for normal and shear stresses of strips, the elastic - plastic law for shear and compressive stresses of wall elements, and the elastic - brittle law for tensile stresses of wall elements.

The strength conditions are ($\sigma_{c,t}$ = compressive, tensile stress):

$$(1)\quad \begin{cases} \tau \leqslant \tau_k \sqrt{1+\dfrac{\sigma_c}{1.5\,\tau_k}} \\ \sigma_c \leqslant \sigma_k \\ |\sigma_t| \leqslant k\,\tau_k \end{cases}$$

where:

τ_k = characteristic value of the shear strength ($\tau_k \geqslant 0$)
σ_k = characteristic value of the compressive strength.

The strength verifications of the strips are carried out through eqs(2) (s = thickness of wall):

$$(2)\quad \begin{cases} V \leqslant 2rs\,\tau_k \\ M = \dfrac{Vc}{2} \leqslant \dfrac{2}{3}\,sr^2\,k\,\tau_k \end{cases}$$

The shear forces on the strips are evaluated starting from the shear forces on wall elements by idealising the entire wall as shown in fig. 4. The shear V_i on the i^{th} part of the strip (positive if clockwise) is given by:

$$(3)\quad V_i = [\alpha_i S_i(d_i+r_i) + \alpha_{i+1} S_{i+1}(d_{i+1}+r_{i+1})]/(2\,l_i)$$

The axial force R_i on the i^{th} wall element (positive if compressive) due to the shear forces V_i is then given by:

$$(4)\quad R_i = V_i - V_{i-1}$$

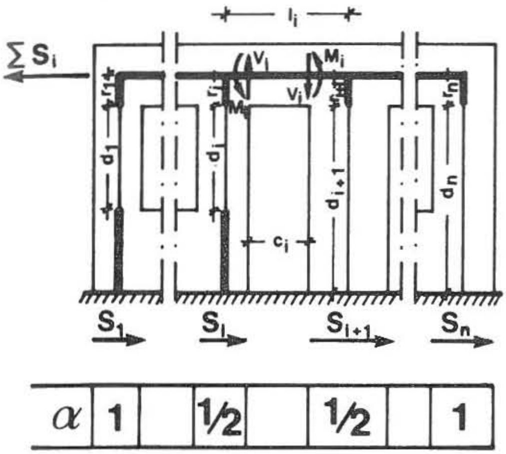


Fig. 4 - Static scheme of a wall with opening and α - values.

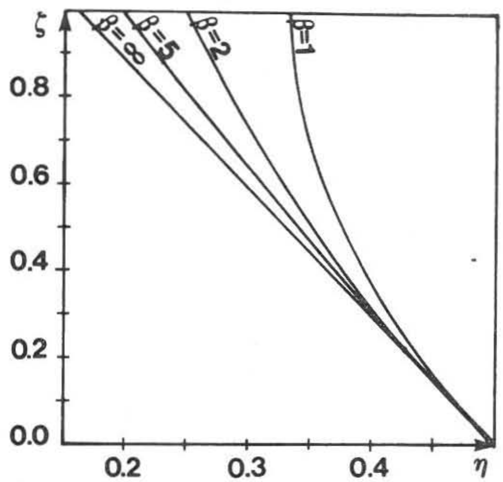
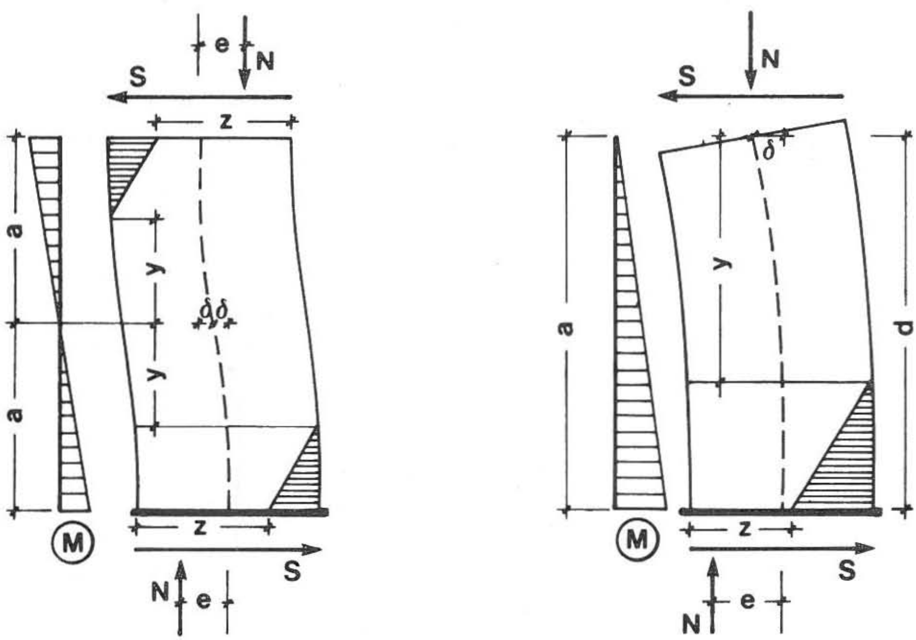


Fig. 6 - Depth of the reacting part of the section as a function of the eccentricity of the axial force (eq. 2).



a) Unyielded strip

b) Yielded strip

Fig. 5 — Possible cracking states of the wall elements (the cracked regions are hatched).

As for the wall elements, the strength verification in terms of shear stresses is carried out through eq. (5):

$$(5) \qquad S \leqslant bs\,\tau_k \sqrt{1 + \frac{\gamma}{1.5}\,\frac{\sigma_k}{\tau_k}}$$

where:

$$\gamma = \frac{N}{b\,s\,\sigma_k}$$

The strength verification in terms of normal stresses, in case of tensile force ($N < 0$) is satisfied when the whole section is reacting; i.e.:

$$(6) \qquad \eta \leqslant \frac{1}{6}\left(1 + \frac{1}{\beta}\right)$$

where (see fig. 5):

$$\beta = \frac{N}{bsk\tau_k}\,, \qquad \eta = \frac{|e|}{b}$$

In case of compressive force ($N > 0$), the strength verification is expressed by eq. (7), for an entirely reacting section ($\eta \leqslant 1/6(1+ + 1/\beta)$), and by eqs. (8), for a partially reacting section:

$$(7) \qquad \eta \leqslant \frac{1}{6}\left(\frac{1}{\gamma} - 1\right)$$

$$(8) \qquad \begin{cases} \eta \leqslant 1/2 \\ \beta \leqslant 1/2 \\ \zeta \geqslant -\gamma + \sqrt{\gamma^2 + 3\gamma(1-2\eta)} \end{cases}$$

where:

$$(9) \qquad \zeta = z/b = \beta - \sqrt{\beta^2 + 3\beta\,(2\eta - 1)}$$

Conditions (8) are obtained by equating the characteristic values of tensile and compressive strengths to the stresses at the edges of the reacting part of the section.

The stiffness of cracked wall elements is evaluated by taking into account the constrain condition with the upper strip (see fig. 5a,b), and by assuming a linear relationship between the width of the reacting part of the section and the eccentricity of the axial force N. Such an assumption is justified by eq. (9), which is obtained by equating the tensile stress at the edge of the reacting part of the section to $k\tau_k$. As can be seen in fig. 6, in fact, when $\beta \geqslant 5$ (which is a condition nearly always satisfied in actual situations), ζ varies almost linearly with η.

The translational stiffness of the wall element is given by:

$$(10) \qquad K = 1/(\psi\delta)$$

where:

$$(11) \qquad \delta = \frac{1}{Gs}\left[\frac{2}{3}\left(\frac{y}{b}\right)^3 + 2\,\frac{L}{D^3} - f(a) + f(y) + \chi\left(\frac{y}{b} + \frac{L}{D}\right)\right]$$

$$L = \ln\left(\frac{z}{b}\right)\,, \qquad f(\xi) = \frac{\xi\,(2C + 3D\,\xi)}{2D^2\,(C + D\,\xi)^2}$$

$$y = \frac{1}{6}\,\frac{a}{\eta}\left(1 + \frac{1}{\beta}\right)\,, \quad \eta = \frac{e}{b}\,, \quad C = \frac{ab - zy}{a - y}\,, \quad D = \frac{z - b}{a - y}$$

$$\psi = \begin{cases} 1 & \text{hinged upper end} \\ 2 & \text{fixed upper end} \end{cases}$$

The physical meaning of the quantities given by eqs. (10),(11) is shown in fig. 5.

3. NUMERICAL APPLICATIONS

The above described method of analysis has been codified in FORTRAN V by utilizing the frame of the POR program; the procedure obtained by introducing the above listed modifications has been called PORFLX.

The program works by increasing the displacement of the stiffness centre of the considered floor and by evaluating, for each step, the stress state of strips and wall elements. if, passing from one displacement to the next, the strength conditions are no longer satisfied for one or more elements, the increment is reduced until the strength conditions are fully satisfied for all the above said elements but one, which have them satisfied as equalities. At this point the static scheme is updated by accounting for the new situation of the element which has attained yielding or failure and a new displacement increment is imposed.

The wall element stiffnesses, which depend on their cracking state, are updated at each step, and are assumed constant along the single step.

The displacement is increased until the reactive force results to be stationary or decreasing for a fixed number of steps. The maximum value of the reactive force during the displacement history is then assumed to be the ultimate strength of the considered floor for an equivalent seismic force which acts in the direction and the way of the imposed displacement.

Each floor is tested by imposing displacements, one at a time, in two orthogonal directions and in both ways for each direction. In fact, considering changes of axial forces in wall elements, due to the frame behaviour of strip - wall element assemblage, implies that the maximum value of the reactive force changes when changing the way of the displacement.

The safety against collapse of the considered floor is evaluated by comparing the resistance of the floor to the sum of the equivalent static forces being applied to the considered floor and to those above it.

The input data are the geometrical and mechanical characteristics of wall elements and strips and the vertical loads applied to the wall elements. Starting from these data, the program calculates the coordinates of the centroid and of the centre of stiffness, and performs the entire analysis.

The output data are, for each step, the values of the centroid displacement, of the reactive force and of the overall stiffness. There is also an option which allows the program to print, when some element attains the yielding or the failure state, the data relevant

to the cracking situation of all wall elements and strips and their stress and strain state.

In order to show the extent of the differences between POR's and PORFLX's results, the structure shown in fig. 7 was examined. The calculated strengths of every floor are listed in tabs 1, 2.

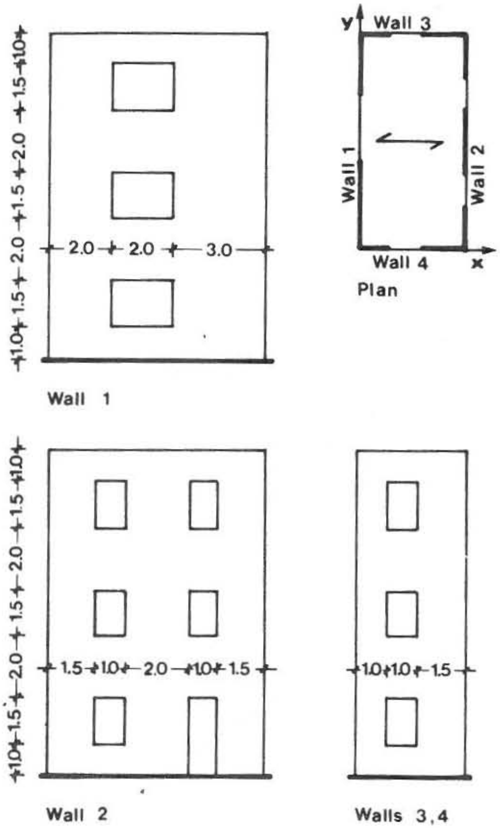


Fig. 7 - Masonry structure model.

Floor	PORFLX from left	PORFLX from right	POR
3rd	12.5	12.5	34.5
2nd	39.7	38.0	76.6
1st	55.9	53.1	85.9

Tab. 1 - Ultimate strengths (t.), for displacement in x-direction.

Floor	PORFLX from left	PORFLX from right	POR
3rd	5.4	3.5	23.6
2nd	20.5	16.3	64.0
1st	24.1	32.6	68.0

Tab. 2 - Ultimate resistance (t.) for displacement in y-direction.

The cracks in the walls at the collapse of each floor are depicted in fig. 8, for displacements in x-direction (a) and in y-direction (b). In figs. 8a and 8b the reactive force - centroid displacement diagrams relevant to the ground floor are also represented.

Tabs. 1 and 2 emphasize that the resistance values provided by POR are strongly non - conservative, as they are two to four times greater than those provided by PORFLX. The reason of such differences can be identified, at least for that example, in the failure of the strips, the failure in flexure of the wall elements whose mean plan is orthogonal to the direction of the force, the cracks and subsequent failures in flexure of the wall elements whose mean plan is parallel to the direction of the force. All that is well emphasized by the force - displacement diagrams of fig. 8a,b, where three curves are represented, which show the results obtained by POR (1) and PORFLX (2,3). In particular, curve 2 was obtained by assuming a deformable height equal to the floor height for the wall elements

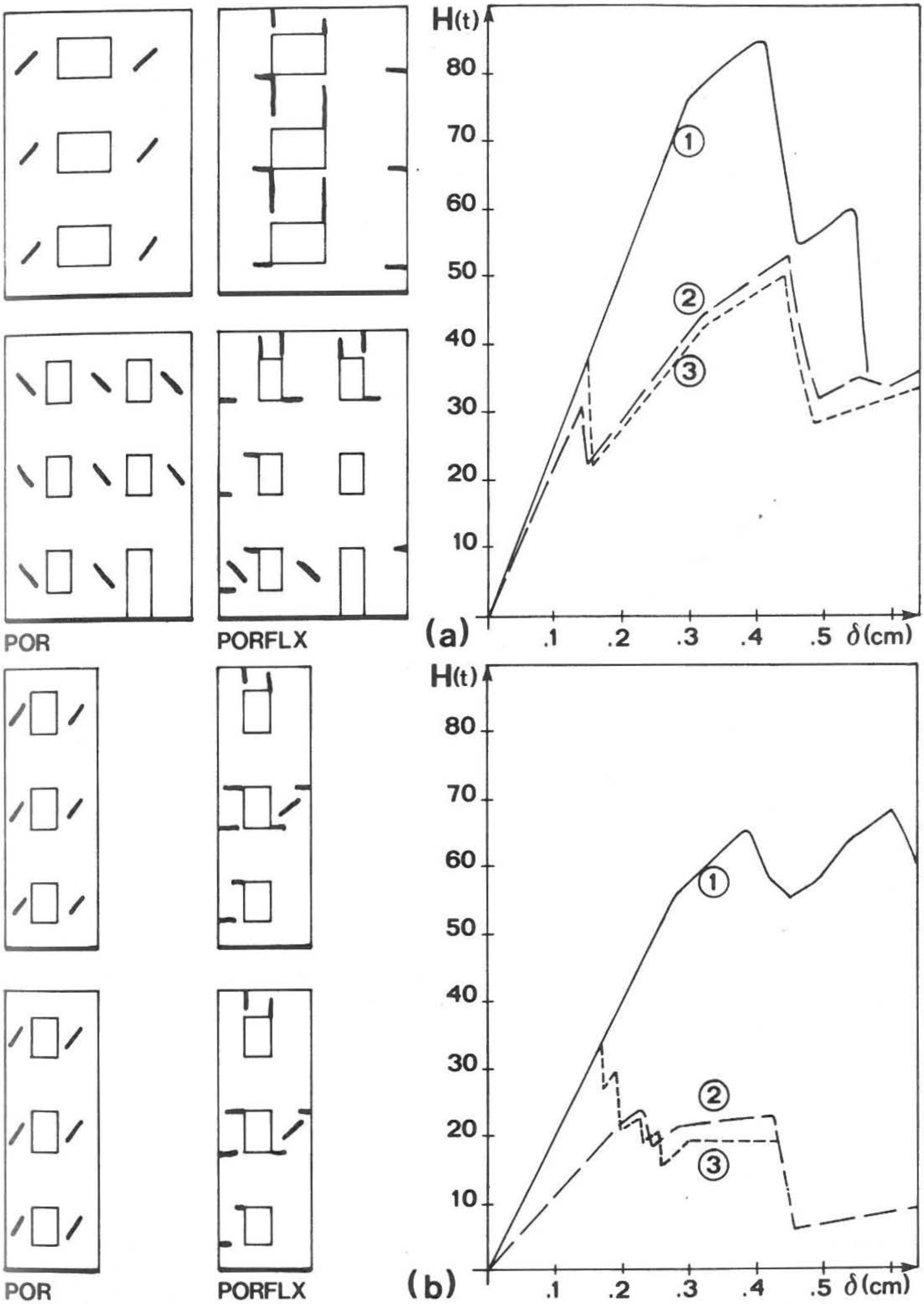


Fig. 8 – Cracking situation at the collapse point of each floor and reacting force - centroid displacement diagrams for seismic force in x - direction (a) and y - direction (y).

orthogonal to the seismic force, while curve (3) was obtained by assuming, for the same quantity, the height of the openings, as POR does. As can be seen (curves 1 and 2), after a first elastic part with different slopes, curve 2 diverges strongly in both cases from curve 1, because of the failure of a part of a strip, for displacements in x - direction, and because of the collapse in tension and flexure of two wall elements parallel to the force, for displacements in y - direction. All that, along with the subsequent cracks in the wall elements, makes the results of PORFLX decidedly different from those of POR, which shows an elastic behaviour well beyond the first failure point of PORFLX and attains the maximum reactive force when the wall elements parallel to the seismic force are yielding in shear. Finally curve 3 emphasizes that a correct evaluation of stiffness and strength of wall elements orthogonal to the force has a great importance on the evaluation of the overall resistance. Assigning too large values to such stiffness can lead (see fig. 8b) to considerable overestimations of the overall resistance of the floor. Such a trend is particularly marked in POR method, as it neglects the failure in flexure of the wall elements orthogonal to the displacements, while they actually show this type of failure as a rule.

4. CONCLUSION

As previouly said, the procedure herein presented does not aim at being a definite answer to the problem of the analysis of masonry structures subject to seismic actions, but it is proposed as an improvement of the simplified methods at present available, from which it inherits the ease of application, while providing more reliable results. On the other hand the numerical results corroborate some perplexities on the approach adopted by these simplified methods as well as by the proposed method. In particular it appears questionable to analyse the floors of the structure separately; such an approach, even though it is the basic simplification, makes it impossible to take "exactly" into account the effects of the failure of the strips and of the redistribution of the axial forces due to the frame behaviour. Such phenomena, indeed, are not negligible at all, as they lead to considerable reduction in the resistance of the structure. That has been shown by the numerical results obtained through PORFLX, which accounts for them, even if approximately. The PORFLX program, however, could turn out to be excessively conservative in the evaluation of the effects of the two phenomena and to give evaluations of the ultimate resistance which are too pessimistic.

It seems then advisable, before accepting a systematic use of POR and PORFLX, to accurately define the ranges of applicability of these procedures. They can be assessed by performing parametric analyses which compare the results of POR to those of PORFLX and the results of PORFLX to those of a more sophisticated program (e.g. a F.E.M. program) to determine the applicability ranges of POR and PORFLX respectively.

5. REFERENCES

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