Failure Mechanism of Centrally Loaded Masonry Walls

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ABSTRACT

This paper discribes the results of a theorethical study on the failure mechanism of a centrally loaded masonry wall. A finite element model - including non-linear mechanical mortar and/or concrete properties - allowed us to determine the influence of several parameters on the internal stress distribution : relative height of masonry unit/joint, non-linearity of joint material, variation of the mechanical properties, presence of reinforcing steel, etc.

1. THE FINITE ELEMENT-PROGRAM

A computer program - based on the finite element displacement method - has been developed. This program takes into account :

- a linear behaviour of bricks,
- a non-linear behaviour of mortar and concrete,
- a (non-) linear behaviour of steel,
- the development of cracks,
- crushing failure.

By means of this program the influence of several parameters - such like external load, relative joint height, mechanical properties of the composing materials, steel reinforcement, etc ... - on the internal stress distribution in a masonry wall can be studied.

The masonry wall is devided into 'micro-elements', i.e. typical, recurring parts of a masonry construction (see figure 1).

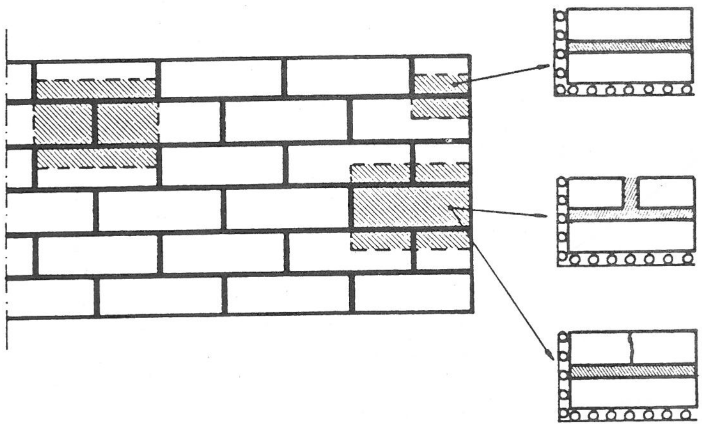

Fig.1 - Micro-elements in a masonry construction.

2. MATERIALS

2.1. Stress-strain relations

A linear stress-strain curve has been assumed for clay bricks. This supposition is based on several series of uniaxial compression and tension tests.

For mortar and concrete a non-linear stress-strain relation, based on a octahedral theory has been adopted in the compression zone ($\varepsilon_o < 0$). This model takes into account the increase of the crushing strength as a function of the confining pressure. According to several researchers (Kupfer, Weigber & Becker, Kotsovos & Newman, Probst, Khoo & Hendry, etc ...) the hydrostatic stress is univocally related to the volumetric strain ($\sigma_o - \varepsilon_o$) and also the deviatoric stress and the corresponding strain ($\tau_o - \gamma_o$). Here we use a model with bulk- and shearmodulus varying as a function fo the resp. volumetric and deviatoric strain :

$$K = K_o \; (a + b \; \varepsilon_o + c \; . \; d^{\varepsilon_o}) \qquad (1)$$

$$G = G_o \; (a' + b'\gamma_o + c' \; . \; d'^{\gamma_o}) \qquad (2)$$

These functions - already used by Cedolin et al. [ref.1] - give a good aggreement with experimental results (see figure 2.). The constants a to d and a' to d' are material constants.

In the tension zone all deformations are linear-elastic up to failure.

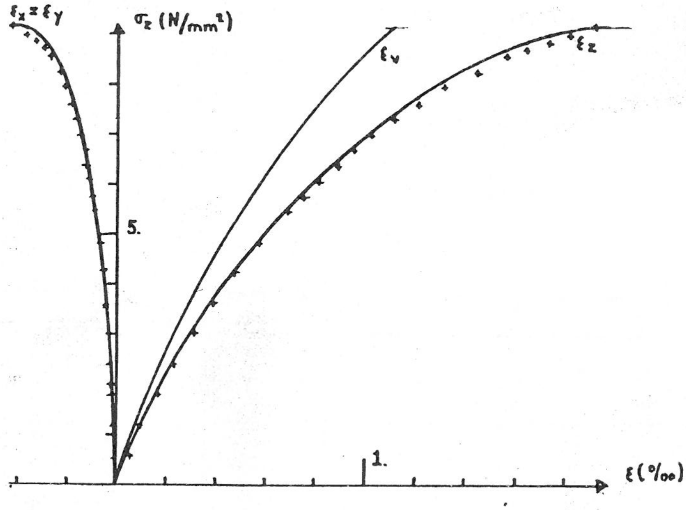

Fig.2 - Theoretical σ - ε relation and experimental curve for mortar test.

2.2. Failure criteria

We accept the principle of total independence of the two principal tensile stresses in the tension-tension zone, and a linear relation between the two principal stresses in the tension-compression zone. When a stress-point falls outside the failure envelope, a crack develops perpendicular to the principal

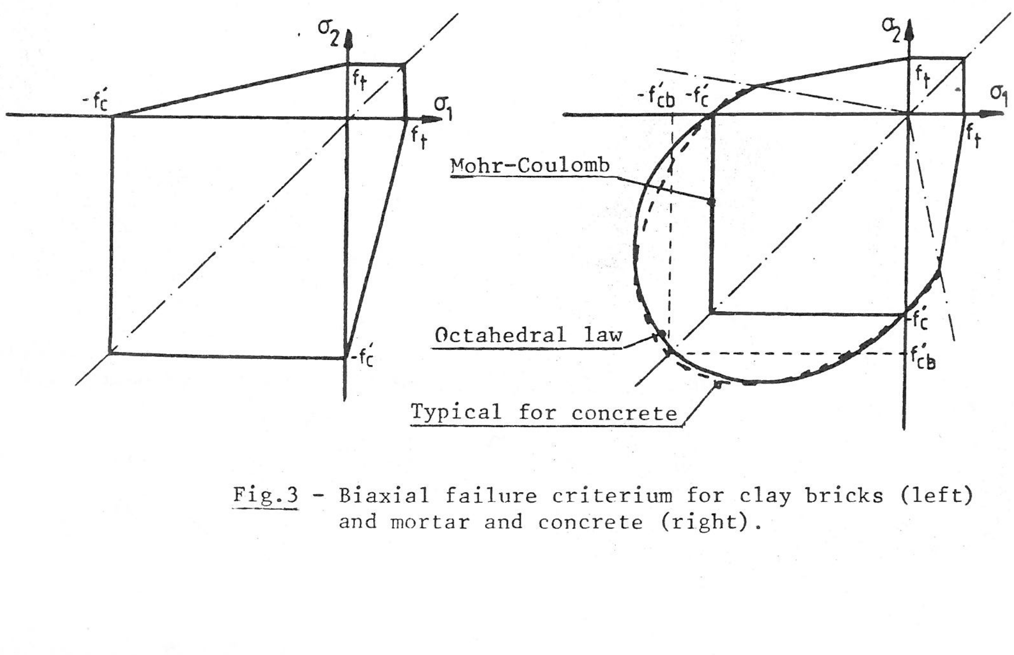

Fig.3 - Biaxial failure criterium for clay bricks (left) and mortar and concrete (right).

tensile stress. After cracking no tensile force can be transmitted in the direction perpendicular to the crack. In the direction of the crack the material keeps its original characteristics. The shear stress transmitted through the crack will be reduced to :

$$\tau' = a \, . \, G \, . \, \gamma \qquad (0 < a < 1) \qquad (3)$$

In the compression-compression zone failure of a brick occurs when a principal stress exceeds the uniaxial compression strength. For mortar and concrete we assume and octahedral form of the Coulomb friction law :

$$\tau_o = c + n \, \sigma_o \qquad (4)$$

c and n are experimentally determined material constants. This gives much better results as the usually adopted Mohr-Coulomb-theory (see figure 3 for the failure criteria). After crushing we take σ_o as constant and adapt τ_o so that this law (4) will be satisfied.

3. STRESS DISTRIBUTION IN A 'LINEAR-ELASTIC' WALL

3.1. Definitions

Basic stress (σ_b) : the minimal horizontal tensile stress appearing anywhere in the masonry units.

Maximal stress (σ_M) : the maximal horizontal tensile stress appearing in a masonry unit at the free edge just above the joint.

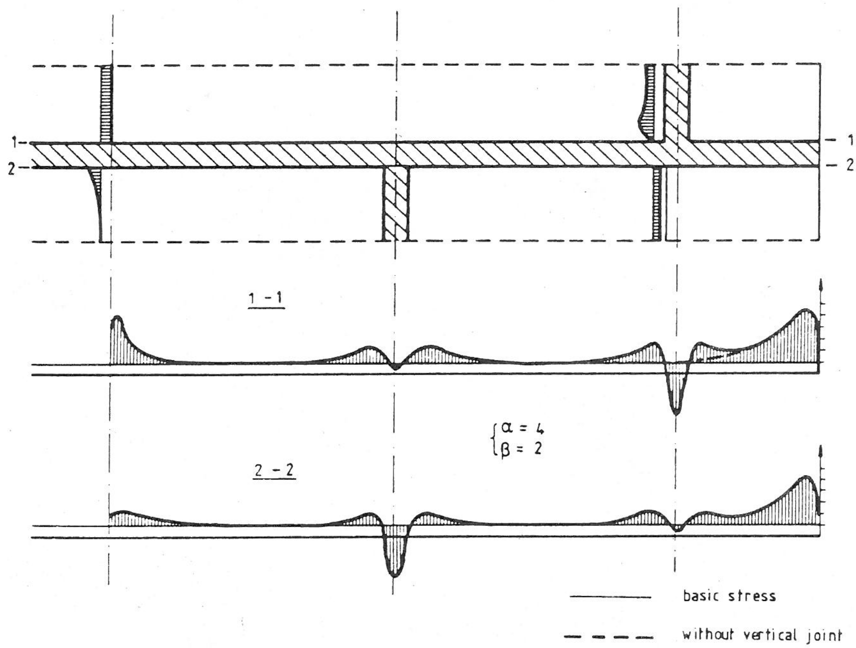

Fig.4 - Internal stress distribution in a masonry wall (linear material characteristics).

Four parameters have an influence on those two stresses :

$$\alpha = \frac{E_b}{\nu_b} * \frac{\nu_m}{E_m} \qquad (5)$$

$$\beta = E_b/E_m \qquad (6)$$

$$\xi = h_b/h_m \qquad (7)$$

and ν_m (or ν_b)

Other parameters - like height/length ratio of the masonry units - have little or no influence.

3.2. Internal stress distribution

In the case of linear elastic materials figure 4. (already published by Probst [ref.2].) gives a good idea of the internal distribution of the horizontal stresses in a masonry wall around free edges, vertical and horizontal mortar joints and vertical cracks. The maximum stress will always be localised near a free edge or a vertical slit.

3.3. The maximum stress

For a constant value of ν_m the maximum stress varies linearly with α and decreases with increasing values of β. The absolute value of the material characteristics has no importance. The parameter ξ has little influence, a variation of ξ from 7 to 28 gives only a very slight decrease of σ_M (about 1 to 2 %) independently of the value of β). A variation of ν_m, however, causes an almost proportional variation of σ_M.

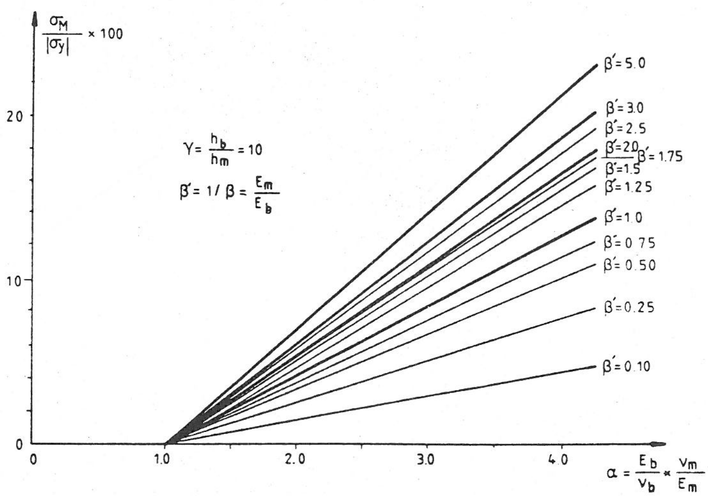

Fig.5 - Maximum horizontal tensile stress as a function of α and β.

3.4. The basic stress

This stress depends in the same way on α, β and ν_m as the maximum stress (see figure 6). The parameter ξ has now a great influence. A good estimation of the basic stress is given by the following formula (Probst [ref.3] p.60). For a biaxial stress state we have :

$$\sigma_b = \frac{E_b \cdot \nu_m - E_m \cdot \nu_b}{E_m + E_b \cdot h_b/h_m} * p \qquad (8)$$

and for a triacial stress state :

$$\sigma_b = \frac{E_b \cdot \nu_m - E_m \cdot \nu_b}{(1-\nu_b)\, E_m + (1-\nu_m)\, E_b h_b/h_m} * p \qquad (9)$$

with p = the uniform vertical pressure.

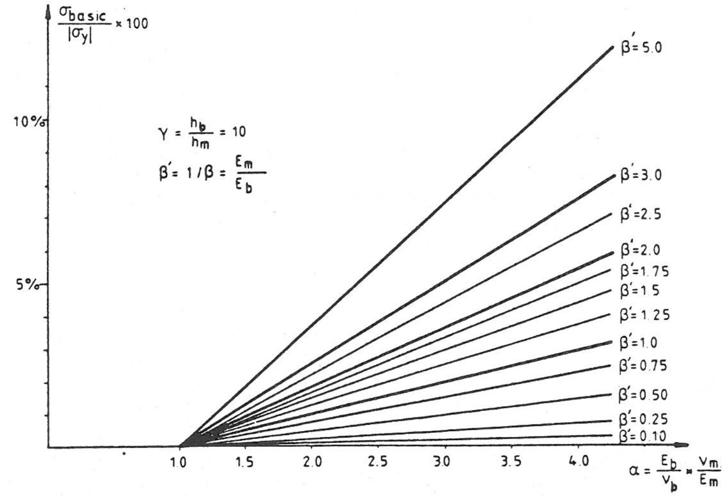

Fig.6 - Basic horizontal tensile stress as a function of α and β.

The influence of ξ ($=h_b/h_m$) on the basic stress can also be deduced from figure 7. giving the 'stress concentration' ($c = \sigma_M/\sigma_b$) as a function of β and ξ.

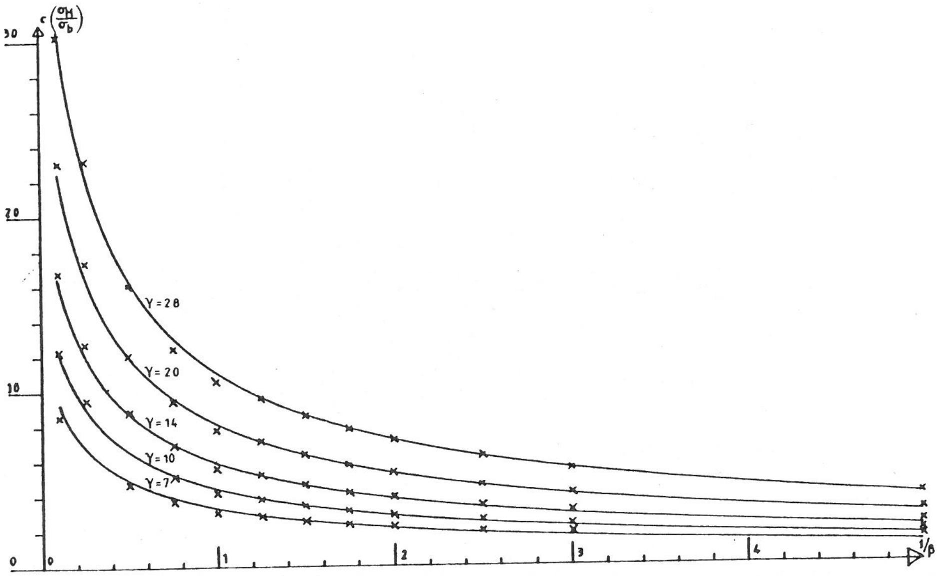

Fig.7 - Stress concentration factor c ($= \sigma_M/\sigma_b$) as a function of β (E_b/E_m) and ξ (h_b/h_m).

This concentration factor is independent of α and can be approximated by :

$$c = \frac{\sigma_M}{\sigma_b} = \frac{0.894 + 0.414\ h_b/h_m}{(E_m/E_b + 0.181)^{0.705}} \tag{10}$$

3.5. Conclusions

High values of β together with low values of ξ (or masonry units much stiffer than the joint material and thick (joints) lead to an early failure caused by vertical splitting of the masonry unit. The maximum stress increases very rapidly and the basic stress approaches more and more the maximum stress.

The masonry unit/joint height ratio has little influence on the maximum stress but thick joints (and thus low ξ-values) lead to high basic stresses 'punishing' every local loss of strength.

By decreasing values of β the stress peak will be wider and less sharp and the localisation of the maximum stress moves away from the free edge. Low values of ξ push this localisation further away from the edge.

Practical values of those parameters are :

h_b = 70 ... 400 mm		ξ = 7 ... 40
h_m = 8 ... 15 mm		
E_b = 5000 ... 30000 N/mm^2		β = 0.25 ... 12.0
E_m = 2500 ... 20000 N/mm^2		
ν_b = 0.10 ... 0.20		α = 0.15 ... 30
ν_m = 0.15 ... 0.25		

However, a combination of a very stiff mortar with a weak masonry unit doesn't seem very logic, so the practical range for β will be from 0.5 to 3.0 and for α from 1.0 to 3.0.

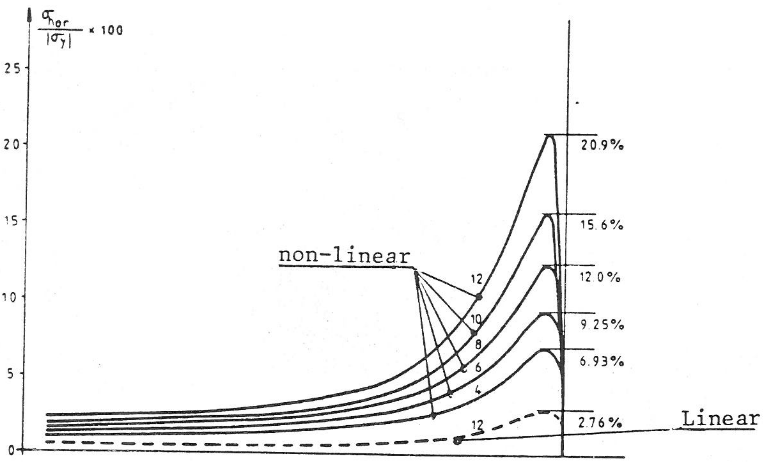

Fig.8 - Horizontal tensile stresses in a masonry unit as a function of the vertical load (p).

4. NON-LINEAR MATERIAL CHARACTERISTICS

4.1. Generalities

Only the influence of a non-linear mortar joint will be discussed. The masonry units are supposed to behave linearly. This non-linearity has two effects on the maximum stress :

- The increasing stress level causes a weakening of the joint material and an increase of the joint deformations. As a direct result of this the basic stress increases more than linearly with increasing load (figure 7 and table 1).
- At the height of the maximum stress the deformations of the joint material are even greater than the mean deformations. This causes an increase in the concentration factor c.

Load	Basic stress		Maximum stress		c
$p = \sigma_y$ (N/mm^2)	σ_b	$r = \frac{\sigma_b}{\lvert\sigma_y\rvert}$	σ_M	$r' = \frac{\sigma_M}{\lvert\sigma_y\rvert}$	σ_M/σ_b
-12	0.052	0.42 %	0.331	2.76 %	6.38
- 4	0.042	1.05 %	0.277	6.93 %	6.59
- 6	0.081	1.35 %	0.555	9.25 %	6.85
- 8	0.132	1.65 %	0.961	12.0 %	7.26
-10	0.196	1.96 %	1.56	15.6 %	7.96
-12	0.274	2.28 %	2.51	20.9 %	9.16

Table 1 - Variation of basic and maximum stress as a function of the vertical load (p). (α = 1.64; β = 1.04; ξ = 14).

4.2. Failure mechanism

The increase of the horizontal tensile stresses makes a crushing failure unlikely. Failure will be caused by vertical splitting of the masonry unit. At high load levels there will appear high shear stresses in the mortar-brick interfac. A shear failure of this connection will cause a decrease of the

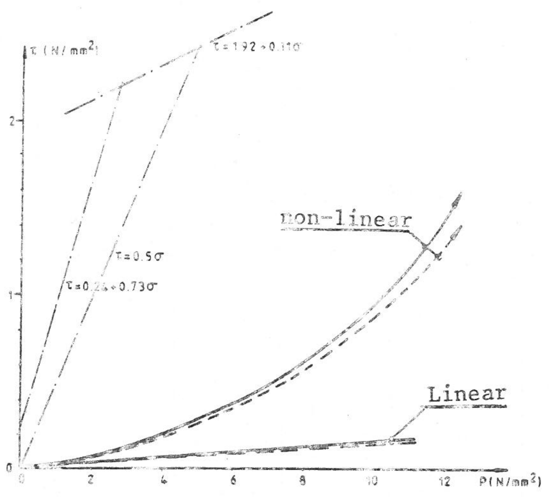

Fig.9 - Shear stress in the brick-mortar interface versus shear failure criterium of mortar (A. Page).

horizontal tensile stresses and thus have a favourable effect on the ultimate

strength of the wall (under static load). A breakdown of this shear connection, however, seems very unlikely if we compare the calculated shear stress with a failure criterium for mortar joints (A. Page [ref.7] fig.9).

4.3. Influence of vertical joints and/or slits

Just like in the case of a free edge all horizontal stresses around a vertical joint are higher with non-linear than with linear joint characteristics but

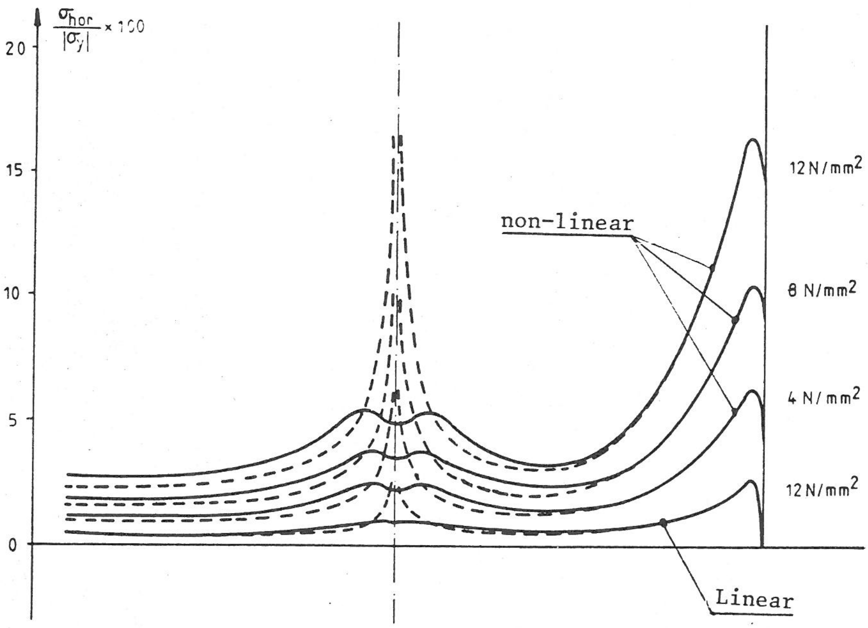

Fig.10 - Horizontal tensile stresses around a vertical slit as a function of the vertical load (p).

much lower than at a free edge. Around a vertical slit, however, we find the same stresses as at a free edge. The stress peak is now much sharper (see figure 10). A 'precracked' masonry unit increases the risk on an early failure.

4.4. Cracking

The appearance of vertical cracks causes a flattening of all stress peaks. The 'lost' tensile forces are recovered as a general increase of the basic stress and so the risk on the formation of new cracks will be increased.

σ_y (N/mm^2)	non-reinforced		reinforced	
	σ_b	σ_M	σ_b	σ_M
-2.0	0.010	0.090	0.010	0.085
-4.0	0.030	0.265	0.025	0.250
-6.0	0.075	0.435	0.060	0.410

Table 2 - Variation of the basic and maximum stress as a function of the vertical load (p) with and without reinforcement (2 ∅ 4 mm/joint).

4.5. Reinforcing steel

A reinforcing bar in an horizontal joint will increase the stiffness of this joint and thus decrease the horizontal tensile stresses in the masonry unit (see table 2). An efficient use of this reinforcement supposes very large deformations of the joint material and thus cracked masonry units. As a result of this the cracking load will only be increases slightly.

After the appearance of the first crack the 'lost' tensile forces are now concentrated in the reinforcing vars and so the propagation of cracks is seriously obstructed (see figure 11).

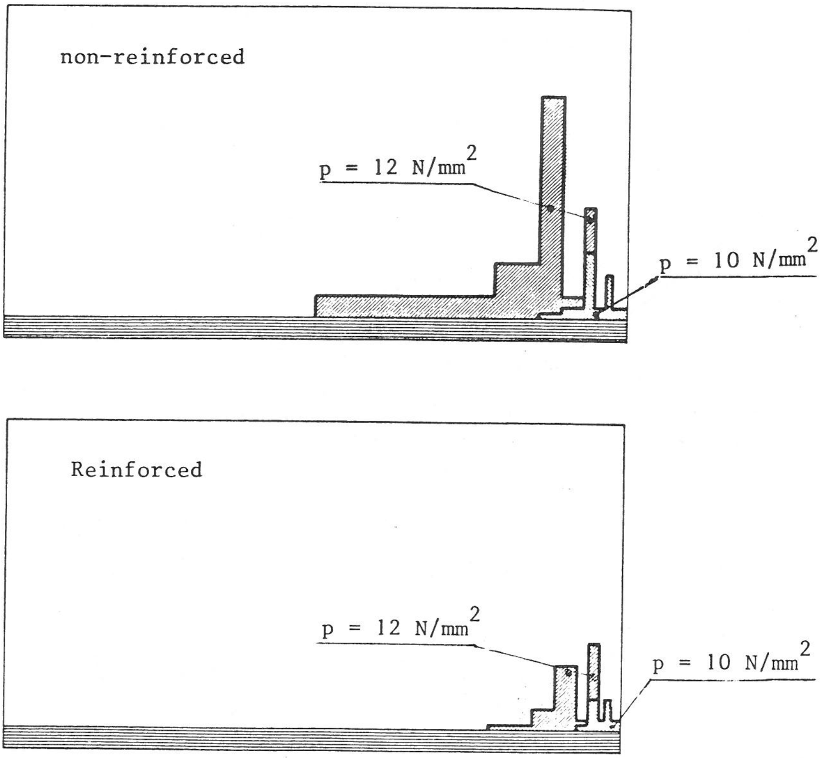

Fig.11 - Influence of the horizontal reinforcement on the crack propagation.

Appendix 1 (symbols used)

K (K_o) : Bulkmodulus (at $\varepsilon_o = 0$)

G (G_o) : Shearmodulus (at $\gamma_o = 0$)

ε_o : Volumetric strain [$= (\varepsilon_1 + \varepsilon_2 + \varepsilon_3)/\sqrt{3}$]

γ_o : Deviatoric strain

[$= \sqrt{(\varepsilon_1 - \varepsilon_2)^2 + (\varepsilon_2 - \varepsilon_3)^2 + (\varepsilon_3 - \varepsilon_1)^2 / 3}$]

$\varepsilon_1\ \varepsilon_2\ \varepsilon_3$	:	Principal strains
$\sigma_1\ \sigma_2\ \sigma_3$	:	Principal stresses
a,b,c,d,a',b',c',d'	:	Material constants
E	:	Young's modulus
ν	:	Poisson's coefficient
indes b	:	bric or concrete block
m	:	mortar joint.

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