A Principal Stress Failure Criterion for Brickwork in Biaxial Bending

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SUMMARY

This paper expresses the failure criterion for masonry in biaxial bending in terms of principal moments. Three stages are necessary in the theoretical development. In the first, a relationship is established between the principal moment at failure and orientation for one-way bending. Secondly, a failure criterion is established in biaxial bending where the principal moments are aligned in the vertical and horizontal directions. Finally, the previous two stages are combined to obtain a general criterion of failure for biaxial bending where the principal moments are at an inclination to the vertical and horizontal axes.

NOTATION

Subscripts

v	Vertical Direction.
h	Horizontal Direction.
p	Maximum Principal Moment Direction.
q	Minimum Principal Moment Direction.
i	Joint Location in Panel Height.
j	Joint Location il Panel Length.

Quantities

w	Applied Uniformly Distributed Load
t	Thickness of Masonry Panel.
Z	Section Modulus $Z=t^2/6$
M_v, M_h, M_p, M_q	Applied Bending Moments per unit length.
F_v, F_h, F_p, F_q	Extreme Fibre Stresses in Biaxial Bending.
F'_v, F'_h, F'_p, F'_q	Moduli of rupture in one-way bending with no applied axial stress.
F''_v, F''_h, F''_p, F''_q	Effective Moduli of rupture in one-way bending with applied axial stress.
k_v, k_h, k_p, k_q	Bending Moment Coefficients M=kw
R, R', R''	Orthogonal Strength Ratios
fa, fa	Vertical Axial Compressive Stresses.
F'_b	Modulus of Rupture of Brick.
J, Jij	Modulus of Rupture of Joint in Vertical Flexure.

1 GENERAL

Practical brickwork panels usually resist wind by combined bending in the vertical and horizontal directions. In addition, the brickwork is subjected to an axial compressive load due to

its own weight and any surcharge that may be present. At a particular location in a brickwork panel the material is subjected to a combination of vertical moment, horizontal moment and vertical compressive force. In addition, torsional moments may be present.

This combination changes from point to point in the panel. Whatever method of analysis is used to calculate these moments and forces, the assessment of the panel strength is dependent upon the criterion adopted for the failure of the material.

Material properties under various combinations of vertical and horizontal moments and axial stress are first considered. From these a failure criterion is derived for a joint subjected to principal moments not oriented in the vertical and horizontal directions. This failure criterion is then developed to a form suitable for application to two-way panels.

2 MATERIAL PROPERTIES

This section deals with the orthogonal strength ratio in horizontal and vertical flexure, the interaction of vertical and horizonal moments, and the effect of an axial stress.

2.1 Orthogonal Strength Ratio

The orthogonal strength ratio for masonry, R, is defined as the ratio of the modulus of rupture in horizontal flexure to the modulus of rupture in vertical flexure. Brickwork spanning in the horizontal direction has a greater flexural strength than when spanning in the vertical direction and therefore the orthogonal strength ratio is always greater than unity. This is because when spanning vertically, failure occurs by tensile bond in the bed joints but when spanning horizontally, the overlapping bricks in alternate courses cause the bricks to break or the mortar bed joints to fail in torsional shear. Panel and joint specimens subjected to horizontal flexure are shown in figures 1 and 2. By considering these two failure modes in horizontal flexure the author suggested (1) that for the practical range of brickwork ($F'v$ greater than 0.3MPa) the orthogonal stength ratio, R may be taken as the lesser of -

$$R = \frac{1}{9}\left(4\frac{F_b'}{F_v'} + 5\right) \quad \text{and} \quad R = \frac{2.75}{\sqrt{F_v'}} \qquad \ldots\ldots 1$$

These relationships are shown in Figure 3.

2.2 Interaction of Vertical and Horizontal Moments

The analysis of the previous section is applicable to one-way bending, that is, when the brickwork is subjected to either a vertical or horizontal moment. This section deals with the case of two-way bending, that is, when the brickwork is subjected to both vertical and horizontal bending moments simultaneously.

Because it is very difficult to apply simultaneous vertical and horizontal moments to specimens of brickwork such as that shown in Figure 1, tests have been carried out on "single joint" specimens, Figure 2. This "joint" approximately reproduces the behaviour of brickwork when subjected to both vertical and horizontal moments.

Tests reported previously (1) have shown that the failure criterion for combined vertical and horizontal moments is given approximately by the elliptical expression:

$$\left[\frac{F_v}{F'_v}\right]^2 + \left[\frac{F_h}{F'_h}\right]^2 = 1 \qquad \ldots\ldots 2$$

This is shown in Figure 4.

2.3 Effect of Axial Stress

The effect of a compressive stress on the failure criterion of joint specimens subjected to combined vertical and horizontal bending has also been investigated (1). The applied compressive stress increased both the vertical and horizontal bending strength and the interaction curve remained elliptical.

The increase in horizontal modulus of rupture was such that the orthogonal strength ratio R was the same with or without the applied compressive stress. Where failure occurs in the mortar joints, one could expect an increase in horizontal strength because the increased shear strength of the mortar due to the applied compressive stress leads to a greater moment of resistance of the bed joint. Where horizontal bending strength is determined by brick strength, no increase could be expected since a small compressive stress would probably have negligible effect on the bending strength of the brick.

3 FAILURE CRITERION FOR A JOINT

From the foregoing work the following failure criterion is obtained where the principal moments are applied about the vertical and horizontal axes:

$$\left[\frac{F_v}{F''_v}\right]^2 + \left[\frac{F_h}{F''_h}\right]^2 = 1 \qquad \ldots\ldots 3$$

where $F''_v = F'_v + f_a$

f_a = applied axial stress

and F''_h is obtained as follows

If F'h is measured by test and failure occurs through the bricks:

$$F''_h = F'_h \qquad \ldots\ldots 4$$

If F'h is measured by test but failure is through the mortar joints then F"h is the lesser of:

$$F''_h = \frac{F'_h}{F'_v} F''_v \quad \text{or} \quad \frac{1}{9}(4F'_b + 5F'_v) \qquad \ldots\ldots 5$$

but not less than F'h.

If F'h is not measured then F"h is the lesser of:

$$F''_h = \frac{2.75}{\sqrt{F'_v}} F''_v \quad \text{or} \quad \frac{1}{9}(4F'_b + 5F'_v) \qquad .....6$$

For any orientation of the principal moments a general failure criterion may be obtained as follows:

Firstly the effective moduli of rupture in vertical and horizontal flexure F"h and F"v may be determined as described above.

Secondly, the effective moduli of rupture in the two inclined principal directions F"p and F"q may be found from the relationship.

$$\left.\begin{aligned} F'' &= F''_v\,(1 + \beta(R' - 1)) \\ \text{for } F''_p,\ \beta &= \beta_p = 1 - \frac{\theta}{90}\left[\frac{5}{3} - \frac{\theta}{135}\right] \\ \text{for } F''_q,\ \beta &= \beta_q = \frac{\theta}{90}\left[\frac{1}{3} + \frac{\theta}{135}\right] \end{aligned}\right] \qquad .....7$$

$$\text{where } R' = \frac{F''_h}{F''_v}$$

This empirical relationship was derived from the parabola of best fit to results of one-way bending tests reported by Hedstrom (2), Satti and Hendry (3) and Cajdert (4) and summarised in Figure 5.

Thirdly, the elliptical interaction diagram established for vertical and horizontal principal moments may be assumed to hold also for other orientations of principal moments. That is:

$$\left[\frac{F_p}{F''_p}\right]^2 + \left[\frac{F_q}{F''_q}\right]^2 = 1 \qquad .....8$$

These assumptions determine the failure criterion when the two moments both produce tension on the same face of the masonry, that is, when both moments are positive. This elliptical criterion is shown in the first quadrant in Figure 6. To obtain a general failure criterion the diagram should be completed for the other quadrants. The second and fourth quadrants correspond to the cases where one moment produces tension and the other moment produces compression on the same face of the masonry. It can be argued that masonry fails in flexure due to tensile stress and the presence of the compressive stress in the orthogonal direction would not reduce the modulus of rupture in the direction of the tensile stress. This leads to the full line interaction diagram shown.

Symmetry may be used in establishing the dotted portion of the interaction diagram as this portion simply corresponds to failure conditions occurring on the opposite face of the masonry. The rectangular interaction diagrams in the second and fourth quadrants are probably conservative.

4 FAILURE CRITERION FOR A PANEL

Consider a joint in a two-way action panel subjected to the principal moments Mp and Mq that produce ultimate extreme fibre stresses Fp and Fq. If it is assumed that there is a linear distribution of stresses over the thickness of the joint, it follows that:

$$Mp = Fp\ Z$$

$$Mq = Fq\ Z$$

For a given panel the bending moments at the joint will be proportional to the applied load and may be expressed as:

$$Mp = kp\ w$$

$$Mq = kq\ w$$

where the coefficients kp and kq may be determined by elastic analysis. Combining equations 9 and 10 gives:

$$F_p = \frac{k_p w}{Z} \quad \text{and} \quad F_q = \frac{k_q w}{Z} \qquad \text{.....11}$$

If both stresses are of the same sign then the failure criterion is:

$$\left(\frac{F_p}{F''_p}\right)^2 + \left(\frac{F_q}{F''_q}\right)^2 = 1 \qquad \text{.....12}$$

Substituting equation 11 gives

$$w = \frac{Z}{\sqrt{\left(\frac{k_p}{F''_p}\right)^2 + \left(\frac{k_q}{F''_q}\right)^2}}$$

or

$$w = \frac{t^2 \cdot F''_q}{6\sqrt{\left(\frac{k_p}{R''}\right)^2 + k_q^2}} \qquad \text{.....13}$$

where from equation 7

$$R'' = \frac{F''_p}{F''_q} = \frac{1 + \beta_p\ (R'-1)}{1 + \beta_q\ (R'-1)} \qquad \text{..... 14}$$

Also, substituting $F''q$ from equation 7 in equation 13 gives

$$w = \frac{t^2}{6} \frac{(1 + \beta q(R'-1)}{\sqrt{(\frac{k_p}{R''})^2 + (k_q)^2}} .F''_v$$

or

$$w = \frac{t^2}{6} \frac{F''_v}{C} \quad \text{where } C = \frac{\sqrt{(\frac{k_p}{R''})^2 + k_q^2}}{1 + \beta_q (R'-1)} \qquad \ldots\ldots 15$$

Substituting that $F'' = J + fa$

Failure conditions will exist at the joint if

$$w \geqslant \frac{t^2}{6} \frac{J + f_a}{C} \qquad \ldots\ldots 16$$

The above failure criterion applies where the principal stresses on a face of the masonry are both tensile. Where one stress is tensile and the other compressive the appropriate failure criteria are obtained from the 2nd and 4th gradients of Figure 6.

If the maximum princpal stress $F''p$ is critical then failure conditions will exist if

$$k_p w \geqslant F''_p Z$$

or

$$w \geqslant \frac{t^2}{6} \frac{1 + k_p (R' - 1)}{k_p} F''_v$$

or

$$w \geqslant \frac{t^2}{6} \frac{J + f_a}{C} \qquad \ldots\ldots 17$$

$$\text{where } C = \frac{k_p}{1 + \beta_p (R' - 1)}$$

Similarly if the minimum principal stress $F''q$ is critical

$$w \geqslant \frac{t^2}{6} \frac{J + f_a}{C} \qquad \ldots\ldots 18$$

$$\text{where } C = \frac{k_q}{1 + \beta_q (R' - 1)}$$

Equations 16, 17 and 18 are identical except for the value of C.

Hence, in general, failure conditions will exist at a joint ij when:

$$w \geqslant \frac{t^2}{6} \frac{J_{ij} + f_{ai}}{C_{ij}} \qquad \text{..... 19}$$

where C_{ij} is given by:

$$\frac{\sqrt{\frac{k_p}{R''} + k_q}}{1 + k_q(R' - 1)}$$

when both principal stresses on a face are tensile, or by:

$$\frac{k_p}{1 + k_p(R' - 1)} \quad \text{or} \quad \frac{k_q}{1 + k_q(R' - 1)} \quad \text{whichever is less}$$

when one of the principal stresses is tensile and the other compressive.

5 CONCLUSIONS

1. For the general case of masonry subjected to principal moments not aligned in the vertical and horizontal directions a failure criterion has been developed, based on an empirical relationship between modulus of rupture and orientation and on the elliptical interaction diagram established for the vertical and horizontal orientation of principal stresses.

2. The criterion has been developed to a stage suitable for application to masonry panels subjected to general out-of-plane flexure.

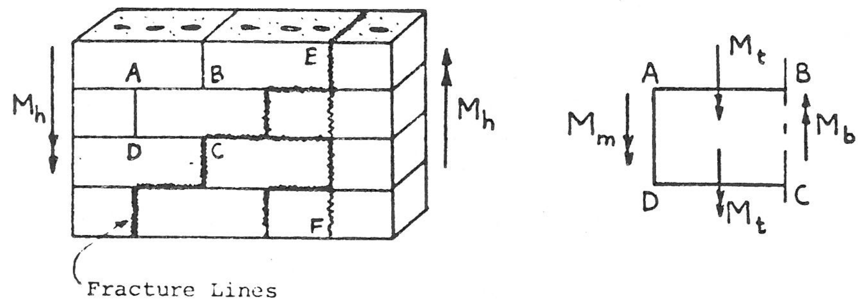
6 REFERENCES

(1) Baker, L.R. "A Failure Criterion for Brickwork in Bi-axial Bending". Fifth International Brick Masonry Conference, Washington 1979.

(2) Hedstrom, R.O. "Load Tests on Paterned Concrete Masonry Walls". A.C.I. Journal Proceedings, Vol57, P.C.A. Development Department Bulletin D.41, April 1961.

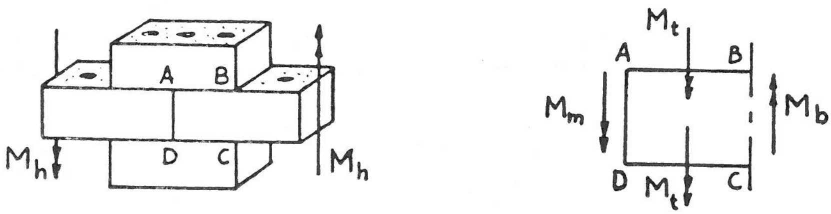
(3) Satti, K.M.H. and Hendry, A.W. "The Modulus of Rupture of Brickwork". Third International Brick Masonry Conference, Essen 1973, pp 155-160.

(4) Cajdert, A. "Laterally Loaded Masonry Walls". Chalmers University of Technology, Publication 80:5, Goteborg, 1980, p283.



(a) (b)

FIGURE 1: MASONRY SUBJECTED TO HORIZONTAL FLEXURE.



(a) (b)

FIGURE 2: JOINT SPECIMEN SUBJECTED TO HORIZONTAL FLEXURE.

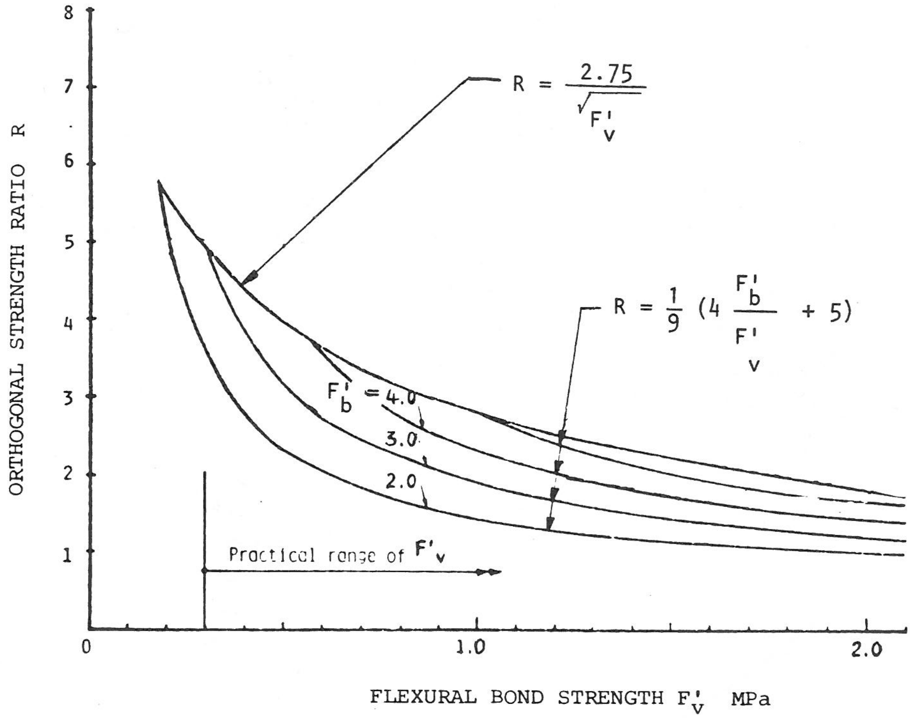


FIGURE 3: STRENGTH RATIO AND FLEXURAL BOND STRENGTH.

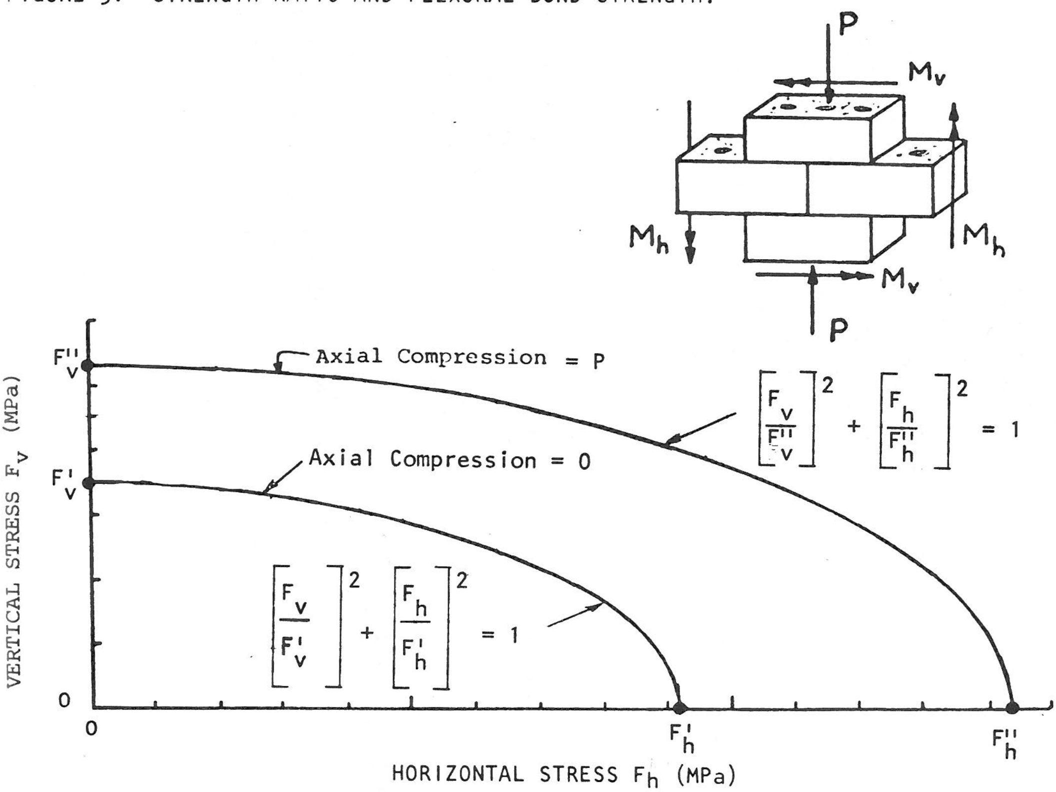


FIGURE 4: INTERACTION OF VERTICAL AND HORIZONTAL MOMENTS ON A JOINT.

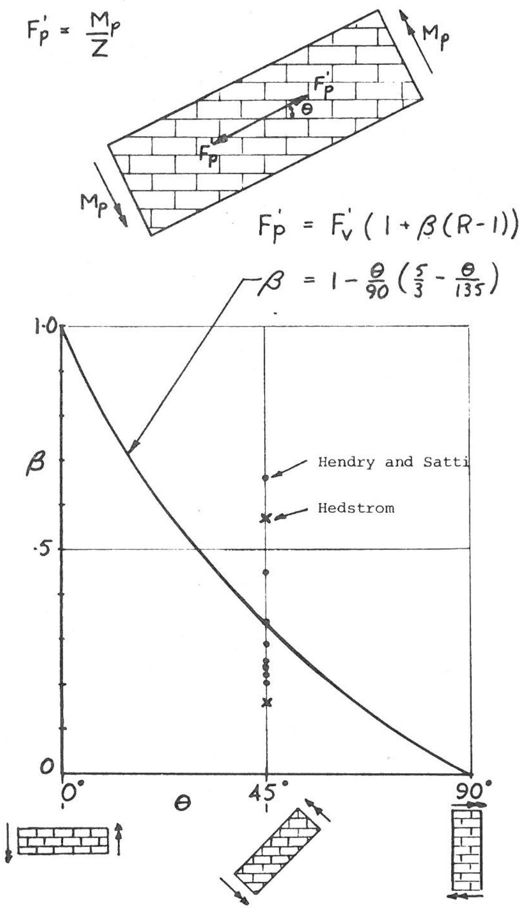


FIGURE 5: FLEXURAL STRENGTH AND ORIENTATION FOR ONE-WAY BENDING.

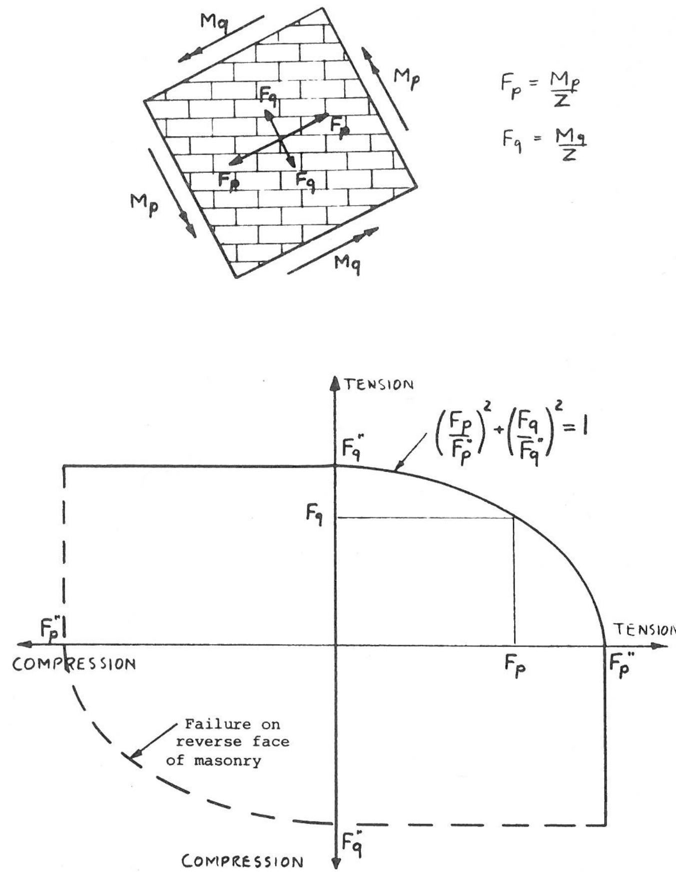


FIGURE 6: GENERAL FAILURE CRITERION IN BIAXIAL BENDING.