Stability and Fracture Analysis of Eccentrically Loaded Brick Masonry Walls (*)

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SOMMARIO

Scopo della presente memoria è di illustrare un procedimento numerico, basato sul metodo delle differenze finite, per calcolare la capacità portante limite di una parete in muratura, soggetta a presso-flessione, con comportamento geometricamente non-lineare.

Il materiale costituente la colonna si considera reagente sia a compressione che a trazione, nelle ipotesi di elasticità lineare e conservazione delle sezioni piane, considerando altresì la formazione di fessure (cracks) in corrispondenza dei giunti di malta.

La distribuzione delle tensioni all'apice della fessura è conforme al "fictitious crack model" (F.C.M.), nel quale è preso in conto il "softening" dei materiali fragili.

La trattazione consente di valutare l'influenza della snellezza della parete sul carico limite e sulla configurazione ultima della struttura.

ABSTRACT

The purpose of this paper is to present a method of analyzing unreinforced load-bearing masonry walls subjected to compressive axial loads and end-moments. The behaviour of the wall, made of bricks and joints of mortar, will be represented by an eccentrically loaded beam-column model, in the geometrically non-linear range. The material is assumed to be linearly elastic both under compression and under tensile stress. The rectangular plane section will be supposed to be plane after deformation, and cracking will occur only in the mortar layers.

The distribution of stress at the crack tip is in agreement with the "fictitious crack model" [1], in which the softening of non-yielding materials is considered.

This approach allows the evaluation of the influence of slenderness on the limit load and the ultimate geometric configuration.

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1. INTRODUCTION

Although brick masonry has been largely used in the past, there is no recognized theory which predicts the load-bearing capacity in terms of mechanical properties of the constituent materials and of the interface bond strength. Brick masonry may be considered a "composite material" in which the constituents are bricks and joint mortar. The behaviour of a load-bearing wall may be represented by a "beam-column" strut. A crack in the interface brick mortar reduces the flexural rigidity of the beam-column and its load-carrying capacity. It will be assumed, in this paper, that cracks occur in the interface between bricks and mortar, i.e. in the weakest link. Although the toughness of the interface has been considered by several Authors [2, 3], the concept of the "fictitious crack model" (F.C.M.), as introduced by Hillerborg et al. [1] and recently developed by Løland [4], will be used in order to analyse the partially cracked sections.

In order to predict the behaviour of masonry under eccentric load, a procedure of successive corrections to the equilibrium and compatibility equations was formulated. A computer programme based on the finite difference technique gives, for assigned values of the load and eccentricity, the deflections in a number of sections along the strut, taking into account the geometric non-linearity. The procedure is repeated until two sets of deflections converge. Material failure occurs when the internal force cannot be increased in order to balance the applied forces. In section 2 the fictitious crack model and the cracking of brittle materials are discussed. In section 3 the basic relationships and the analysis of cracked sections are illustrated. In section 4 the particular case of a two-hinged column with a constant initial load eccentricity is studied. In section 5 an outline of the solution procedure is reported. Lastly, in section 6, an illustrative example is presented.

2. CRACKING IN NON-YIELDING MATERIALS

Generally the masonry wall behaves in one of three ways depicted in Fig. 1 depending on the axial eccentric force and on the height of the wall [6]. In case (a) the elastic solution occurs; in case (b) cracking takes place in the tension fibres, whereas in case (c) crushing occurs in the extreme fibres subjected to compressive stress. In Ref. [7] three different types of section crisis were considered: fracture crisis, crushing crisis and traction crisis.

In order to evaluate the distribution of stress over a cracked section, the fracture zone which develops in front of the crack tip should also be taken into consideration. It is well known that when a cracked speciment of linearly elastic material is subjected to load, the stress field near the crack tip has a singularity in the order of $1/\sqrt{r}$, where r measures distance from the tip of the crack (see solid line in Fig. 2).

In the case of real materials, the stress distribution in front of a crack tip is affected by a fracture

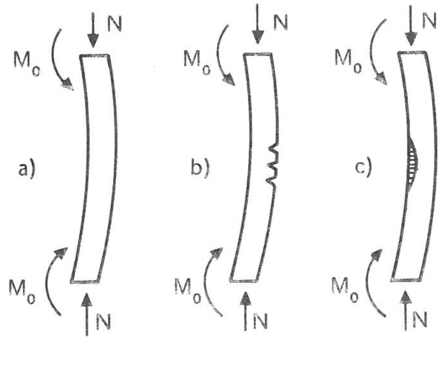


Fig. 1 - Modes of masonry behaviour [6].

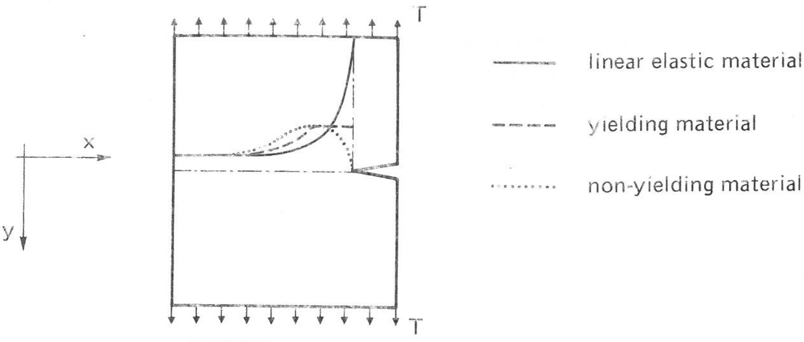


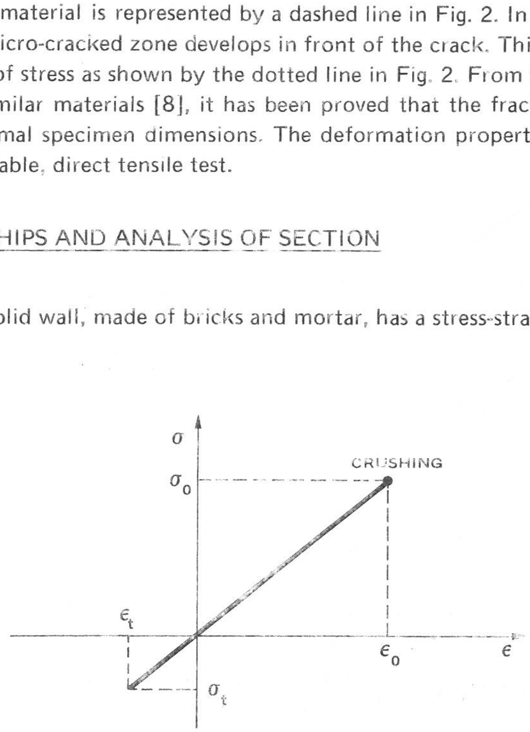
Fig. 2 - Probable σ_y stress distribution in front of a crack [8].

zone [8]. In the case of yielding material, plastic flow will take place at the crack tip and the probable stress distribution of this material is represented by a dashed line in Fig. 2. In the case of non-yielding materials, e.g. mortar, a micro-cracked zone develops in front of the crack. This reduces the concentration and the distribution of stress as shown by the dotted line in Fig. 2. From the strain measurements taken for concrete and similar materials [8], it has been proved that the fracture zone is never small when compared with normal specimen dimensions. The deformation properties of the fracture zone may be obtained from a stable, direct tensile test.

3. BASIC RELATIONSHIPS AND ANALYSIS OF SECTION

3.1 Basic relationships

It is assumed that a solid wall, made of bricks and mortar, has a stress-strain relationship as shown in Fig. 3.



$f = \sigma_t/\sigma_0 = \epsilon_t/\epsilon_0$
$f < 0$
$\lambda = \sigma/\sigma_0$

Fig. 3 - Assumed stress-strain relationship for the brick-mortar system.

Generally the bricks and mortar have different properties. Consequently, the assumed stress-strain relationship is to be taken as the unit average of the brick and mortar system.

Maximum tensile and compressive stress are denoted as σ_t and σ_0 respectively, and $f = \sigma_t/\sigma_0$. The dimensionless parameter $\lambda = \sigma/\sigma_0 = \epsilon/\epsilon_0$ can take any value within the interval $f \leqslant \lambda \leqslant 1$. It is suppose that the $\sigma - \epsilon$ relationship represents the properties of the brick-mortar system only outside the fracture zone.

3.2 Analysis of section

The load-carrying capacity of the cross section of the masonry wall, under eccentric axial loading,

may be determined if the stress distribution over the cross section at the point of failure is known. It is assumed that plane sections remain plane after deformation. The stress distribution of a typical cracked cross section, is shown in Fig. 4.

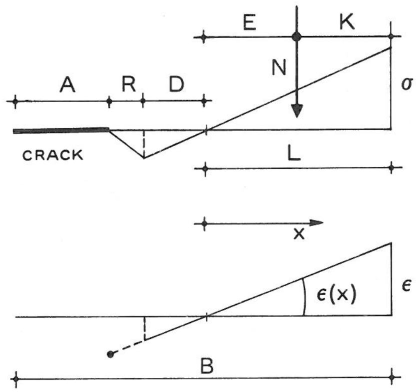


Fig. 4 - Cracked section of masonry under eccentric vertical load.

It may be observed that when tensile stress reaches its maximum value, a cross sectional area is not capable of carrying more load, and micro-cracks will develop inside a small volume of material or fracture zone. Then the stress on the material inside this fracture zone decreases elastically when strain is increased. In Fig. 4, A represents the length of the crack, R is the width of the fracture zone, E is the load eccentricity (defined with respect to the zero stress point), and K is the distance of N from the free edge.

The total force on the section, per unit of thickness, and the moment force around the neutral axis, can be expressed as: (see Fig. 4)

$$\frac{\sigma L}{2} - \sigma_t \frac{R+D}{2} = N \tag{1}$$

$$\frac{\sigma L^2}{3} - \frac{\sigma_t D^2}{3} - \frac{\sigma_t R}{2}\left(\frac{R}{3} + D\right) = EN \tag{2}$$

where $D = \sigma_t \, L/\sigma_0$. With the following non-dimensional quantities:

$$d = \frac{D}{B}, \quad \ell = \frac{L}{B}, \quad r = \frac{R}{B}, \quad f = \frac{\sigma_t}{\sigma_0}, \quad \lambda = \frac{\sigma}{\sigma_0}, \quad m = \frac{M}{B^2 \sigma_0}, \quad n = \frac{N}{B\sigma_0} \tag{3}$$

the above equations of equilibrium assume the following aspect:

$$\lambda \ell - f\left(r + \frac{f}{\lambda}\ell\right) = 2n \tag{4}$$

$$\frac{\lambda \ell^2}{3} - \frac{f^3 \ell^2}{3\lambda^2} - \frac{fr}{2}\left(\frac{r}{3} + \frac{f\ell}{\lambda}\right) = m \tag{5}$$

where $m = n(\ell - k)$, in which $k = K/B$; the constants f, r depend on the material, whereas n, k depend on the loading. By combining equations [4] and [5], that are non-linear in the λ, ℓ unknowns, in order to eliminate parameter ℓ, the following is obtained:

$$g(\lambda) = C_4\lambda^4 + C_3\lambda^3 + C_2\lambda^2 + C_1\lambda + C_0 = 0 \tag{6}$$

where:

$$
\begin{aligned}
C_0 &= -2f^3(2n+rf)^2 - r^2f^5 + 3f^4r(2n+rf) + 6nkf^4 \\
C_1 &= 6nf^2(2n+rf) \\
C_2 &= f^2[2fr^2 - 3r(2n+rf) - 12nk] \\
C_3 &= (2n+rf)(2rf-2n) \\
C_4 &= 6nk - r^2f
\end{aligned}
\tag{7}
$$

Equation (6) was solved for λ using a computer. In the particular case of zero tensile stress ($f = 0$), the solution is $\lambda = 2n/3k$. This value of λ has been used in order to define the end-points of the interval, which contains one zero of the real valued function $g(\lambda)$.

4. ANALYSIS OF STRUT

The procedure used in this paper is based on the finite difference method. The simple case of the two-hinged column with a constant initial eccentricity, was considered. The strut (Fig. 5) is divided into 20 parts of equal length "h". In an arbitrary section, the deflection is y_i. The curvature, χ_i, at this section may be approximately calculated as follows:

$$\chi_i = -\frac{d^2y}{dz^2} = -\frac{y_{i+1} - 2y_i + y_{i-1}}{h^2} \tag{8}$$

Since the curvature will be different at stations 1 — 10, the following system of equations was obtained:

$$[A]\{y\} = \{b\} \tag{9}$$

where:

$$[A] = \begin{bmatrix} 2 & -1 & & & & & & & & \\ -1 & 2 & -1 & & & & & & & \\ & -1 & 2 & -1 & & & & & & \\ & & -1 & 2 & -1 & & & & & \\ & & & -1 & 2 & -1 & & & & \\ & & & & -1 & 2 & -1 & & & \\ & & & & & -1 & 2 & -1 & & \\ & & & & & & -1 & 2 & -1 & \\ & & & & & & & -1 & 2 & -1 \\ & & & & & & & & -2 & 2 \end{bmatrix} ; \quad \{y\} = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \\ y_5 \\ y_6 \\ y_7 \\ y_8 \\ y_9 \\ y_{10} \end{bmatrix} ; \quad \{b\} = \begin{bmatrix} \chi_1 \\ \chi_2 \\ \chi_3 \\ \chi_4 \\ \chi_5 \\ \chi_6 \\ \chi_7 \\ \chi_8 \\ \chi_9 \\ \chi_{10} \end{bmatrix} h^2 \tag{10}$$

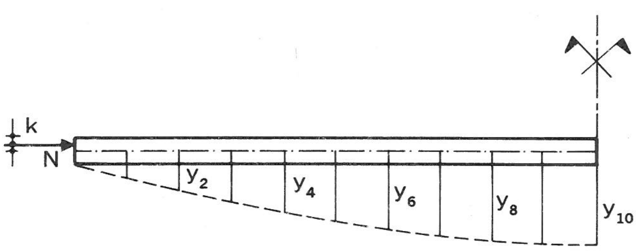
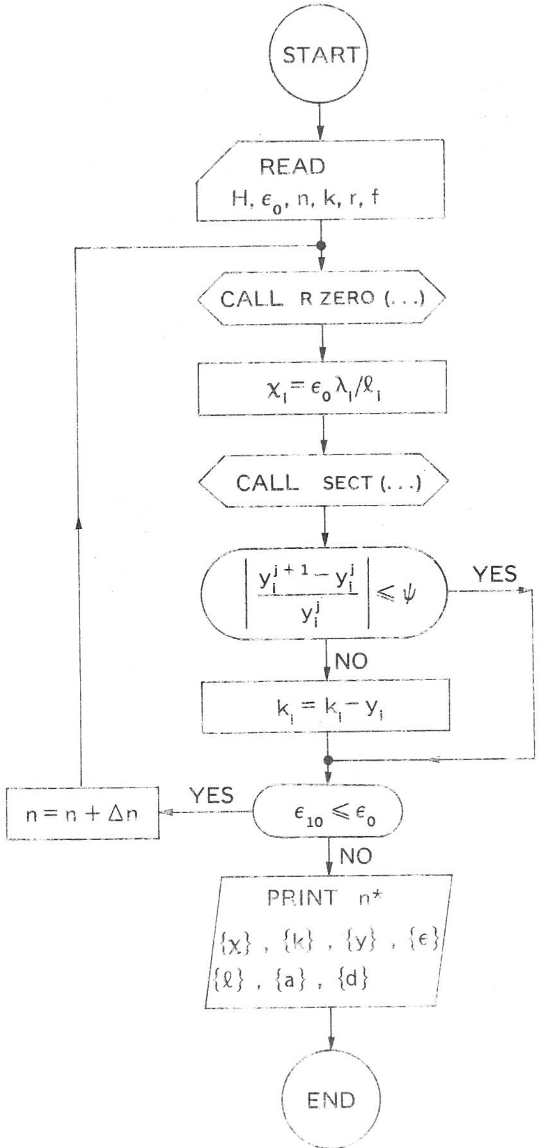


Fig. 5 - Division of strut into 20 segments.

Once the displacements $\{y\}$ have been determined, it is possible to obtain the deflection profile of the strut. It will be noted that on deflection, geometrical non-linearity results since load eccentricity changes at every station; this, is known as the "second order effect". In order to take the geometrical non-linearity into account, a process of successive correstions is required.

5. NUMERICAL SOLUTION

The procedure used is schematically illustrated in Fig. 6. In order to find out if a state of equilibrium is possible for a given value of the axial load, "n", and a given eccentricity, "k", an iterative pro-



COMMENTS

See notation (APPENDIX)

Compute λ, ℓ satisfing the equilibrium equations (4) - (5).

Compute curvatures at each station.

Compute $\{y\} = [A]^{-1}\{b\}$

Convergence test.

New-values of eccentricity.

Strain test and load increments.

n^* = dimensionless critical load
$\{\chi\}$ = vector of curvatures
$\{y\}$ = vector of deflections
$\{\epsilon\}$ = vector of strains
$\{k\}$ = eccentricity vector
$\{a\}$ = crack length vector
$\{d\}$ = vector of traction part of section
$\{\ell\}$ = vector of compressive part of section

Fig. 6 - Flow chart for analyzing unreinforced load-bearing masonry walls.

cess can be started by assuming that curvatures χ_i , at all positions, are the same, as determined from the following equation:

$$\chi_i = \epsilon_0 \lambda / \ell \tag{11}$$

where i = 1, 2, . . . , 10. Then, equation (6) is solved and deflections $\{y\}$ at the ten stations are obtained according to equation (9). So, in the arbitrary section, the eccentricity k_i is modified :

$$k_i = k - y_i \tag{12}$$

The equation (6) is solved with eccentricity k_i in each section. This leads to new curvatures $\{\chi\}$ from equation (11) and new deflections $\{y\}$ from equation (9). Once convergence has been achieved, the maximum strain of the middle section is compared with ϵ_0. Then load N is increased, in order to obtain a new state of equilibrium, until $\epsilon_{10} = \epsilon_0$, which is the limit value of N for the value of eccentricity considered. Using this procedure, numerical results for a set of strut geometries, are obtained.

Figure 7 shows the dimensionless ultimate load $n^* = N/B\sigma_0$ versus slenderness ratio H/B, for the load eccentricity k = K/B = 0.2. Two different types of brickwork, of different strengths were considered.

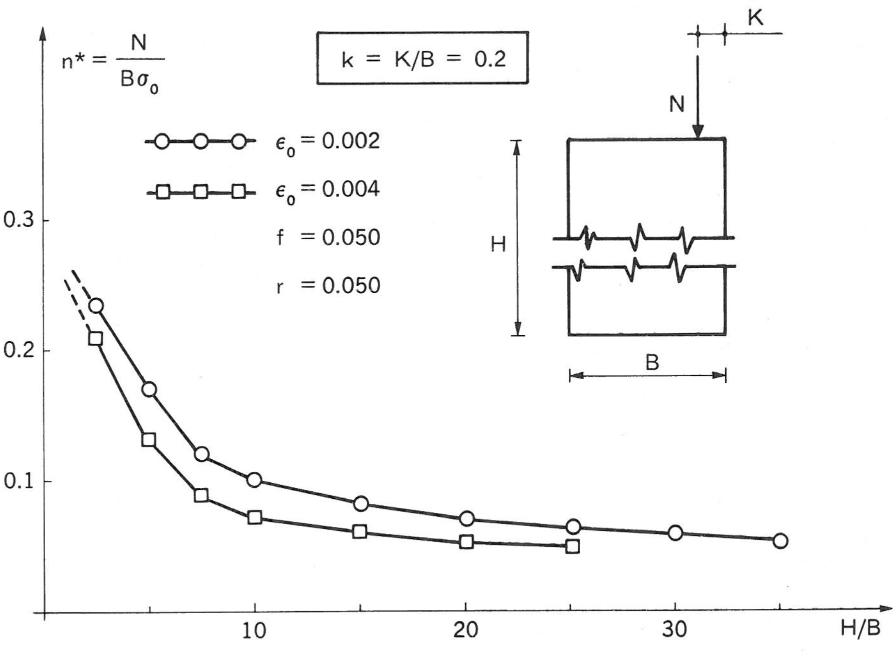


Fig. 7 - Slenderness ratio effect on the dimensionless ultimate load.

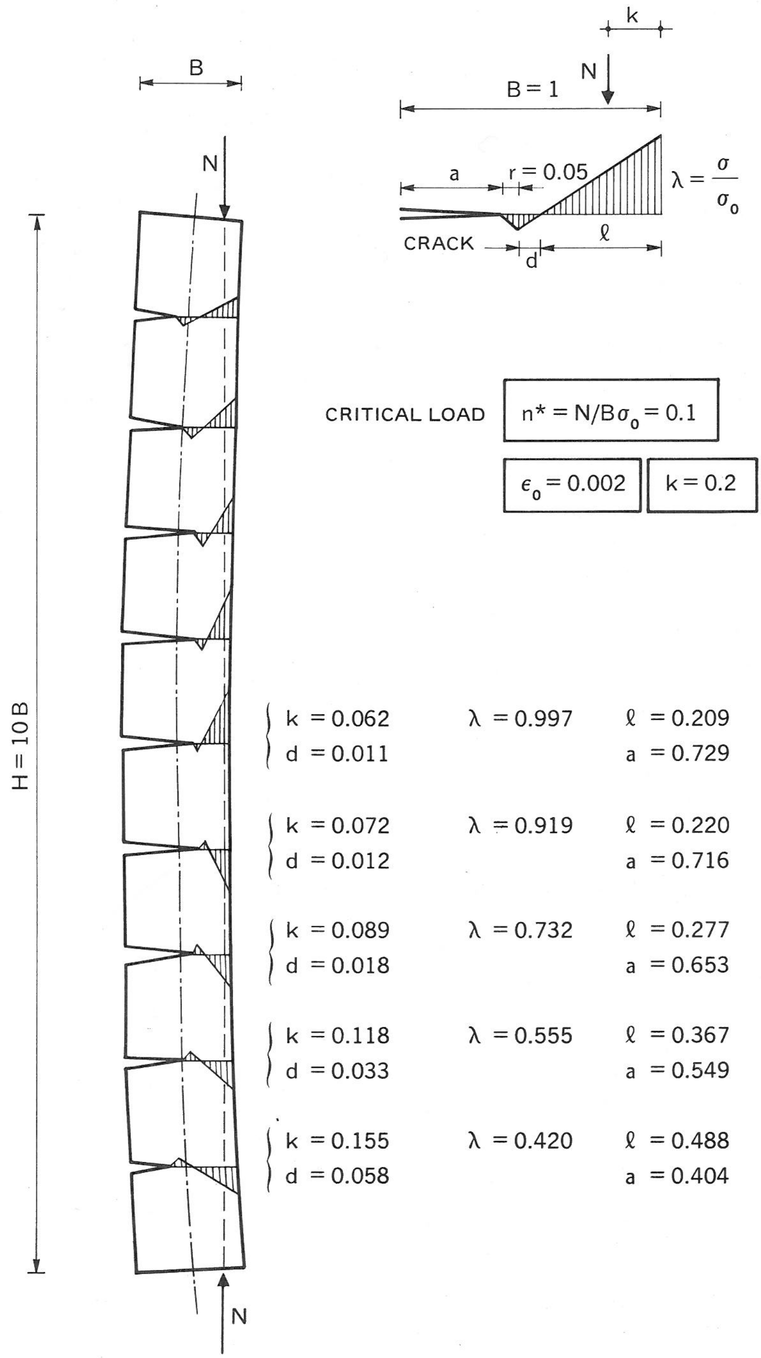


Fig. 8 - Limiting values of eccentrically loaded strut.

6. ILLUSTRATIVE EXAMPLE

Let us consider the simple supported wall of unit width $B = 1$ and eight $H = 10B$ shown in Fig. 8. The line of application of the external load N is fixed at $k = 0.2$ from the free edge. In order to find the dimensionless critical load $n^* = N/\sigma_0 B$ and the ultimate geometric shape of the column, $\epsilon_0 = 0.002$ is assumed for the brick-mortar system. The computer programme developed gives $n^* = 0.1$ and also the numerical results presented in Fig. 8., which reproduce the crack configuration and the distribution of stress in each cracked section.

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APPENDIX I

The following symbols are used in this paper:

K	=	distance of N from free edge
N	=	eccentric applied load
B	=	total depth of cross section
k	=	K/B
A	=	length of cracked part of section
a	=	A/B
R	=	dimension of the process zone
r	=	R/B
L	=	depth of the compressive cross section
ℓ	=	L/B

σ_t, σ_0	=	tensile and compressive strength
f	=	σ_t/σ_0
n	=	$N/\sigma_0 B$ dimensionless axial load
m	=	$N/B^2\sigma_0$ dimensionless bending moment
y	=	lateral deflection
χ	=	curvature
E	=	load eccentricity from zero stress axis
e	=	E/B
X	=	distance from neutral axis
D	=	traction part of section
d	=	D/B
H	=	total height of the wall
n*	=	dimensionless ultimate load
ϵ, ϵ_0	=	current and maximum strain
σ	=	current stress
λ	=	$\epsilon/\epsilon_0 = \sigma/\sigma_0$
$\{ \}$	=	column matrix
[ ]	=	square matrix
$[\ ]^{-1}$	=	inverse of a matrix