Characterization of Brick Masonry Stiffness by Numerical Modelling and in situ Flat-Jack Test Results

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Abstract- A large scale, non destructive, in-situ test for determining the mechanical properties and the stress state in masonry walls has recently been developed at ISMES. The technique is based on inserting flat-jacks between the brick courses. The relative displacements of various points located on the wall surface close to the flat-jacks are measured as the pressure is increased.

In the paper a numerical simulation procedure is presented. The purpose is to determine the material property values which minimize the discrepancies between computed and measured displacements. Masonry is modelled as an elastic orthotropic material. In order to reduce the number of independent unknowns in the characterization procedure a method of relating the overall orthotropic stiffness coefficients of masonry to the properties of bricks and mortar is developed.

1. INTRODUCTION

Structural analysis of masonry buildings requires masonry to be modelled as a homogeneous material.

It is however quite difficult to obtain mechanical properties for actual masonry. In principle it would be sufficient to take an undisturbed sample of masonry, of adequate size, and apply standard laboratory tests, disregarding the intrinsic nonhomogeneity of the material. In practice such an obvious procedure cannot always be applied. Typically it is usually impractical to obtain large samples of the masonry of existing buildings (because of the high cost, of the structural risk or of the respect due to ancient historic monuments).

The problem has motivated the development and successful application of a new non destructive in-situ large scale testing technique. The new testing procedure is based on the application of two flat jacks, inserted between brick courses. The masonry deformations under known pressure loadings are measured in order to derive all needed information on the mechanical properties of masonry. Since two mortar layers only are destroyed, the procedure may be ap-

plied at all points of interest to the structural engineer without the restraints due to aesthetic, cultural, economical or technical considerations.

In the paper the testing procedure is sketched very briefly. A complete treatment is available elsewhere [1] . The paper is centred on a computer procedure to be used for interpretation of the experimental results.

The interpretation is based on the comparison between in-situ deformations and those computed by a three dimensional finite element model of a masonry portion around the flat-jacks. In the computations the masonry is modelled as an equivalent orthotropic elastic continuum.

There are two reasons for anisotropic behaviour of brick masonry. The first one is simply the texture of the two component materials. As brick and mortar layers are located at regular intervals along three mutually orthogonal directions, an orthotropic material model seems very appropriate for large scale behaviour of masonry. The second reason is that bricks often exhibit a markedly orthotropic behaviour, due to the extrusion and lamination processes. In the following anyway both mortar and bricks are considered isotropic. In fact we have in view the application of the proposed technique to old buildings, where molded bricks are used.

Orthotropic materials are characterized by 9 independent elastic coefficients. The identification of such a large number of unknowns is computationally expensive and error prone because of the limited amount of experimental data generated by a single test. On the other hand the assumption that brick masonry behaves isotropically may be unnecessarily crude. The paper presents an analytical procedure which yields the 9 coefficients of orthotropic elasticity as a function of the material properties of bricks and mortar and of the thickness ratios of mortar and brick layers in the three principal directions of orthotropy.

Since for a given masonry wall the thickness ratios can be very easily measured, the parametric search depends only on the elastic moduli and the Poisson's ratios of mortar and bricks. In practice considerable changes in the value of the Poisson's ratio have a limited effect on the results, therefore the elastic moduli of mortar and bricks may be assumed as the primary unknowns to be identified from the experimental data. Since the interaction between mortar and bricks is too complex to be taken into account exactly, the moduli determined from the experimental data should be considered only as "equivalent" moduli which are likely to be influenced by the thickness ratios between mortar layers and bricks. Once the equivalent moduli of mortar and bricks are determined, the above mentioned analytical procedure yields the desired equivalent orthotropic material stiffness of masonry. It is expected that the error in the equivalent masonry stiffness be much smaller than the error in the elastic moduli of mortar and bricks.

2. TESTING PROCEDURE

The in-situ testing procedure is comprised of two stages: a) determination of the state of stress in the masonry; b) determination of strength and stiffness characteristics.

In the first stage of the test a horizontal cut (wide 40 cm, deep 20 cm and high 1,5 cm) is made in the masonry wall by removing a mortar layer

between two bricks courses.The consequent stress redistribution in the masonry determines a partial closure of the cut, which is monitored by measuring the relative displacements of previously established reference points on the wall surface.Then a flat-jack is inserted in the cut and an increasing pressure is applied until the reference points go back to their initial location. This procedure provides the local average vertical stress. Occasional discrepancies in the reference points locations are caused by local collapse of the masonry during unloading or to difficulty of restoring exactly the initial distribution of vertical pressures.

In the second stage another flat-jack is inserted about 50 cm above or under the first one. The two jacks, connected in parallel to an hydraulic pump, undergo several loading/unloading cycles. At regular load intervals, the relative displacements of all reference points are measured.

Although the flat-jacks pressure is not increased up to the level of masonry collapse, the strength of the masonry can be estimated by extrapolation from the load deformation diagrams.

The stiffness of the masonry can be determined in two ways. First a physical model, with known material properties, may be used to calibrate the testing procedure. This approach is illustrated by Rossi in a companion paper presented to this conference [1] . The second approach, namely interpretation by computer simulation, is dealt with here.

The above mentioned testing procedure has been devised and first applied in view of the static restoration of the Palazzo della Ragione, an old masonry building in Milan [2] . The mechanical properties of the masonry of external walls was needed for finite element analysis of the deformations caused by differential foundation settlements.

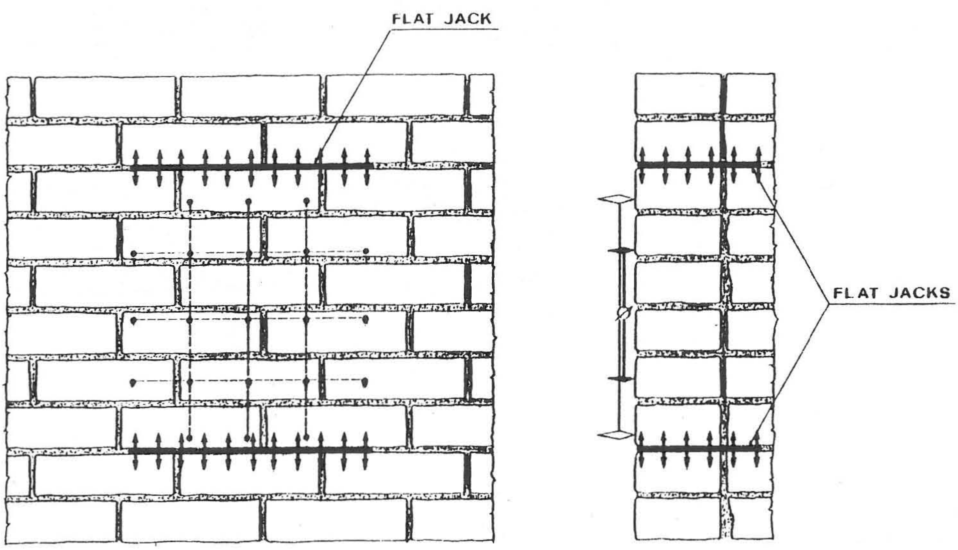

FIG.1 - Flat-jack test: scheme of the 2nd stage

3. EQUIVALENT MASONRY STIFFNESS MODEL

The "micro-scale" state of stress of masonry under vertical loads is three dimensional because of the different deformability of the component materials [3]. The lateral deformation of mortar is usually restrained by shearing forces applied by the adjacent stiffer bricks. Therefore mortar is in compression in all the principal directions, while bricks are subjected to vertical compression and horizontal tractions. At a finer level of detail the state of stress is very complex because the transfer of shearing forces from bricks to mortar determines a complicated stress state which depends on the relative stiffness of the two materials and on the geometric pattern of masonry. Hence it is very difficult, if at all possible, to relate the exact material properties of the component materials to the exact "large scale" stiffness of masonry [4]. The aim here is different because the exact value of the elastic modulus of mortar and bricks is not seeked. The aim is identification of large scale properties of masonry and it is desired to exclude from the parametric search the possible orthotropic material stiffness matrices which are not compatible with the known microstructure of masonry. In fact masonry is a special orthotropic material comprised of only two isotropic components arranged at regular and known geometric intervals.

Therefore the 9 stiffness coefficients of orthotropic elasticity will be related to other 9 coefficients: 3 parameters determined by the geometric pattern of masonry and mortar texture, and 3 material constants for each of the two component materials. The three geometric parameters are the three ratios between the brick dimensions a_i, $i = 1,2,3$ and the corresponding thicknesses of mortar layers t_i, $i = 1,2,3$. The material constants are obviously the Young's moduli (E), the Poisson's ratios

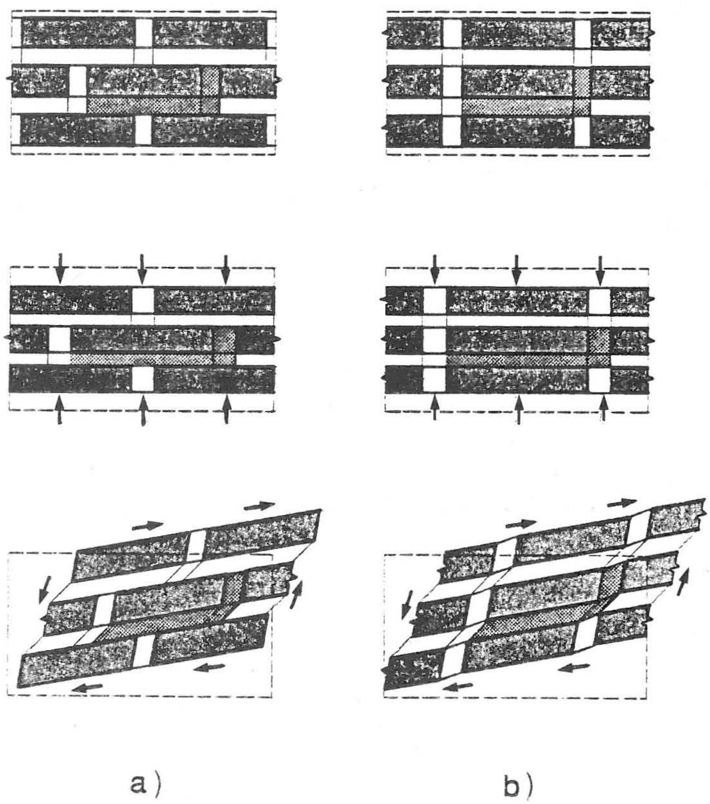

FIG.2 - Deformation of brick masonry in compression and in shear: a) Real masonry texture; b) Fictitious masonry texture.

(ν) and the shear moduli (G) of mortar and bricks. In this paper only the Young's moduli are varied because the Poisson's ratios have a limited impact on the final results and their value is assumed a-priori. Moreover the shear moduli are dependent variables because of the isotropy of the component materials and are computed as:

$$G = \frac{E}{2(1+\nu)} \tag{1}$$

In conclusion the Young's moduli of mortar and bricks are the only independent variables to be identified in the following. As discussed earlier they should be viewed as equivalent moduli. In order to determine the equivalent masonry rigidity it is necessary to isolate a repetitive module in the masonry pattern. Here we choose a single brick supplemented by layers of mortar on three orthogonal faces. This module is considerably more complex than the ones used by other authors [5,6,7]. Still such a simple building module does not distinguish between the real masonry pattern (fig.2a) and the fictitious one indicated in fig.2b. On the other hand the two geometric patterns lead to the same large scale behaviour, as long as axial stiffness is considered. Of course a discrepancy may occur for the shearing stiffness because real masonry behaves more like a horizontally stratified material. In fact shearing deformation is mainly due to the mortar layers and the mortar between bricks is restrained by the higher shearing rigidity of the bricks. The equivalent shearing stiffness for a stratified material is already available in the technical literature [8]. The shearing stiffness provided by the model proposed here is a lower-bound. The real stiffness is certainly higher, but still lower than the value provided by a stratified material model. As indicated in the exploded view (fig.3) the reference moduli can be viewed as comprised of 8 blocks. One of the blocks is the brick, the other are portions of the mortar layers determined by the planes of the brick faces inside the reference module.

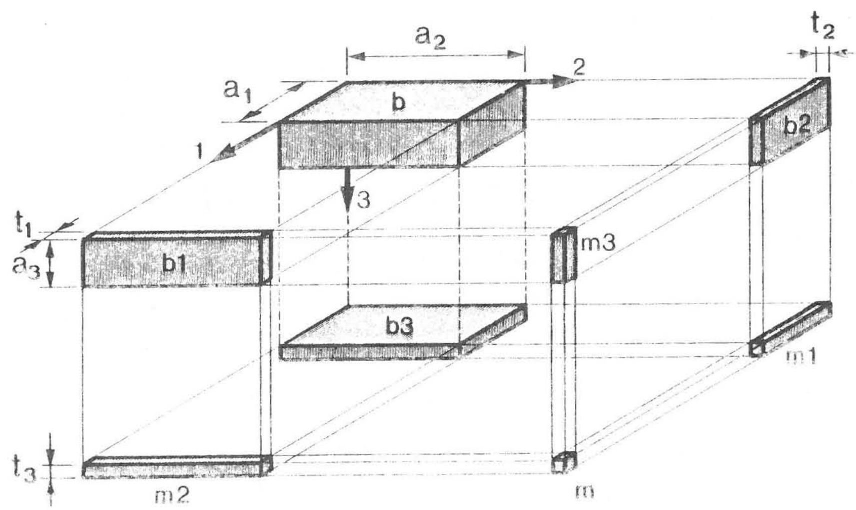

FIG.3 - Decomposition of the repetitive module into one brick (b) and 7 mortar blocks.

The following assumptions are made:

1. Each block is in a state of constant equivalent strain. The strain is called "equivalent" because the actual strain and stress state is more complex, as discussed earlier.

2. Compatibility of equivalent strains of the various blocks is enforced exactly.

3. The equivalent strain of each block is related to a constant state of average stress by the above-mentioned equivalent properties of brick and mortar.

4. Normal and shearing forces applied to the module are in equilibrium with the resultant of the internal forces supplied by the blocks.

5. Equilibrium of resultant normal and shearing forces is also enforced across the three internal planes which generate the internal subdivision in 8 blocks.

The above procedure may be applied independently to shear stresses and strains and to normal stresses and strains. In fact the two sets of equations are uncoupled. This is due to the orthotropy of the equivalent material stiffness matrix. The equations corresponding to the above assumptions are not given in detail here [9].

3.1 Normal stiffness of masonry

Let us first consider normal strains and stresses. There are 24 constitutive equations, 18 compatibility equations and 6 equilibrium equations. The unknowns are the 24 normal stress components (three for each block) and the corresponding 24 normal strains. Compatibility equations for elongations in the i-th direction simply state that the four blocks located at the same distance from the j-k plane have the same elongation (three equations). Six such equations may be written for each direction, as two groups of four blocks each are generated by each of the above-mentioned internal planes.

The deformation of the repetitive module is governed by the 6 axial strains of the brick (b) and of the little mortar block (m). The axial strains of the other 6 mortar blocks are eliminated by means of the compatibility equations.

Therefore the average strain of the module is computed by means of the strains of the brick (b) and of the mortar block (m):

$$\varepsilon_i^+ = \frac{a_i\ \varepsilon_i^b + t_i\ \ \varepsilon_i^m}{a_i + t_i} \qquad (2)$$

The six unknown strain components can be determined by means of the equilibrium equations. For instance the two equilibrium equations for the i-th

direction are:

$$\sigma_i^+ \ell_j \ell_k = \sigma_i^b a_j a_k + \sigma_i^{bj} t_j a_k + \sigma_i^{bk} a_j t_k + \sigma_i^{mi} t_j t_k \tag{3a}$$

$$\sigma_i^+ \ell_j \ell_k = \sigma_i^{bi} a_j a_k + \sigma_i^{mk} t_j a_k + \sigma_i^{mj} a_j t_k + \sigma_i^m t_j t_k \tag{3b}$$

Where $l_j = a_j + t_j$ and $l_k = a_k + t_k$ are lengths of the sides of the module and superscripts denote the block according to the notation of fig.3. The six equilibrium equations are in general coupled. The coupling is caused by a nonzero Poisson's ratio in the stress/strain relationship for mortar and brick. In general the analytic treatment of the problem is quite complex and is omitted here for brevity [9]. Althought in the following we assume $\nu = 0.15$, the principal elastic moduli for the case $\nu = 0$ are presented here. They can be derived rather easily from the above equations:

$$E_i^+ = E_m \frac{\ell_i \ell_j \ell_k + (\rho-1)\ell_i a_1 a_k}{\ell_i \ell_j \ell_k + (\rho-1) t_i a_j a_k} \tag{4}$$

Where $\rho = E_b/E_m$ is the ratio of Young's moduli of brick and mortar. Equation (4) indicates that the anisotropy is more pronounced for high values of the brick Young's modulus, as expected. Moreover there is an upper limit to the brick masonry moduli determined by the thickness ratios:

$$E_i^{max} = E_m \frac{\ell_i}{t_i} \tag{5}$$

Analogous properties are exhibited by the moduli for Poisson's ratios larger than zero.

3.2 Shearing stiffness of masonry

Again there are 48 unknowns: the 3 shear stresses and the 3 shearing strains of the 8 blocks. The problem is simpler, however, because no coupling between different planes exists.

Let us consider shear deformations in the i-j plane (Fig.4). The first compatibility equation enforces continuity of deformed blocks at the internal point where all blocks meet. This requires the sum of the angular deformations to be zero:

$$\gamma_{ij}^b - \gamma_{ij}^{bi} + \gamma_{ij}^{mk} - \gamma_{ij}^{bj} = 0 \tag{6}$$

The deformation of the mortar layer of thickness t_k is completely governed by the deformation of the other larger

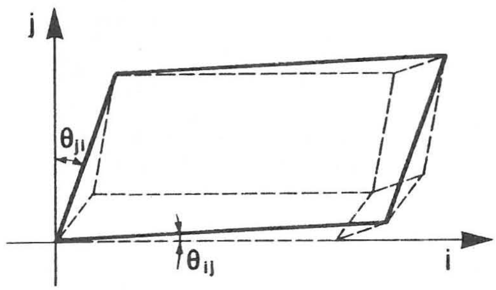

FIG.4 - Equivalent shear deformation of the module

blocks. This implies 4 additional compatibility equations.

Four additional equations result from enforcing equilibrium on the external surfaces of the repetitive module. A typical equation is:

$$\tau^{+}_{ij}\ \ell_j\ \ell_k = \tau^{b}_{ij}\ a_j\ a_k + \tau^{bj}_{ij}\ t_j\ a_k + \tau^{bk}_{ij}\ a_j\ t_k + \tau^{mi}_{ij}\ t_j\ t_k \tag{7}$$

Note the similarity with the traslational equilibrium equations. Here, however, only three equilibrium are independent, because of the overall rotational equilibrium of the module.

The solution of the above equation can be marked out explicitly. In order to deduce the large scale shearing stiffness of masonry it is also necessary to define the average shear strain γ^{+}_{ij} of the module. The shear strain is the sum of the two angles θ_{ij} and θ_{ji} indicated in fig.4. After some computation it turns out:

$$\gamma^{+}_{ij} = (\gamma^{b}_{ij}\ a_i\ a_j + \gamma^{bi}_{ij}\ t_i\ a_j + \gamma^{bj}_{ij}\ t_j\ a_i + \gamma^{mk}_{ij}\ t_i\ t_j)/\ell_i\ \ell_j \tag{8}$$

From the above results the average shearing modulus of masonry is:

$$G^{+}_{ij} = G_m\ \frac{\ell_i\ \ell_j\ \ell_k + (\sigma-1)(a_i\ a_j + a_i\ t_j + a_j\ t_i)\ a_k}{\ell_i\ \ell_j\ \ell_k + (\sigma-1)(a_i\ t_j + a_j\ t_i)\ a_k} \tag{9}$$

where σ is the ratio between the shear modulus of the brick and the shear modulus of mortar.

Equation (9) is a generalization of the formulas provided by Pinto for stratified materials. In fact assuming $t_i = t_j = 0$ or $t_i = t_k = 0$ we obtain respectively the shear modulus in the plane of the strata or the shear modulus in a plane normal to the strata.

As discussed earlier the above model is likely to understimate the shear modulus of masonry. Let the axis 1 to be horizontal in the plane of the wall, the axis 2 be horizontal normal to the wall and the axis 3 be the vertical one. The geometric pattern of masonry in the plane 2-3 is similar to the one indicated in fig.2b. Therefore equation (9) can be applied. In the other two planes the texture is as indicated in fig. 2a. Hence we resort to a different model. Let us assume for the moment a stratified continuum composed of alternate layers of shear moduli G_b and G_m and thickness a and t. The equivalent shear modulus (G*) in a plane normal to the strata is such that:

$$\frac{a+t}{G^{+}} = \frac{a}{G_b} + \frac{t}{G_m} \tag{10}$$

In practice a brick course is not a continuum stratum because of the mortar between adjacent bricks, therefore its stiffness must be reduced accordingly.

After some labor, it is found that the shear moduli in planes 1-3 and 2-3 should be computed as:

$$G^{+}_{ij} = G_m\ \frac{\ell_i\ \ell_j + (\sigma-1)\ \ell_i\ a_j}{\ell_i\ \ell_j + (\sigma-1)\ t_i\ a_j}. \tag{11}$$

4. NUMERICAL PROCEDURE FOR CHARACTERIZATION OF MECHANICAL PROPERTIES OF BRICK MASONRY

The non-destructive flat-jack test was developed and first applied to the mechanical characterization of the brick masonry walls of Palazzo della Ragione, a monumental building in Milan [2].

The building is comprised of a lower part of XIII century with marked cracks due to differential foundation settlements and of an upper part of XVIII century, where poorer masonry materials were used. During the last two centuries up to 1959 the building hosted the Notarial Archives of the town. The heavy load accelerated the deterioration of the building.

In view of the static restoration, an extensive testing program was conducted. The program included 6 flat-jack tests on the masonry. An accurate interpretation of test results can be based on the characterization procedure described in the following. Characterization or identification problems are well known [10]. Structural engineering applications are relatively new, however. See for instance [11, 12].

Two key ingredients are needed: a parametric numerical model and an error function to be used to select the values of the parameters that, when adopted in the numerical model, lead to computed values "as close as possible" to the experimental ones.

Here a 3-D finite element model of a portion of masonry surrounding the flat-jacks was prepared. Because of simmetry only one quarter of the problem was modelled using 1860 degrees of freedom. The model covered an area of 1,50x1,50 square meters. As the Poisson's ratio of both mortar and bricks was assumed 0,15 [5], two parameters only (E_m and E_b) had to be determined.

The error function can be selected simply as:

$$Z = \Sigma_i (u_i^c(E_b, E_m) - u_i^m)^2 \tag{12}$$

where:

u^m is the vector of measured displacements in the in-situ test

u^c is the vector of displacements calculated in the model

On the basis of the assumption of linear elastic behaviour, it can be observed that if vector $\hat{u}^c$ represents the calculated displacements corresponding to a pair of values (E_m, E_b) such that $\sqrt{E_b^2 + E_m^2} = 1$, $\hat{u}^c/\alpha$ represents the calculated displacements corresponding to the elastic moduli αE_m and αE_b. Hence the error function can be expressed in the following way:

$$Z = \Sigma_i \left(\frac{1}{\alpha} \hat{u}_i^c(\rho) - u_i^m \right)^2 \tag{13}$$

where, as earlier, $\rho = E_b/E_m$.

For a given ratio $\bar{\rho}$, the local minimum of the quadratic error function Z in the domain (E_b, E_m) can be found very easily observing that:

$$(14) \qquad \alpha = \frac{\Sigma_i (\hat{u}_i^c(\bar{\rho}) \cdot u_i^m)}{\Sigma_i (\hat{u}_i^c(\bar{\rho}))^2}$$

The search for the absolute minimum value of Z can be performed in a parametric way by choosing a suitable number of ratios ρ , along which a local minimum is calculated.

Due to the different order of magnitude of vertical and horizontal displacements, two indipendent minimizations where performed.

After averaging measurements simmetric with respect to center lines, 4 independent vertical displacements and 4 independent horizontal displacements were obtained for each position in which the flat-jack test was performed. If one tries and applies the characterization procedure using the 4 vertical displacements only, a case of non identifiability arises.

A 'locus' is obtained in the (E_b, E_m) domain characterized by almost the same minimum value of the error function (line A of fig.5).

This is not surprising as the same average vertical modulus can result from different combinations of elastic moduli of the component materials.

Using as measured displacements the horizontal ones only, a second 'locus' of minima is obtained (line B of fig.5). The intersection of lines A and B corresponds to the pair of moduli E_b and E_m minimizing the error functions of both vertical and horizontal displacements. It is interesting to observe the trend of the coefficients of the elasticity matrix corresponding to the points marked on line A and line B. It can be noted from Table I that, even though E_b and E_m are not separately identifiable, the value of the average vertical stiffness E_3 is practically constant for all the points of line A.

The same considerations can be extended to the average horizontal stiffness E_1 corresponding to the points of line B. The intersection of lines A and B provides the best fitting of all measured deformations.

5. CONCLUSIONS

The theoretical model for stiffness of brick masonry developed in this paper is very effective for identification problems, where the number of unknown parameters must be kept to a bare minimum.

So far no attempt was made to correlate the computed equivalent moduli E_b and E_m with the real Young's moduli of bricks

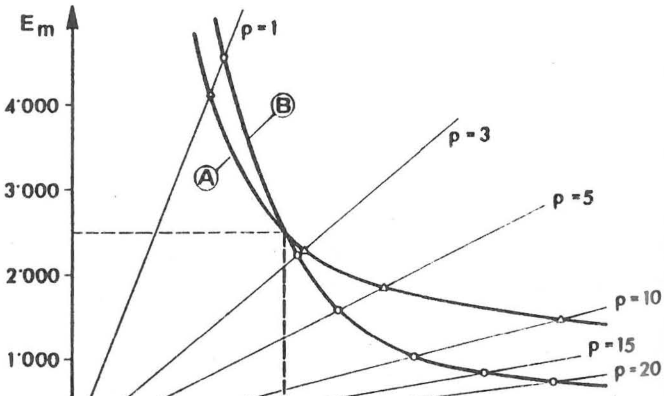

FIG.5 - Parametric identification of equivalent mortar and brick moduli from vertical (A) and horizontal (B) deformations.

TABLE I - Material constants appearing in the orthotropy matrix for line A

E_b/E_m	E_1 (MPa)	E_2 (MPa)	E_3 (MPa)	G_{12} (MPa)	G_{23} (MPa)	G_{31} (MPa)	ν_{12}	ν_{23}	ν_{31}
1	4167	4167	4167	1811	1811	1811	0,150	0,150	0,150
3	4694	4464	4237	2344	1710	1942	0,144	0,139	0,130
5	5376	4784	4291	2721	1623	1946	0,135	0,125	0,109
10	6711	5291	4347	3351	1510	1920	0,118	0,103	0,078
15	7692	5586	4385	3747	1455	1899	0,105	0,091	0,062
20	8403	5780	4397	4021	1423	1884	0,095	0,083	0,053

and mortar. The computed values are likely to overestimate the real ones as in the theoretical model local equilibrium is violated. Shear stresses cause also local axial strains and axial stresses cause also local shear strains, particularly at the interface between different materials.

A possible source of errors in the characterization procedure is the presence of creep effects which may develop during the experimental tests [6].

As a general recommendation for good characterization results it is suggested that the portion of masonry to be tested and the number of displacement measurements should be as large as possible. Inclined displacement measurements (to be interpreted separately) would yield a third independent locus. This could allow a better definition of the intersection point and an indication of the scatter of the identified parameters.

An extensive testing programme using well known component materials has to be developed to assess the validity of the numerical characterization procedures here described and to evaluate the confidence limits of the results.

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