The Calculation of the Load-bearing Capacity of Brick Masonry with Reinforced Network Subject to Compression

CHEN XINGZHI and SHI CHUXIAN
Dept. of C. E. , Hunan University, Changsha, Hunan, China

Abstruct - Basèd on the theoritical analysis of the experimental results of more than 160 test prisms, this paper gives a comprehensive discussion of the compressive behavior of brick masonry with different ratio of reinforcement and varied eccentricity of loading. By using the probability based limit states design methods, the reliability of the brickwork with reinforced net subject to compression is calibrated. Thus the calculation of this type of brickwork construction is further perfected.

1. INTRODUCTION

The laboratory research work done as described in this paper is proposed by the Revision Group of our "National Design Code for Masonry structures" for the purpose of revising the said code. In this report the strength characteristics of the brickwork with reinforced net is clarified and new calculation formulae with explainations for brickwork with reinforced net subject to compression are established in the revised national code. Further, on the basis of revision of the existing calculation formulae and according to the theory of the probability based limit states design, the resistance factor in the expression for the design of coefficient of separate terms is proposed.

2. TESTS AND RESULTS

No.75 bricks with No.25 and No.50 mortars are used for the making of the test prisms. Cold drawn $\phi^b 4$ low carbon steel wire nets are horizontally placed in the mortar joints of the prisms from one to five layers of bricks apart. Ten different volume ratios of reinforcement (μ) are used in two sizes of prisms: 24x37x72 cm. and 37x49x100 cm.

When the test prisms were compressed, the first series of cracks soon appeared, followed by more cracks and finally the test prisms failed. The ratio of the load producing the first series of cracks to the ultimate load is about 0.50 to 0.86 , the average value is 0.67 when μ = 0.067 - 0.334%, the load producing the first series of cracks is higher than the same prism but without reinforcement. when the value of μ = 0.355 - 2.0% the ratio is decreased to 0.37 - 0.59 (average = 0.47), in this case, the load producing the first series of cracks is much lower than prisms without reinforcement.

A part of the test results of prisms subject to axial and eccentric compressive stresses are tabulated in Table 1 and Table 2 (where e_0 being the eccentricity, y being the distance between the centroid of the section and the less compressed side of the prism).

3. THE CALCULATION OF THE LOAD-BEARING CAPACITY

When the brickwork is subjected to vertical compressive load vertical as well as transversal deformations are produced. When a certain amount of reinforced network is placed horizontally in the mortar joints of the brickwork, the transversal deformation of the latter is then restrained due to the combined action of the two materials, thus

the compressive strength of the brickwork is increased. It may be said that this is the fundamental working characteristics of the brick masonry with reinforced network.

3.1. Axial Compression

Under the action of the compressive force N on the brickwork with reinforced net as shown in Fig.1, the vertical deformation is $\varepsilon = \sigma/E$, and based on the Poisson's ratio $\lambda = \varepsilon_n/\varepsilon$, then the corresponding transversal deformation of the prism is $\varepsilon_n = \sigma_n/E = \lambda\sigma/E$. Thus σ_n is like the pressure of a liquid acting on the walls of the prism as diagrammatically shown in Fig.1 and produces axial tension Z_a and Z_b within the prism which are resisted by the transversal reinforcement in the prism.

According to "Strength of Materials":

$$Z_a = \frac{\sigma_n b}{2} S_g \quad , \qquad Z_b = \frac{\sigma_n a}{2} S_g \quad .$$

The total tensile strength is:

$$2 (Z_a + Z_b) = \lambda\sigma (b + a) S_g \qquad (1)$$

When the network is made of wires with cross sectional area equal to a_g, and strength equal to R_g, the size of the square opening equal to C_g and the vertical distance between the network equal to S_g, the total tensile strength which can be sustained by the transversal reinforcement is

$$\left(\frac{b}{C_g} a_g + \frac{a}{C_g} a_g \right) R_g = \frac{a_g}{C_g} (b + a) R_g \qquad (2)$$

As (1) = (2), therefore: $\sigma = \frac{a_g}{\lambda C_g S_g} R_g$ (3)

By transformation we get: $\sigma = \frac{1}{2\lambda} \frac{\mu}{100} R_g$, where $\mu = \frac{2 a_g}{C_g S_g} 100$.

Based on our "National design code of reinforced concrete structures" (TJ10 - 74), let $\lambda = 0.25$, then $\sigma = \frac{2\mu}{100} R_g$ This value of σ is the increased strength of the brickwork after the placement of the transversal reinforcement in comparison with those without reinforcement. Let R be the strength of the latter, then $\sigma = \Delta R$. Thus the strength of the brickwork with reinforced net subject to axial compression is

$$R_p = R + \Delta R = R + \frac{2\mu}{100} R_g \qquad (4)$$

and the formula for the calculation of its load-bearing capacity is

$$K N_p \leqslant \left(R + \frac{2\mu}{100} R_g \right) A \qquad (5)$$

A part of the experimental value N' and the calculating value N are shown in Tab.1.

Based on the experiments, the relationship between R_p/R and μ can be expressed by the linear equation (see Fig.2)

$$R_p/R = 1 + 3\mu \qquad (6)$$

When $\mu = 1\%$, then $R_p/R = 4$, but when $\mu \geqslant 1\%$, the points representing the results are all under the straight line. This means that

the development of the strength of this type of brickwork is limited when the value of μ is more or less greater than 1% . For the sake of safety and economy, the value of $R_p/R \leqslant 4$ is recommended

3.2. Eccentric Compression

When the prism is eccentrically loaded, the values of the stresses in the reinforcement within the compressive area do not vary accordingly with the eccentricity e_o . The value of R_g is still approximately equal to the strength used in the tensile design as specified in the design code. But as e_o increases, the area of compression decreases, the effect of the reinforced net to prevent the transversal deformation is reduced, therefore the value of ΔR should be multiplied with a reduction factor η . Let $\eta = 1 - e_o/y$ then

$$\Delta R = \left(1 - \frac{e_o}{y}\right) \frac{2\mu}{100} R_g \tag{7}$$

According to "Strength of Materials", the formula for the calculation of the load-bearing capacity for eccentric compression is

$$N \leqslant \frac{AR}{1 + \frac{e_o y}{r^2}} = \alpha' AR \tag{8}$$

$$\alpha' = \frac{1}{1 + \frac{e_o y}{r^2}} \tag{9}$$

where r = the radious of gyration of the cross section.

This value of α' may be called the factor of eccentric effect when the formula in "Strength of Materials" is used for the calculation. As the materials used in brick masonry construction possess a definite elastic and plastic properties and its tensile strength is very low, a correction factor $\omega\, e_o/y$ should be applied to the term $e_o y/r^2$ in (8), thus

$$N_p \leqslant \frac{AR}{1 + \omega\left(\frac{e_o}{r}\right)^2} = \alpha_p AR \tag{10}$$

$$\alpha_p = \frac{1}{1 + \omega\left(\frac{e_o}{r}\right)^2} \tag{11}$$

and for rectangular cross section:

$$\alpha_p = \frac{1}{1 + 3\omega\left(\frac{e_o}{y}\right)^2} \tag{12}$$

Based on the experimental results, a value of 1.5 for ω may be used, then the factor of eccentric effect for brick masonry with reinforced network is:

$$\alpha_p = \frac{1}{1 + 4.5\left(\frac{e_o}{y}\right)^2} \tag{13}$$

Experimental and calculating values are shown in Fig.3

To sum up the above analyses, the formula for calculating the load-bearing capacity of brick masonry with rectangular cross section and reinforced network under the action of eccentric loading is:

$$KN_P \leqslant \alpha_P \left[R + (1 - \frac{e_o}{y}) \frac{2\mu}{100} R_g \right] A \qquad (14)$$

A part of the experimental value (N') and calculating value (N) are shown in Table 2.
Based on the results of experiments, when $e_o = 0.5 - 0.7y$, its load-bearing cacity is only equal to 18 - 65% of that under axial compression, in comparison with brickwork without any reinforcement, the load-bearing capacity is increased only a little. Further, when the value of e_o is increased and horizontal cracks are developed in the tensile area, the brickwork will at once loses its load-bearing capacity. Thus it will make the structure unsafe. Therefore the value of e_o/y should not be greater than 1/3.

When the vertical bending factor φ_P is introduced into formulae (5) and (14), then the calculation formula for the load-bearing capacity of brickwork with reinforced net is obtained:

$$KN_P \leqslant \varphi_P \alpha_P \left[R + (1 - \frac{e_o}{y}) \frac{2\mu}{100} R_g \right] A \qquad (15)$$

Since horizontal reinforced network is placed in the mortar joint, the brickwork is separated by the former and the thickness of the mortar joint is increased. According to actual measurement, when the loading ratio is the same, the deformation of the brick masonry with reinforced net is greater than that without any reinforcement, and the deformation increases as the value of μ is increased. The value of φ in our national code for brickwork without reinforcement is $1/(1+a\beta_P^2)$ In consideration of the above stated characteristics, the following values are adopted for brickwork with reinforced net:

$$a = 0.0015 \quad , \quad a_P = 0.0015(1 + 3\mu)$$

then

$$\varphi_P = \frac{1}{1 + a_P \beta_P^2} \qquad (16)$$

Where; β_P = the ratio of the height to the thickness of the brickwork. When β_P is greater than 15, it is not suitable to use this type of construction.

4. ANALYSIS OF RELIABILITY OF BRICKWORK BY 'CALIBRATION'

By considering the first-order second moment method used for the types of distribution of random variables, the calibration analysis of safety index β of brickwork with reinforced net under compression is carried out. The statistical parameters of load are shown in Table 3.

The expression for the average value of resistance of a member is:

$$R_m = R_n \cdot M_m \cdot F_m \cdot P_m \qquad (17)$$

where:

R_m = the mean strength of a member,
R_n = the standard strength of a member,
M_m, F_m and P_m are the average value of the coefficient of effect of uncertainty of the strength of materials, the geometrical chacteristics of members and the calculation models respectively.

Let: $$k_R = R_m/R_n = M_m \cdot F_m \cdot P_m \qquad (18)$$

then the coefficient of variation of k_R is:

$$V_K = (V_M^2 + V_F^2 + V_P^2)^{\frac{1}{2}} \qquad (19)$$

where V_M, V_F and V_P are the coefficients of variation of M_m, F_m and P_m respectively.

4.1. Axial Compression

According to formula (15), where $e_0 = 0$, determine the statistical parameters of the brickwork. The different terms of strength in formula (15) are average values. Actually the average compressive strength of brickwork is higher than the value obtained from test prisms in the laboratory, the ratio is 1.15 . The coefficient of variation of the compressive strength of brickwork without reinforcement is $V_R = 0.20$. The value of M_{mR_g} of the reinforcement is 1.20 . The coefficient of variation of the strength of reinforcement and its cross sectional area is $V_{R_g A_g} = 0.075$. In brickwork with reinforced net, we also have to consider the variation of C_g and S_g, so the value of the coefficient of variation $V_{R_g} = 0.10$ of the strength of reinforcement is used. Further, we take $\mu = 0.1$, 0.2 and 0.3% in order to compare the influence of μ on the reliability of the brickwork. The results are shown in Table 4 .

Generally, the cross sectional area of the prism with reinforced net approaches 49 x 49 cm or over, when this area (49x49cm) is used, we get $F_m = 1.0$, $V_F = 0.0117$.

According to the results of axial compression test and calculations presented in this paper, we get $P_m^I = 1.02$, $V_{PI} = 0.149$. The mean value of the coefficient of the effect of uncertainty of vertical bending for brickwork without reinforcement is used, that is: $\bar{\Phi}_p = 1.0$, its coefficient of variation is $V_{\Phi} = 0.11$. Thus the mean value of the coefficient of the effect of uncertainty of the calculation model of axial compressed brickwork with reinforced net and including the effect of uncertainty of vertical bending is $P_m = 1.02$ and its coefficient of variation $V_P = 0.185$.

Based on the above stated statistical parameters of resistance (Table 4) of test prisms and making it in accord with the normal or log normal distribution, together with the statistical parameters of loading and and their distribution as shown in Table 3 , the β value of axially compressed brickwork with reinforced net can be obtained by using the iteration method and calculated with electronic computer. The results are shown in Table 5 .

4.2. Eccentric Compression

Similarly, determine the statistical parameters of the prism by formula (15). When it is eccentrically loaded, actually, the ratio of the average compressive strength of brickwork to that of the laboratory test prism is 1.10 . The average value 1.20 (M_{mR_g}) of the coefficient of effect of uncertainty of the strength of reinforcement is still adopted, the corresponding values of $V_R = 0.20$ and $V_{R_g} = 0.10$. As shown in Table 4, the effect of the variation of μ to the values of M_m and V_M is very small, therefore we can select a value of μ for analysis. Let $\mu = 0.2\%$, then we can get the values of M_m and V_M for three different values of eccentricity (see Table 4).

As stated above, let $F_m = 1.0$, $V_F = 0.0117$.

Based on the eccentric compression tests and the result of calculation as presented in this paper, we obtain $P_m^I = 1.093$, $V_{PI} = 0.157$ and let $\bar{\Phi}_p = 1.0$, $V_{\Phi} = 0.11$, then $P_m = 1.093$, $V_P = 0.192$

Based on Tables 3 and 4 , the value of β of brickwork with reinforced net under eccentric compression can be obtained by electronic computers (see Table 5).

According to the building code used in our country, when the safety grade for the failure of members due to brittleness is 2 , the value of β should be 3.7 , the corresponding probability of destruction is 1.0×10^{-4} . The above stated analysis and calculation show that the safety index approaches or equals to the required value for the brickwork with reinforced net under compression.

5. CONCLUSION

5.1. The load-bearing capacity of brickwork with reinforced net can be calculated by the following formula:

$$KN_p \leq \varphi_p \; \alpha_p \; R_p \; A$$

where:

- k = factor of safety, a value of 2.3 is generally used,
- φ_p = vertical bending factor, as shown in formula (16), $\beta_p \leq 15$,
- α_p = coefficient of eccentrical effect, according to formula (13),
- R_p = the compressive strength of brickwork with reinforced net:

$$e_o/y \leq 1/3, \qquad R_p = R + (1 - \frac{e_o}{y}) \frac{2\mu}{100} R_g \leq 4R$$

- A = cross sectional area of brickwork.

When the above formula is calibrated, the safety index approaches or equals to the required value.

5.2. When the probability based limit states design method is used and expressed by the coefficient of each term:

$$\gamma_G S_G + \gamma_Q S_Q \leq \frac{R_K}{\gamma_R}$$

Then, if $\gamma_G = 1.2$, $\gamma_Q = 1.4$, and the level of reliability of brickwork with reinforced net under compression as shown in our current design code of brick masonry is maintained, we can adopt $\gamma_R = 1.85$. In the above formula, S_G, S_Q & R_K are the standard values of dead load, live load and the resistance of the structural members respectively. γ_G & γ_Q are the corresponding coefficients of load effect and γ_R is the coefficient of resistance of the structural members.

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Table 1 Load – bearing capacity subject to axial compression

Test NO.	μ %	N' (T)	N (T)	N'/N	Test NO.	μ %	N' (T)	N (T)	N'/N
1	0.067	22.4	18.7	1.198	22	1.000	67.0	67.8	0.988
2		14.3		0.765	23		71.5		1.055
3		12.0		0.642	24	0.126	66.0	61.8	1.068
4		13.2		0.706	25		76.0		1.230
5	0.111	26.5	20.4	1.299	26		64.0		1.036
6		27.5		1.348	27	0.126	66.0	56.5	1.168
7		28.0		1.373	28		69.5		1.230
8	0.332	32.2	33.2	0.970	29		59.5		1.053
9		28.6		0.861	30	0.252	83.5	72.2	1.157
10		31.1		0.937	31		78.0		1.080
11	0.333	34.8	33.2	1.048	32	0.334	79.5	80.0	0.994
12		32.5		0.979	33		72.0		0.900
13		40.0		1.205	34		64.0		0.800
14	0.665	52.0	50.4	1.032	35	0.355	82.5	90.5	0.912
15		40.8		0.810	36		90.0		0.994
16		39.4		0.782	37		81.0		0.895
17	0.665	40.0	50.4	0.794	38	0.355	87.5	85.0	1.029
18		49.0		0.972	39		88.5		1.041
19		31.2		0.619	40		100.0		1.176
20	1.000	53.4	67.8	0.788	41	1.000	154.0	163.0	0.945
21		74.0		1.091					

Table 2 Load – bearing capacity subject to eccentric compression

Test NO.	e_o/y	μ %	N' (T)	N (T)	N'/N	Test NO.	e_o/y	μ %	N' (T)	N (T)	N'/N
1	0.1	0.067	26.6	17.6	1.511	26	0.2	0.332	26.2	25.1	1.044
2			11.5		0.653	27			31.0		1.235
3			21.8		1.239	28			29.4		1.171
4	0.1	0.111	26.1	19.0	1.374	29	0.2	0.665	34.0	36.8	0.924
5			21.2		1.116	30			34.0		0.924
6			27.4		1.442	31			25.0		0.679
7	0.1	0.332	27.0	30.0	0.900	32	0.2	1.000	45.0	48.6	0.926
8			29.5		0.983	33			59.0		1.214
9			22.7		0.757	34			59.8		1.230
10			22.5		0.750	35	$\frac{1}{3}$	0.067	13.0	11.9	1.092
11	0.1	0.665	40.0	44.8	0.893	36			17.6		1.479
12			36.0		0.804	37			19.0		1.597
13			40.0		0.893	38	$\frac{1}{3}$	0.111	20.8	12.6	1.651
14	0.1	1.000	48.0	59.9	0.801	39			17.0		1.349
15			60.5		1.010	40			12.8		1.016
16			53.8		0.898	41	$\frac{1}{3}$	0.332	16.5	18.4	0.897
17			60.1		1.003	42			19.6		1.065
18	0.2	0.067	10.5	15.4	0.682	43			20.2		1.098
19			18.0		1.169	44			19.6		1.065
20			16.0		1.039	45	$\frac{1}{3}$	0.665	25.0	26.0	0.962
21			19.0		1.234	46			24.0		0.923
22	0.2	0.111	16.0	16.5	0.970	47			32.0		1.231
23			26.0		1.576	48	$\frac{1}{3}$	1.000	48.0	33.7	1.424
24			17.6		1.067	49			34.0		1.009
25	0.2	0.332	24.1	25.1	0.960	50			40.0		1.187

Table 3 Statistical parameters of loading

Types of Load	average Value / Standard Value	Coefficient of variation	Types of distribution
Dead load	1.06	0.07	Normal
Live load on office floors	0.70	0.29	Extreme distribution type 1
Live floor load of dwellings	0.86	0.23	
Wind load	1.00	0.18	

Table 4 Statistical parameters of members

Axial compression								
μ %	M_m	V_M	F_m	V_F	P_m	V_P	k_R	V_k
0.1	1.1524	0.199	1.0	0.0117	1.02	0.185	1.1754	0.272
0.2	1.1548	0.199	1.0	0.0117	1.02	0.185	1.1779	0.272
0.3	1.1572	0.199	1.0	0.0117	1.02	0.185	1.1803	0.272
Eccentric compression								
e_0/y	M_m	V_M	F_m	V_F	P_m	V_P	k_R	V_k
0.1	1.104	0.199	1.0	0.0117	1.093	0.192	1.2100	0.277
0.2	1.103	0.199	1.0	0.0117	1.093	0.192	1.2089	0.277
0.3	1.102	0.200	1.0	0.0117	1.093	0.192	1.2078	0.277

Tabel 5 Safety index β by calibration

Classification of members	Live load / Dead load				Average value
	0.10	0.25	0.5	0.75	
Axial compression	3.405	3.547	3.672	3.707	3.583
Eccentric compression	3.437	3.577	3.701	3.738	3.613

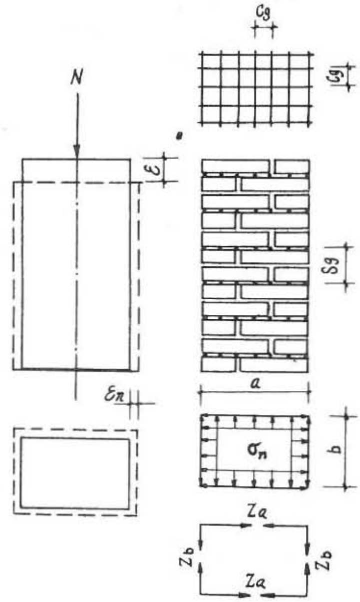

Fig. 1 Graphs used for calculation of brickwork with reinforced net

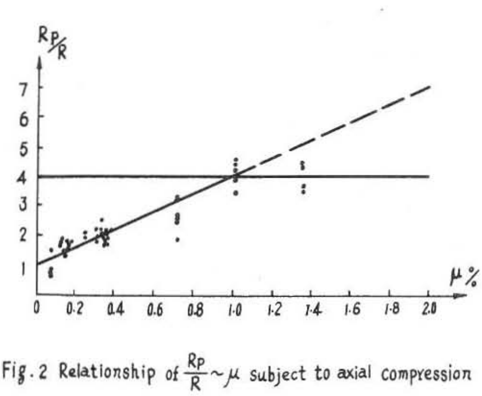

Fig. 2 Relationship of $\frac{R_P}{R} \sim \mu$ subject to axial compression

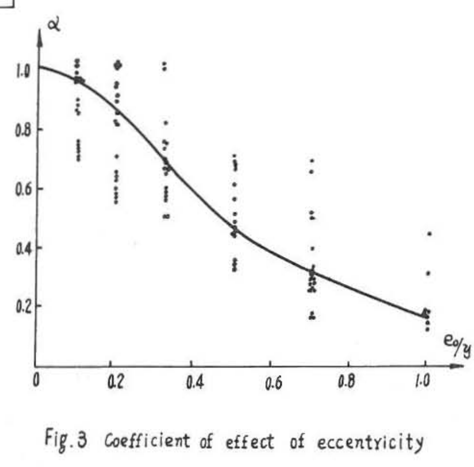

Fig. 3 Coefficient of effect of eccentricity