

# **Numerical Analysis of Walls and Piers with Different End Conditions**

F. Sawko, D.Sc.

K. Towler, Ph.D.

Liverpool University  
Liverpool, United Kingdom

## Summary

The paper compares the numerical results for capacity reduction factors for walls with different support conditions. It concludes that different support conditions have an important influence on load carrying capacity of walls, and that an assumed initial imperfection has an over-riding effect on their behaviour.

## Introduction

From the classical Euler theory of buckling we know that the ultimate load of members in compression is significantly influenced by end support conditions. The behaviour of load bearing walls and piers cannot be equated to the behaviour of elastic struts, but end conditions must nevertheless play a significant part on their load bearing characteristics. This paper discusses the effect of different support conditions and of structural imperfections on the limit state behaviour of brickwork members in compression.

## End Support Conditions

The four different cases of end support conditions are shown in Fig. 1.

Case (a) - Hinged top and bottom, the same eccentricity. This condition is often employed in laboratory testing since hinged condition is easy to achieve with knife edge supports. The condition is not really encountered in practice. However, if one half of the wall is considered, this corresponds to the case of a free cantilever wall with eccentric loading applied to the top as shown in Fig. 1(a).

Case (b) - Hinged top and bottom, zero eccentricity at the base, eccentricity  $e$  at top. This case requires the presence of a horizontal support to deviate the line of thrust from its vertical position to cross the neutral axis at base with zero eccentricity. This case is assumed to occur in practice, and the British Code of Practice<sup>1</sup> bases its Capacity Reduction factors on such a model.

Case (c) - Fixed top and bottom. This occurs in practice in tall continuous walls tied to a rigid frame. This case can be reduced to an axially loaded simply supported case (a) with zero eccentricity.

Case (d) - Fixed base, propped top with eccentric load  $e$ . This is most likely to occur in single and multi-storey buildings where the wall is built on a rigid level base with an eccentric load applied subsequently.

## Method of Analysis

A numerical procedure has been developed by the first author<sup>2,3</sup> to predict the behaviour of walls and piers constructed in brittle (no-tension) material exhibiting a parabolic stress-strain relationship in compression. This has been used to analyse walls with simple support conditions (a) and (b) in Fig. 1. This procedure has now been extended to case (d). This has proved far from straightforward, since the eccentricity at the base cannot be defined from the geometry of the wall and loading, and is a function of material properties and deflected shape of the wall. A numerical model has been developed which successively corrects the deflected geometry of the wall until equilibrium and compatibility are both satisfied. The computer program developed has been used to produce a set of capacity reduction factors and to compare them with case (b) and Code recommendations.

Due to workmanship and other effects, brickwork walls cannot be considered perfectly straight and the Code assumes an imperfection varying linearly from zero at top and bottom of the wall to certain value constant over the central fifth of the wall height. Numerical analysis has been extended to include this effect also in order to assess the effect of the initial imperfection on capacity reduction factors.

### Behaviour of Perfect Wall

Figures (2) and (3) show the deflected profile and strain and stress distributions and line of thrust for a typical stocky and slender wall. It will be observed that the line of thrust crosses the centroidal line at approximately  $1/3$  height above base. The wall above this point resembles case (b) but is not its exact replica, since the relative horizontal movement between ends of wall at (b) is zero, but not between the top of wall and the point of intersection between line of thrust and axis in case (d).

Figures (4) and (5) illustrate a significant difference between the capacity reduction factors relating to the two support conditions.

### Walls with Initial Imperfection

Figures (4) and (5) also illustrate the effect of initial imperfection as defined in the Code. Results for walls constructed with different brickwork are illustrated in Figures 6 and 7. One noticeable effect of imperfections is the bunching together of graphs for different brickwork for axial loading demonstrating the over-riding effect of initial eccentricity. For large applied eccentricity this effect is less pronounced.

### Conclusions

The paper has highlighted significant differences in ultimate load of walls with different support conditions. These differences are particularly noticeable between idealised condition (b) assumed in the Code and the case (d) it is supposed to represent. Numerical modelling is now capable of simulating more accurately the actual behaviour of walls and should lead to a better understanding of their behaviour.

### References

1. British Standards Institution: BS5628 : Part 1 : 1978  
Structural Use of Masonry.
2. Sawko, F. "Numerical Solution of Eccentrically Loaded Struts in No-tension Material" Proceedings of 2nd International Conference on Numerical Methods in Fracture Mechanics, Swansea, July 7-11, p. 1-20.
3. Sawko, F. "Numerical Analysis of Brick Walls under Compressive Loading". Proceedings of Conference of British Ceramic Society, p. 1-10.

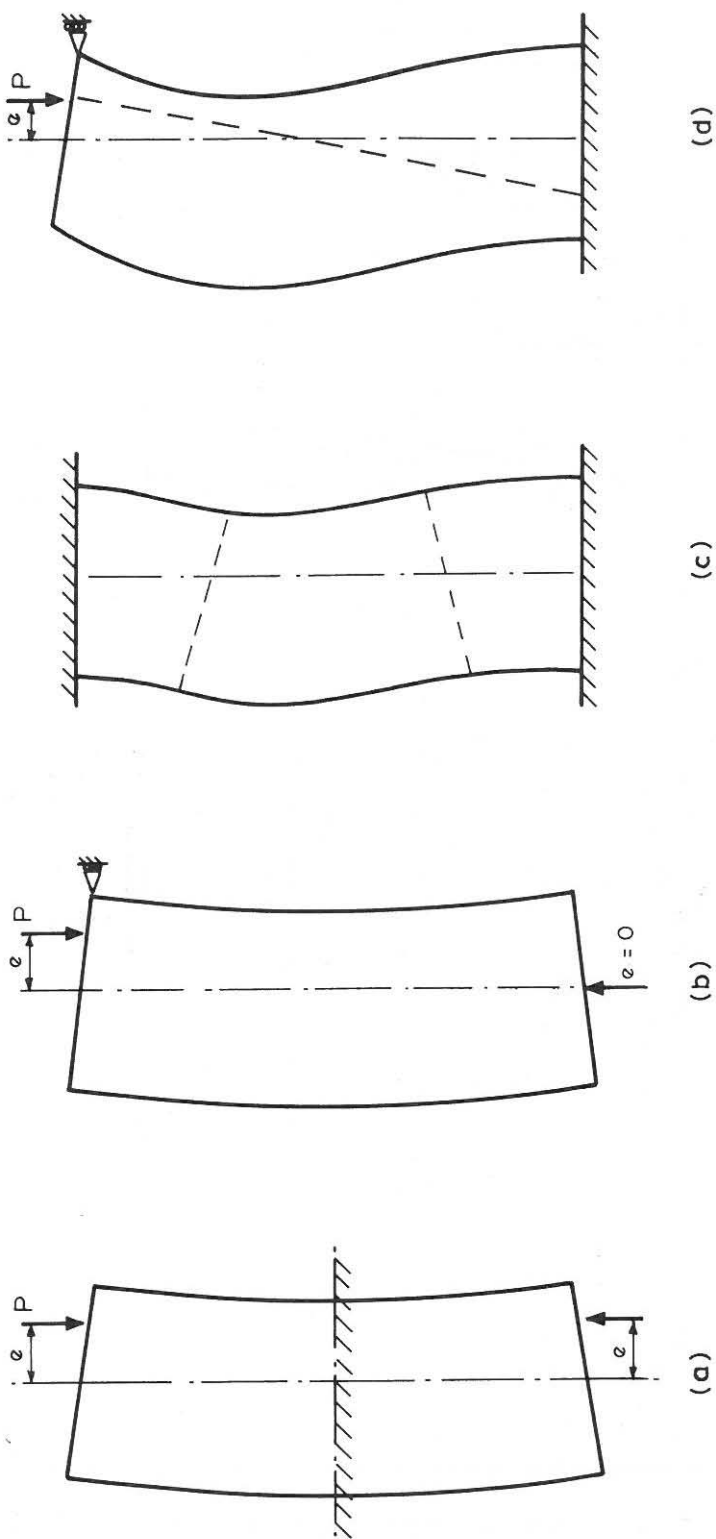


FIG. 1. DIFFERENT SUPPORT CONDITIONS OF WALLS.

$\sigma_m = 10 \text{ N/mm}^2$   
 $\epsilon_m = 0.004$   
 $t = 0.3 \text{ m}$   
 $H = 1.5 \text{ m}$   
 $H/t = 5$   
 $P/t = 136 \text{ N/mm}$

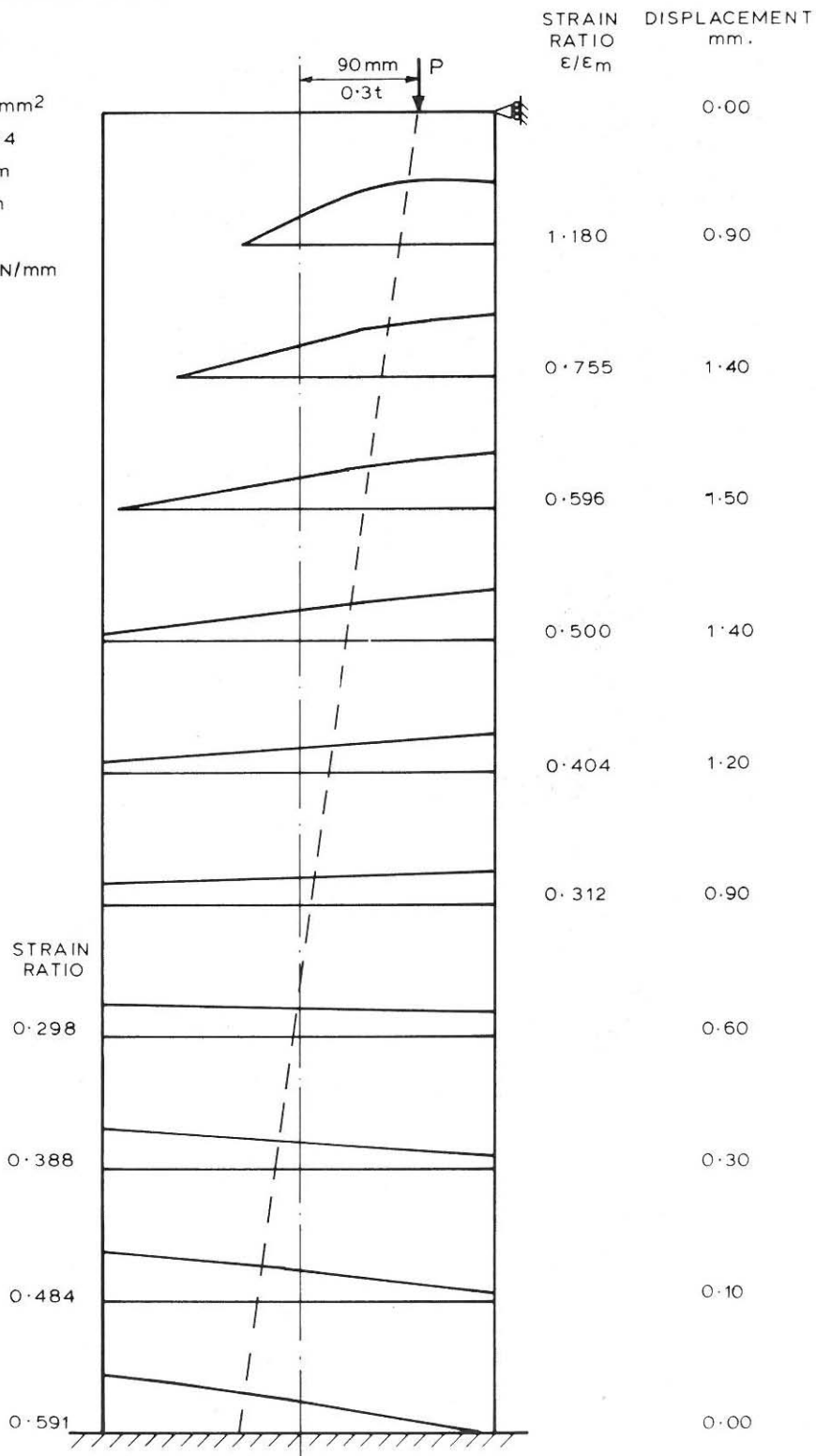


FIG. 2. STOCKY WALL WITH BUILT-IN BASE.

$\sigma_m = 10 \text{ N/mm}^2$   
 $\epsilon_m = 0.004$   
 $t = 0.3$   
 $H = 7.8 \text{ m}$   
 $H/t = 26$   
 $P_{ult} = 80 \text{ N/mm}$

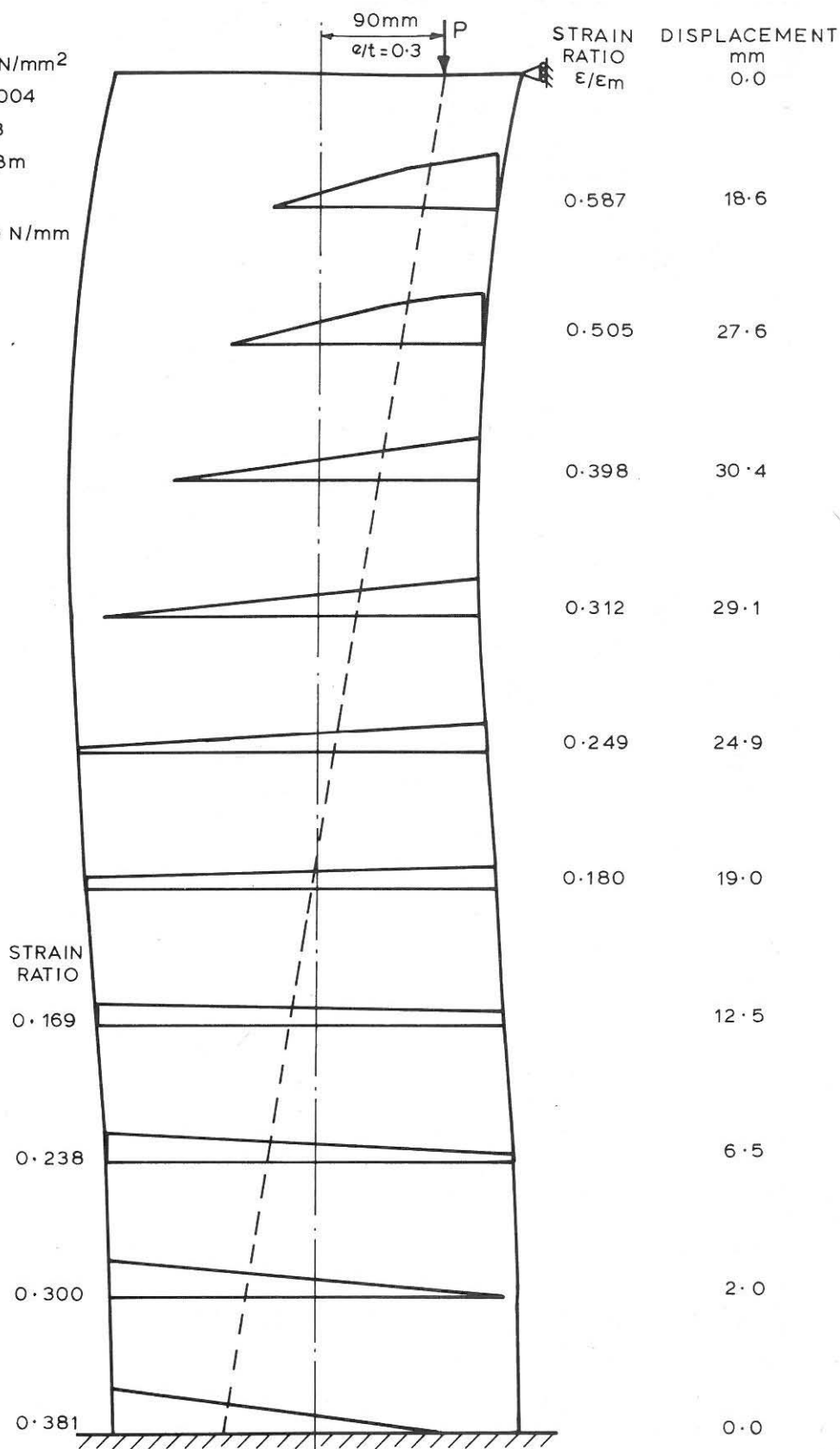


FIG. 3. SLENDER WALL WITH BUILT-IN BASE.

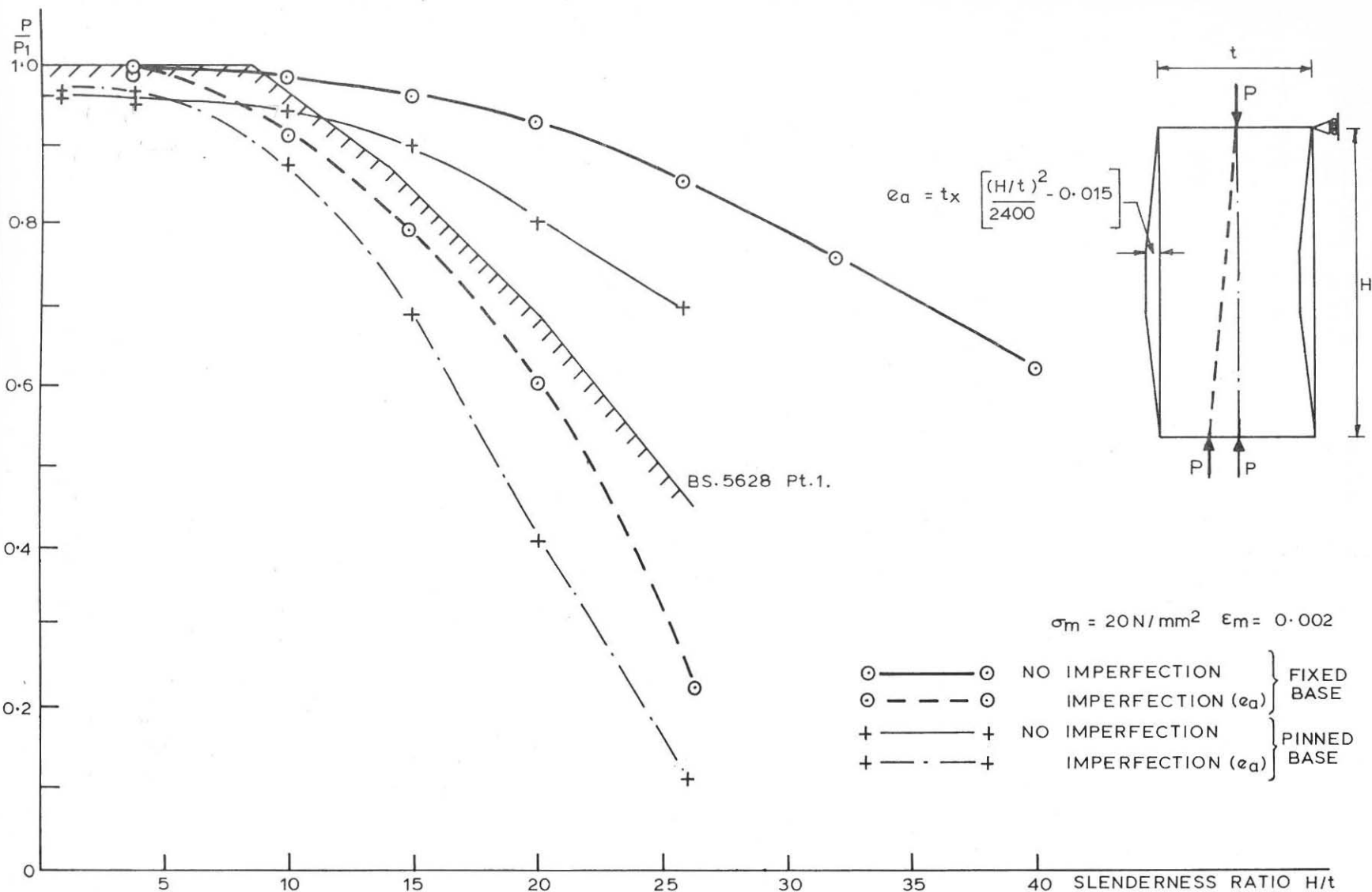


FIG.4. CAPACITY REDUCTION FACTORS FOR PINNED AND FIXED/PINNED SUPPORT CONDITIONS WITH  
 AXIAL LOAD ( $e/t \leq 0.05$ )

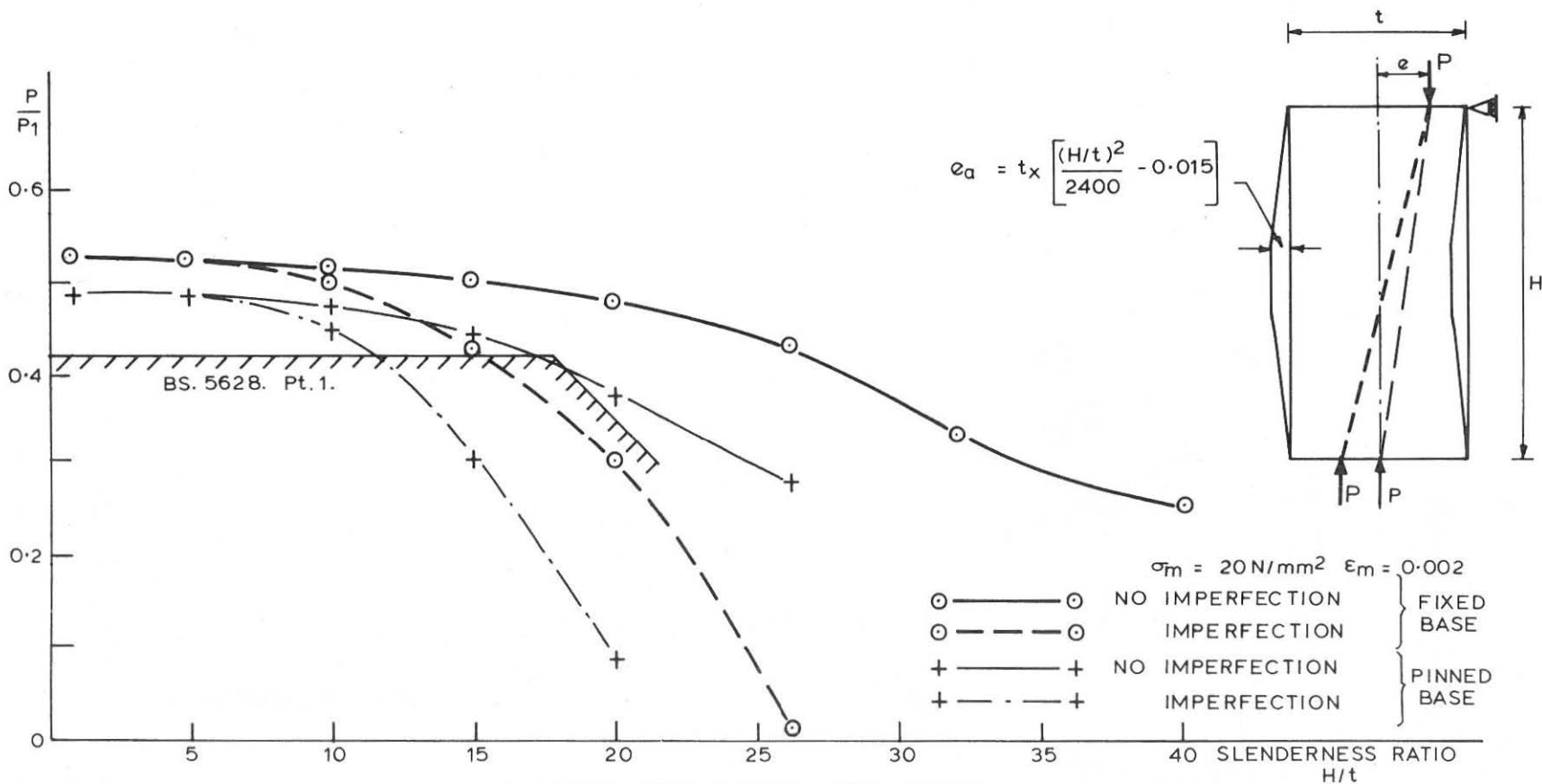


FIG.5. CAPACITY REDUCTION FACTORS FOR PINNED AND FIXED/PINNED SUPPORT CONDITIONS WITH ECCENTRIC LOAD ( $e/t = 0.3$ ).



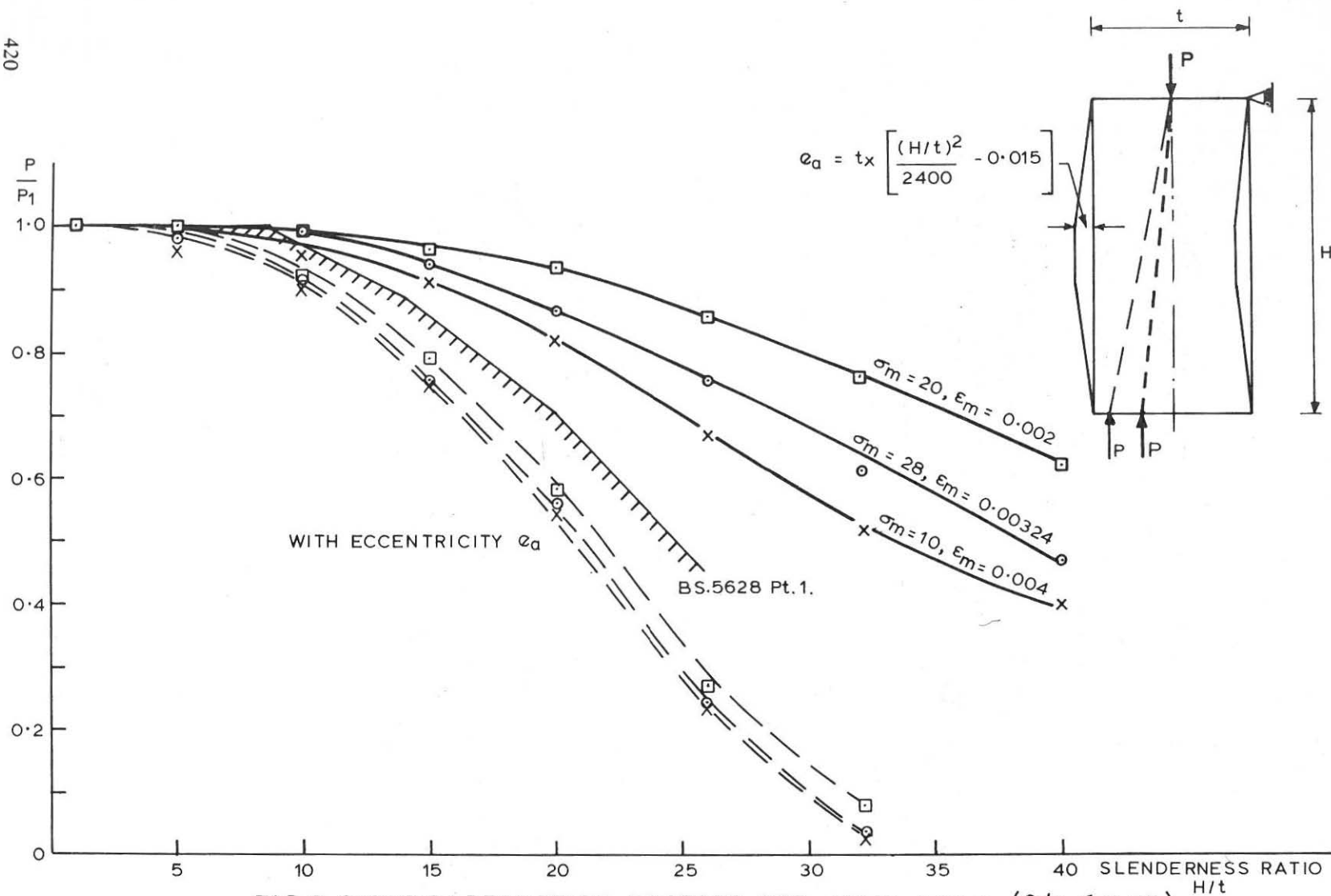


FIG.6. CAPACITY REDUCTION FACTORS FOR AXIAL LOAD ( $e/t \leq 0.05$ ).

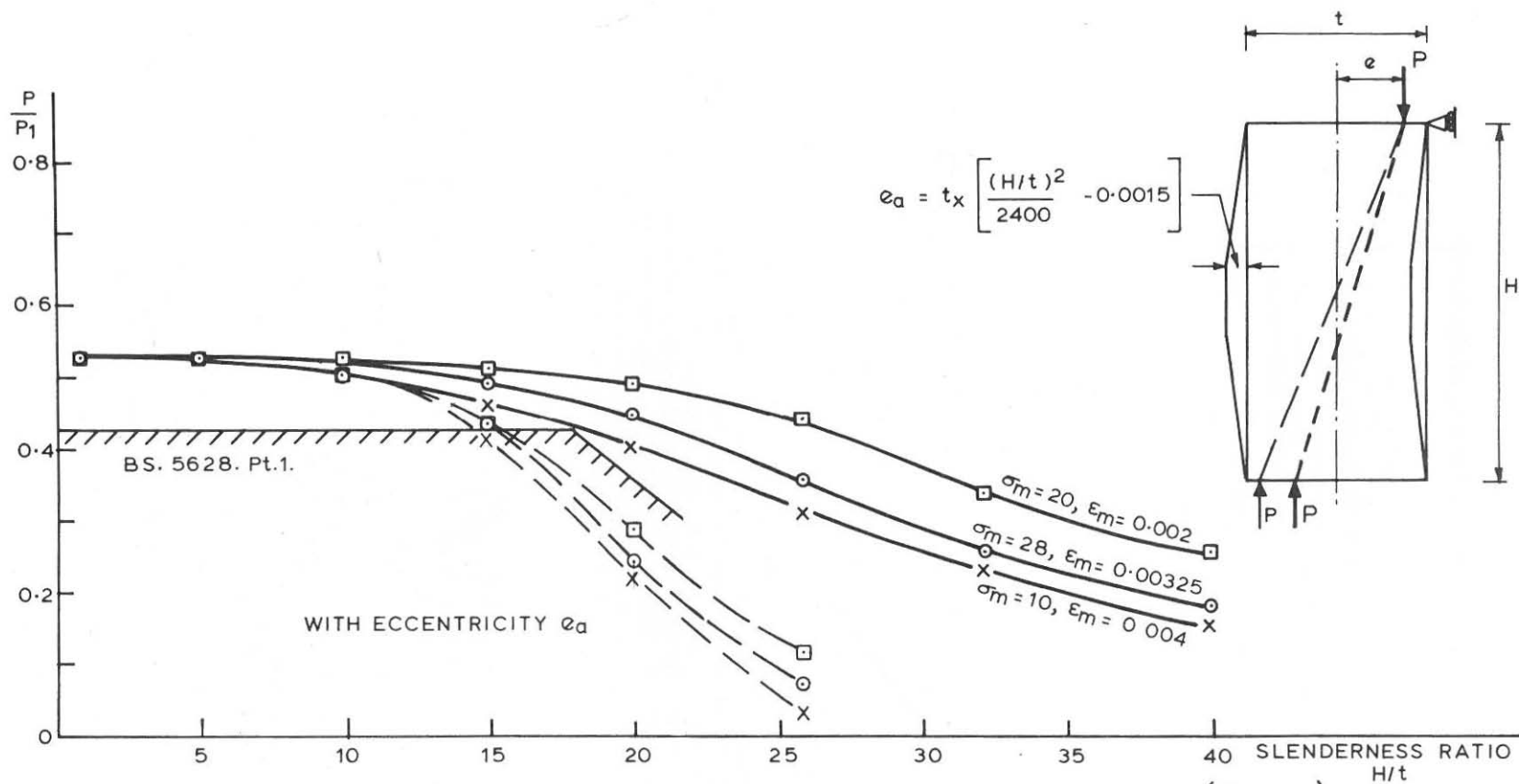


FIG. 7. CAPACITY REDUCTION FACTORS FOR ECCENTRIC LOAD  $\left(\frac{e}{t} = 0.3\right)$ .