An Elastic Principal Stress Theory for Brickwork Panels in Flexure

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SUMMARY

A general theory for the analysis of brickwork subjected to lateral loads is presented. It is consistent with basic concepts of structural behaviour and is based on principal moments in the panel and a realistic failure criterion for the material. Allowance is made for variable joint strengths in the panel and a degree of load sharing between joints. Experimental work supports the theory.

NOTATION

w	Applied uniformly distributed load.
F'v,F'h	Moduli of rupture in vertical and horizontal bending, respectively.
R	Orthogonal strength ratio F'h/F'v
Fa	Axial compressive stress at mid height.
L,H,t	Length, height and thickness of masonry panel.
E	Number of adjacent joints over which averaging occurs.

1 GENERAL

The simplest case of flexure for brickwork panels is that in which the panel spans one-way in the vertical direction. Here the principal moments are statically determined, oriented in the vertical and horizontal directions, and produce failure along a single bed joint in the panel. Previous work (1) has shown the strengths of these panels to be described by a partially plastic theory in which the variable joint strengths occurring in the panel may be averaged over three adjacent joints along a course.

The more general case of masonry panels supported in such a way that lateral loads are resisted by combined vertical and horizontal bending are considered in this paper. These panels ultimately fail by cracking through the perpend joints and also through the bricks and bed joints. Principal moments in two-way panels are statically indeterminate, depend on the deformation characteristics of the masonry materials, and are not generally oriented in the vertical and horizontal directions. In addition, the first crack in a two-way action panel does not necessarily lead to immediate collapse as is the case with vertically spanning panels. Two common practical cases of masonry panels with two-way action are those simply supported on all four edges and those simply supported along the bottom and two sides but free at the top. These two types of support conditions are investigated both theoretically and experimentally in this paper and are shown to be closely related for masonry panels.

Observation of experimental walls simply supported on three sides without in-plane restraints, shows that ultimate load practically corresponds with initial cracking load. On the other hand, panels simply supported on four sides have, in general, two stages of behaviour. In the first stage the panel remains uncracked and the central deflection is approximately proportional to the lateral applied uniformly distributed load. The end of this stage is marked by a horizontal crack occurring along a bed joint at approximately mid-height of the panel as the moment capacity in vertical flexure at this level is reached. This initial phase is similar to that of a panel in vertical flexure except that the moments result from two-way rather than one-way action.

In the second phase the horizontal crack rapidly extends for practically the full length of the course and the wall then behaves as two sub-panels. It can be argued that since the crack occurs near mid-height and extends the full length of the wall there will be very little shear transferred across this crack and the two sub-panels will behave as panels simply supported on three sides and free along the cracked edges.

These concepts are developed theoretically using the partially plastic failure c iterion and compared with experimental results.

2 THEORY

The above observations point to two cases for consideration. Firstly, the initial cracking load of a panel supported on four sides and secondly, the initial cracking load of a panel supported on three sides. The latter solution provides an estimate of ultimate load for panels supported on three sides and also for panels supported on four sides.

For each of these cases a principal moment failure criterion in biaxial bending, as derived in a companion paper (2), must be applied. In this theory it is assumed that conditions may be averaged over E adjacent joints such that failure conditions exist over those joints when

$$w \geqslant \frac{t^2}{6} \quad \frac{\Sigma(J_{ij} + fa_i)}{\Sigma C_{ij}}$$

where the summations are taken over the E adjacent joints.

- fa_i is the axial stress at the i th course in the panel
- J_{ij} is the modulus of rupture of the joint in vertical flexure at location ij in the panel
- C_{ij} is a coefficient dependent on the magnitude, sense and orientation of the principal moments and the material properties at location ij and is derived in reference (2).

Whereas in vertical flexure the E adjacent joints are along a single course and form a horizontal crack pattern, in a two-way panel they may be on a vertical or stepped pattern.

2.2 Panels Supported On Four Sides

Since the observed initial cracking of panels supported on four sides is along the bed joint of a single course of brickwork near mid-height, a horizontal crack pattern of E adjacent joints is the appropriate one to use. Also the concept of a joint consisting of a brick length of bed joint is appropriate. Because the maximum principal stresses are both tensile and occur at the centre of the panel in the vertical and horizontal directions the coefficient Cij may be expressed in terms of vertical and horizontal moment coefficients. Also the summations extend over E adjacent joints in the same course.

A computer program has been developed that calculates moment coefficients by elastic plate analysis using a series solution, assigns random joint strengths from a normally distributed population with negative values truncated , searches the panel to find the combination of E adjacent joints that gives the lowest value of w, and repeats the simulation 500 times to find the mean panel strength and its coefficient of variation. Mean panel strength is expressed non-dimensionally as a vertical strength ratio or as a horizontal strength ratio. The vertical strength ratio is the uniform load to cause initial cracking of the panel divided by the load to cause failure of a vertically spanning strip of the panel based on the stress at mid-height.

$$\text{Failure load} = (\text{Vert Ratio}) \times \frac{4}{3}\,\frac{(F_v' + F_a)}{(\frac{H}{t})^2}$$

where Fa is the axial compressive stress at mid-height.

Similarly the horizontal strength ratio is the uniform load to cause initial cracking of the panel divided by the load to cause failure of a horizontally spanning strip

$$\text{Failure load} = (\text{Hor Ratio}) \times \frac{4}{3}\,\frac{R\,F_v'}{(\frac{L}{t})^2}$$

Some theoretical results are illustrated for panels simply supported on four sides, 40 courses high and up to 50 bricks long. Bricks having a length to height ratio of 2.77 have been assumed, resulting in panels having aspect ratios up to about 4.5. Isotropic properties are used with Poisson's ratio equal to 0.15.

Figure 1 shows both vertical and horizontal strength ratios obtained from the program for an orthogonal strength ratio of 3. As the aspect ratio of the panel approaches zero the vertical ratio curves (shown dotted) tend to infinity while the horizontal span ratio curves tend to unity. That is, the strength of the

panel is substantially determined by the one way action in the horizontal direction. At the other extreme, as the aspect ratio of the panel tends to infinity, the vertical span ratio tends to unity and the horizontal span ratio tends to infinity. For this case the panel strength is essentially determined by one-way action in the vertical direction. Since portions of the curves that tend to infinity cannot be adequately represented graphically it is more convenient to represent the panel strength by either the vertical span ratio or the horizontal span ratio whichever is less. It can be easily shown that these two ratios are equal when the aspect ratio of the panel is equal to the square root of the orthogonal strength ratio of the material.

This figure also shows the effect of the variability of joint strengths (coefficient of variation=0.25) and the effects of averaging effects over adjacent joints (E=1, 2, 3). It is interesting to note that except for the case of E=1, the variability of panel strength is practically independent of its length. In addition, load sharing between 2 or 3 adjacent joints results in a marked reduction in the variability of the panel strength.

Where the length of a panel is small compared with its height, an initial horizontal crack may not develop along the bed joint near mid-height. For these cases a vertical crack will develop and an analysis along the lines described in the next section is appropriate. Such an analysis has been included in the program . The results of this vertical cracking mode are printed out if they are more critical than the horizontal cracking mode.

2.3 Panels Supported on Three Sides

The theory above can be extended to the case of panels supported on three sides. In applying the failure criterion in this case however, it has been found essential to use the general failure criterion. Text books present elastic solutions of these plates in terms of coefficients for moments in the vertical and horizontal directions. There are several difficulties in using these moment coefficients for three-sided plates.

Firstly, the horizontal and vertical moments are always a maximum on the centre line of the panel but failure does not always occur there. For a panel with a long free edge compared with its height, failure occurs well away from the centre line. In addition, for these large aspect ratio panels, the horizontal and vertical moment coefficients predict a panel strength much greater than predicted by plastic theory. This is contrary to basic concepts of structural behaviour. Both of these anomalies disappear if the principal stress criterion is used. Torsional effects produce maximum principal stresses along the lines where failure cracking is observed in practice. An illustration of the distribution of principal streses in three sided panels of various aspect ratios is shown in Figure 2. It can be seen that

for the higher aspect ratios the maximum principal stresses do not occur on the centre line and are not oriented in the vertical and horizontal directions. In addition the maximum principal stress is tensile and is accompanied by a compressive principal stress in the orthogonal direction. On this basis elastic analysis predicts a lower strength than plastic theory and is therefore consistent with basic structural theory.

In making allowance for the inclusion of weak joints and load-sharing between adjacent joints, the concept of a joint must be redefined to allow a stepped cracking pattern to occur. A stepped crack pattern rises one brick height every half brick length. Hence a joint may be considered as a unit one brick high and half a brick in length or alternatively a unit one brick long and 2 bricks high. It can be seen from Figure 3 that both the "small" and "large" joint units defined above allow all the crack patterns observed in practice to occur. When searching for the critical combination of E adjacent joints in a panel the program covers the combinations shown in Figure 4 for E=2 and E=3.

The program randomly assigns joint strengths from a given population to locations in the panel, calculates principal stresses and their directions at each joint by a finite difference program, calculates the failure load for the critical combination of E adjacent joints in the panel, and repeats the process for 500 simulated panels to find the mean panel strength and its coefficient of variation. The mean panel strength is expressed non-dimensionally in terms of the horizontal span ratio as defined for four-sided support panels.

Some theoretical relationships are illustrated using panels simply supported on three sides and free at the top having 20 joints in their lengths. That is, these panels are half the height of the previous panels supported on four sides. The same bricks and material properties are assumed. The "large" joint units have been used in the program in obtaining the relationships. Figure 5 shows how the theory modifies the usual elastic theory that is based on vertical and horizontal bending moment coefficients. It can be seen that the principal stress criterion leads to a drastic reduction in predicted strength for the panels having high aspect ratios. A further reduction in predicted strength occurs when the variability of joint strengths is considered but some of this strength is regained as load-sharing over 2 or 3 adjacent joints occurs. Also, the reduction in variability of panel strength due to load-sharing is shown. The analysis is insensitive to orthotropic elastic properties as only a small spread of values was obtained for elastic moduli ratios varying from 0.5 to 1.5.

3 EXPERIMENTAL WORK

To test the above theories a total of 30 panels were built and tested under applied uniform lateral pressure. Panels supported on four sides were 23 courses high and had either 6, 11

or 19 bricks in their lengths to produce aspect ratios of 0.67, 1.33 and 2.33 respectively.

Similar lengths were used for panels supported on three sides but only 13 courses were used in their heights to produce aspect ratios of 1.19, 2.37 and 4.14 respectively. Five replicate panels were made for each of the above conditions. Companion small specimens were made with the walls to allow the measurement of joint strengths. A five-high stack-bonded pier and four horizontal joint specimens (as described in reference 2) were made with each wall to provide a total of 120 joints for testing in vertical flexure and 120 joints for testing in horizontal flexure.

Extruded clay bricks containing three 30.2mm diameter cores were used. Measurements of twenty bricks averaged 74.9mm high x 71.1mm thick x 223.5mm long. The modulus of rupture of these bricks averaged 65 MPa with a coefficient of variation of 0.21. Mortar containing portland cement, hydrated lime, and sand in the properties 1:1:6 by volume was used. Materials were weigh batched and water added by the bricklayer to give a uniform consistency. The flow of each batch was measured before use and averaged 82% with a coefficient of variation of 0.025.

All specimens were tested at an age of between 27 and 30 days. The stack bonded specimens when tested in vertical flexure gave an average modulus of rupture for the joints of 0.87 MPa with a coefficient of variation of 0.24. Horizontal joint specimens failed when the average extreme fibre stress in the brick was 5.80 MPa. The orthogonal strength ratio calculated from these values was R=3.50 (2,4). The panels were tested in a loading frame. Pipe supports were plastered against the panel and tightened to it at 625mm centres, by 25mm bolts passing through holes left in the panels. A polythene air bag the same size as the panel under test was inflated between the backing board and the panel to apply pressure to the panel. After each increment of pressure, measured by a water manometer on the outlet side of the bag, deflection measurements were read from dial gauges. Results are summarised in Table 1 and typical load-deflection graphs shown in Figures 6 and 7.

In addition to the above tests some of the tests carried out by the author on one-third scale model brick-work and reported in his MEngSc thesis (3) may be re-examined in terms of the current theory. Results of the wall tests are summarised in Table 2.

Average initial cracking loads of panels built from full sized bricks are compared with theoretical values in Figure 8, together with the theoretical and measured variabilities. It can be seen that the mean cracking strength was closely predicted by the theory that assumed load sharing over three adjacent joints, E=3. The coefficient of variation of panel strengths corresponded more with the assumption of load sharing over two adjacent joints, E=2. It should be noted, however, that estimating the coefficient of variation from only five results is

not very reliable.

Ultimate failure loads of the panels built from full sized bricks are plotted against aspect ratio in Figure 9. It can be seen that the ultimate strength of a panel supported on four sides is practically the same as that of a similar panel supported on three sides but of half the height. This gives quantitative verification that after initial cracking a panel supported on four sides may be treated as two sub-panels each supported on three sides and free along the cracked edge. One would expect the sub-panels to have a slightly greater strength than a single three sided panel because of the enhanced self weight effects in the former and this in fact was observed in the experimental graphs of Figure 9.

Consequently the ultimate failure loads for the panels supported on four sides and also those supported on three sides are evaluated together in Figures 10 and 11. The experimental strengths are well below the predictions of conventional elastic and plastic theories as shown in Figure 10. Elastic theory using the principal moment criterion also predicts too high a strength unless the variability of joint strengths is taken into account. All of the experimental results lie between the two curves obtained using the principal moment criterion and assuming the coefficient of variation of joint strengths is zero or 0.25 with no load-sharing between adjacent joints. Assuming load sharing between joints gives better agreement with experimental results but the degree of load sharing required is not the same for all aspect ratios. The effects of using the "small" joint unit in the theory rather than the "large" joint unit is shown in Figure 11.

Also in these figures are shown the experimental and theoretical coefficients of variation of panel strength. The theory assuming load sharing over three adjacent joints, E=3, gives the closest predictions.

Analysis of initial cracking load and ultimate failure load for the one-third scale model panels is shown in Figure 12. Initial cracking load was conservatively predicted assuming load sharing over three adjacent joints, E=3, and ultimate failure load was reasonably predicted assuming load sharing over two adjacent joints, E=2.

4 CONCLUSIONS

1. A theory has been presented to predict the strengths of two-way action panels based upon an elastic principal moment failure criterion. From known joint strength properties it predicts both the mean panel strength and its variability taking into account the variability of joint strengths and their load sharing capacity.

2. The initial cracking loads of full sized and one-third sized panels simply supported on four sides were conservatively predicted by the theory assuming load sharing over three adjacent joints. Variability of these strengths were approximately predicted by assuming load sharing over two adjacent joints.

3. Ultimate failure loads of full-sized panels supported on four sides were shown to be approximately the same as similar panels of half the height supported on three sides but free at the top.

4. Average ultimate failure loads of full sized and one-third sized panels supported on three or four sides were adequately predicted by the theory assuming load sharing over two adjacent joints. Variability of these panel strengths were closely predicted by assuming load sharing over three adjacent joints.

5. In general, the proposed elastic theory based on the principal moment criterion, that allows for variability of joint strengths and load sharing between adjacent joints adequately predicts both the mean strength of a panel and its variability. Load sharing generally takes place between 2 or 3 adjacent joints.

5 REFERENCES

(1) BAKER, L.R. "The Failure Criterion of Brickwork in Vertical Flexure" Proceedings,Sixth International Symposium on Load-bearing Brickwork,December 1977,London.

(2) BAKER, L.R. "A Principal Stress Failure Criterion for Brickwork in Biaxial Bending" Paper presented at this conference.

(3) BAKER, L.R. "Brickwork Panels Subjected to Face Wind Load" M.Eng.Sc. thesis. University of Melbourne, 1972

(4) BAKER, L.R. "The Flexural Action of Masonry Structures Under Lateral Load" Ph.D Thesis. Deakin University, Geelong 1981

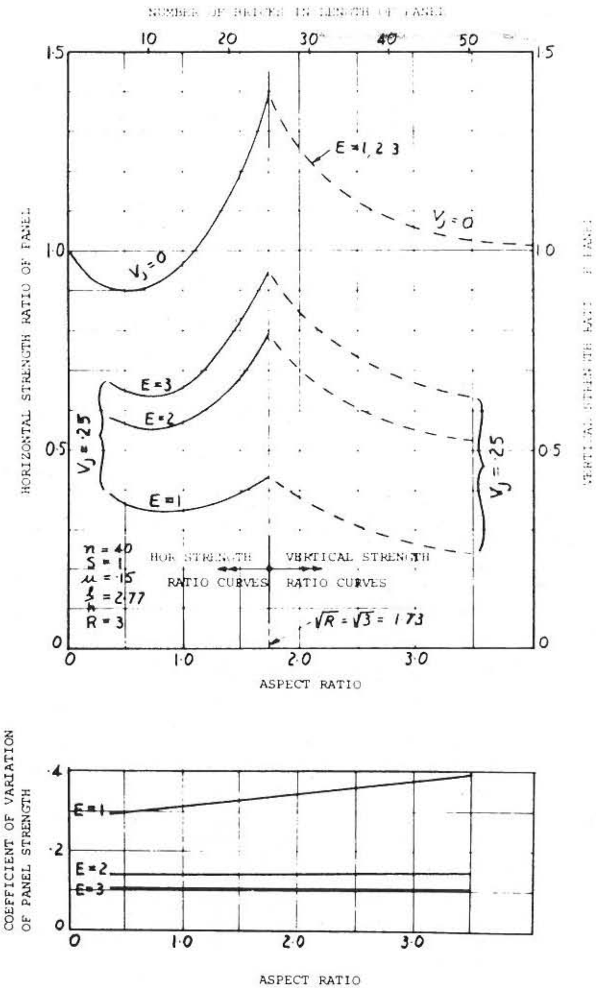

FIGURE 1: INITIAL CRACKING STRENGTH OF PANELS SUPPORTED ON FOUR SIDES.

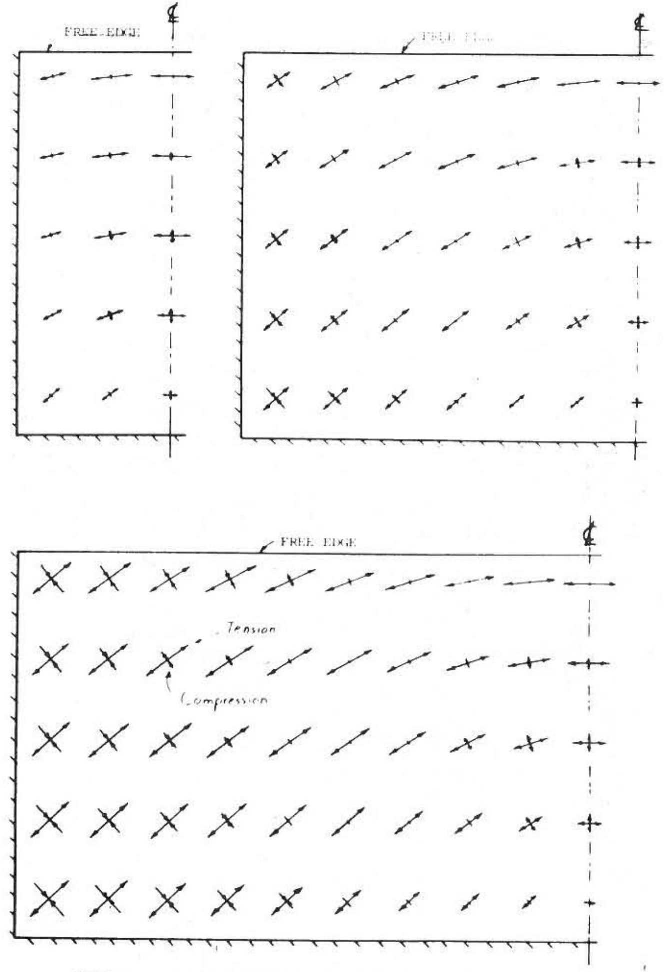

FIGURE 2: PRINCIPAL STRESS DISTRIBUTION.

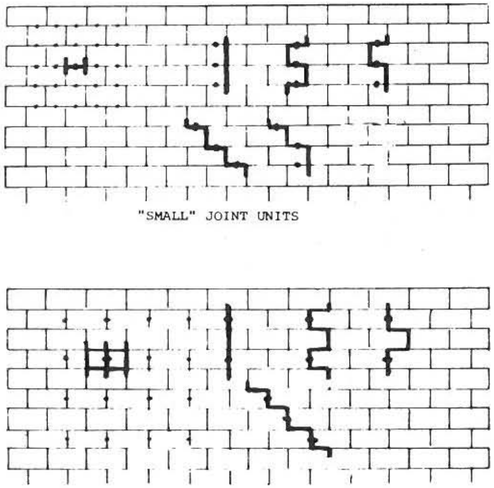

FIGURE 3: JOINTS AND CRACKING PATTERNS.

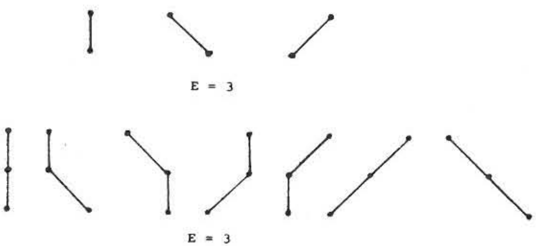

FIGURE 4: JOINT COMBINATIONS INVESTIGATION.

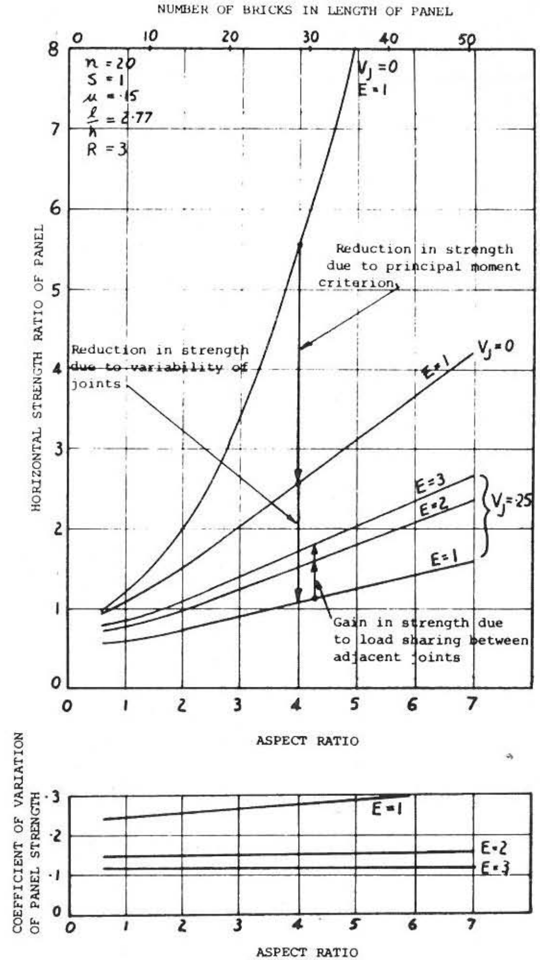

FIGURE 5: ULTIMATE STRENGTH OF PANELS SUPPORTED ON THREE SIDES.

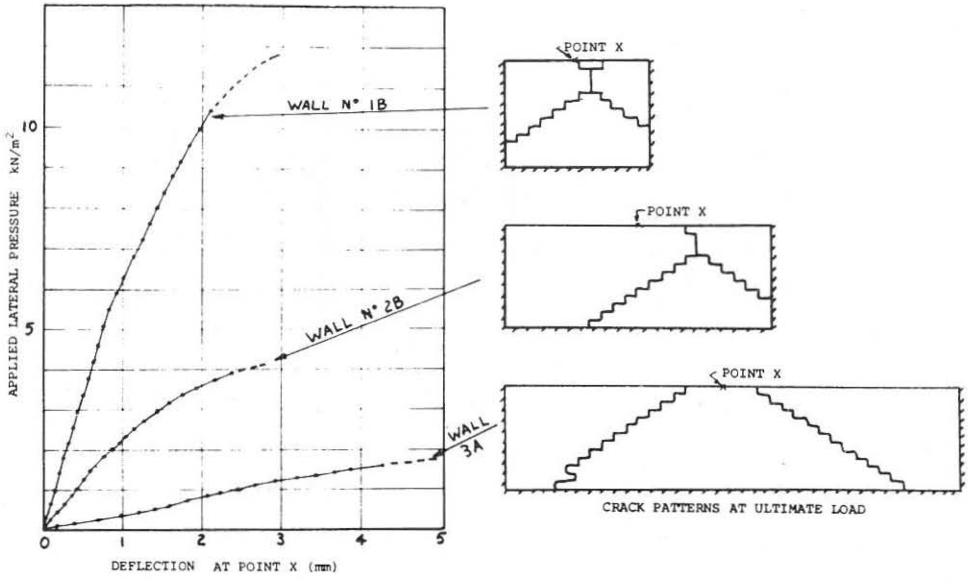

FIGURE 6: TYPICAL RESULTS FOR FULL SIZED PANELS SUPPORTED ON THREE SIDES.

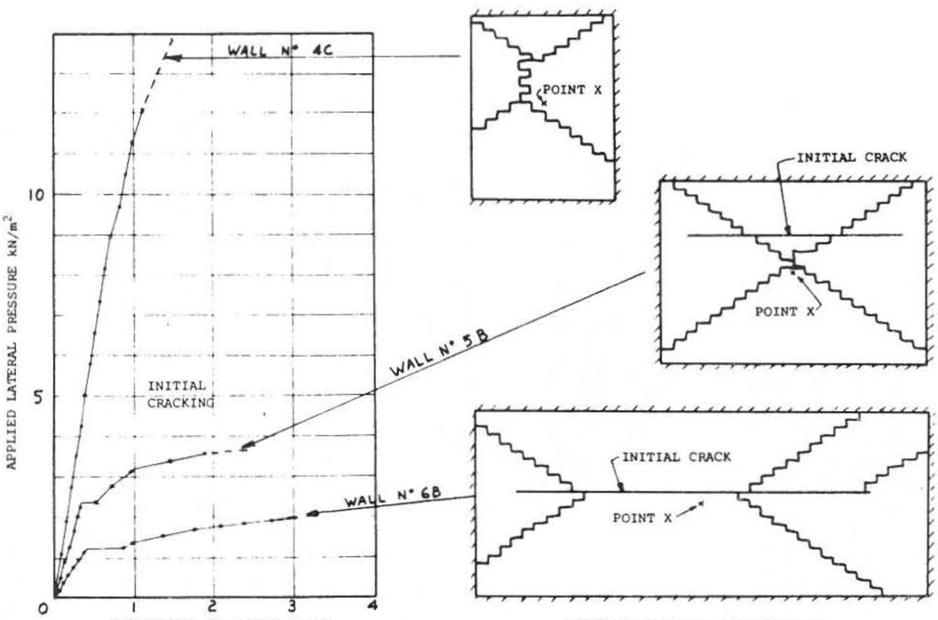

FIGURE 7: TYPICAL RESULTS FOR FULL SIZED PANELS SUPPORTED ON FOUR SIDES.

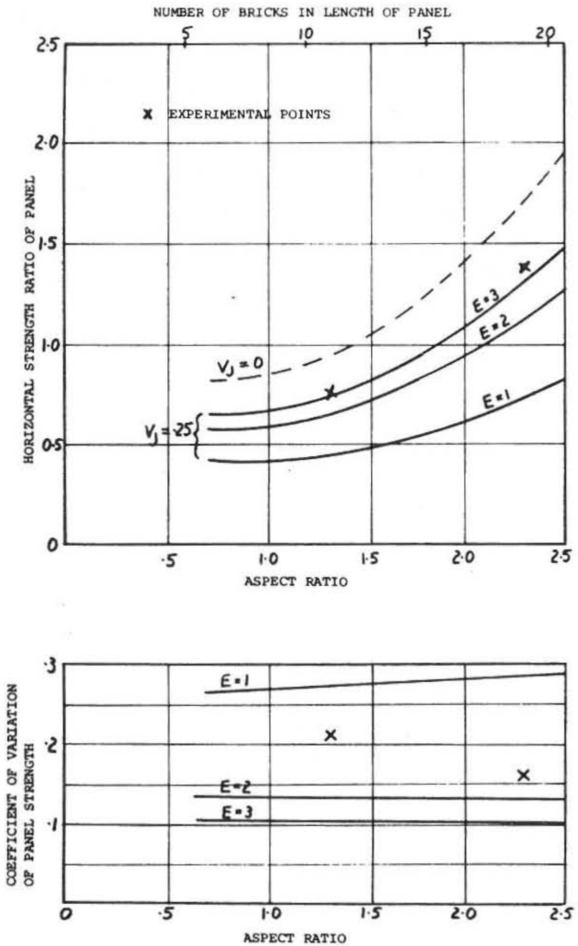

FIGURE 8: CRACKING STRENGTHS OF PANELS SUPPORTED ON FOUR SIDES.

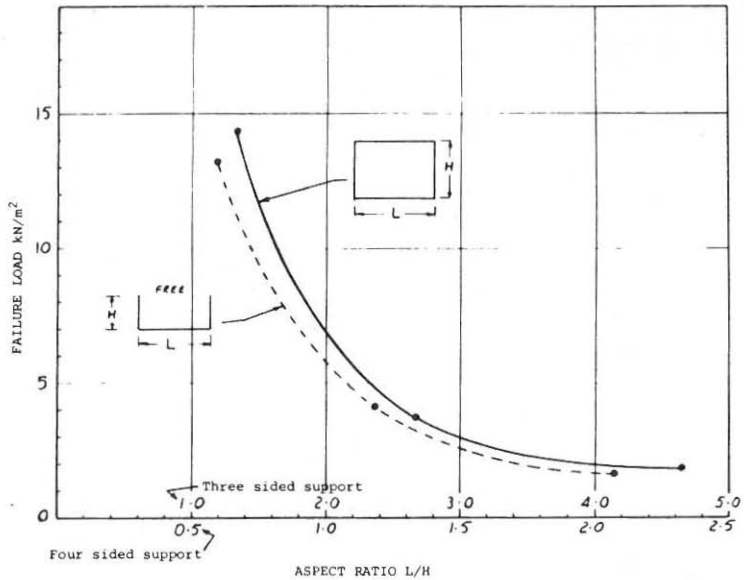

FIGURE 9: STRENGTHS OF PANELS SUPPORTED ON THREE AND FOUR SIDES.

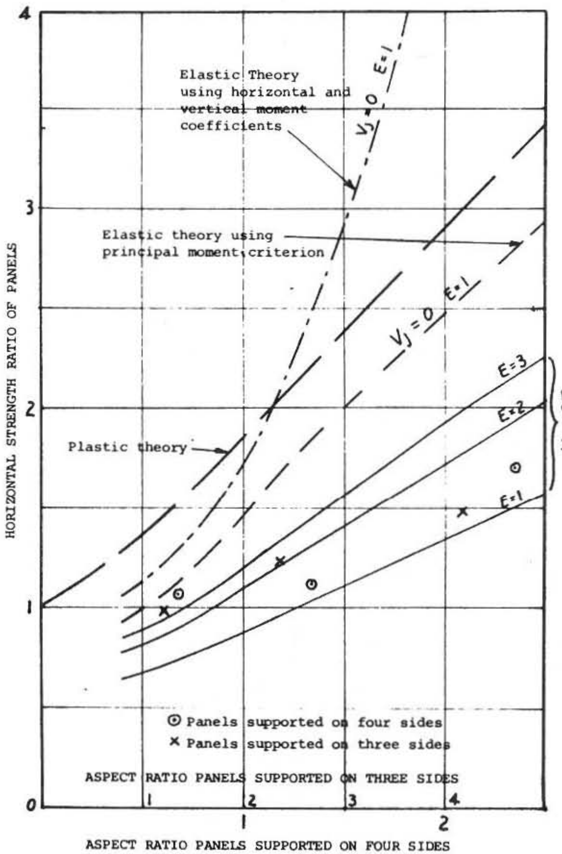

FIGURE 10: STRENGTHS OF FULL SIZED PANELS USING "LARGE" JOINT UNITS.

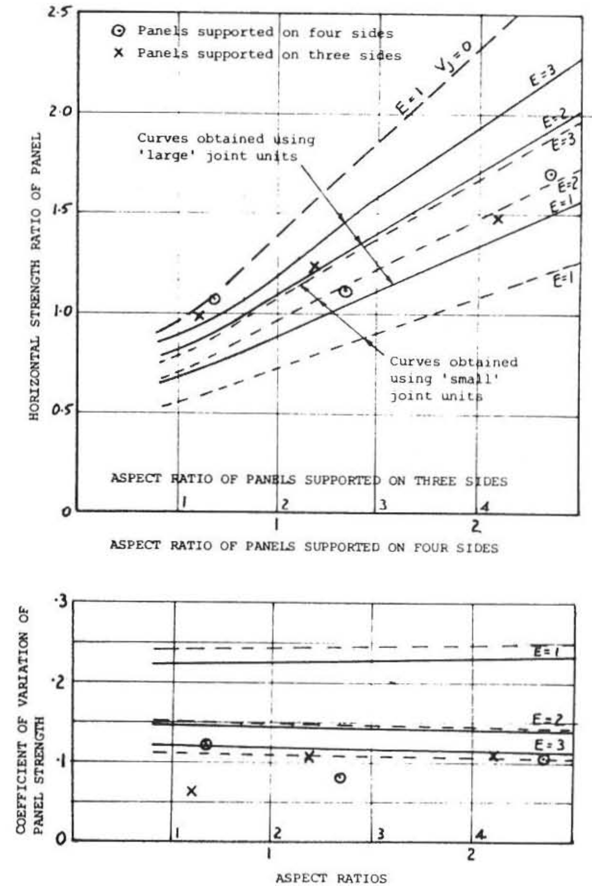

FIGURE 11: STRENGTHS OF FULL SIZED PANELS SHOWING EFFECT OF USING "SMALL" JOINT UNITS.

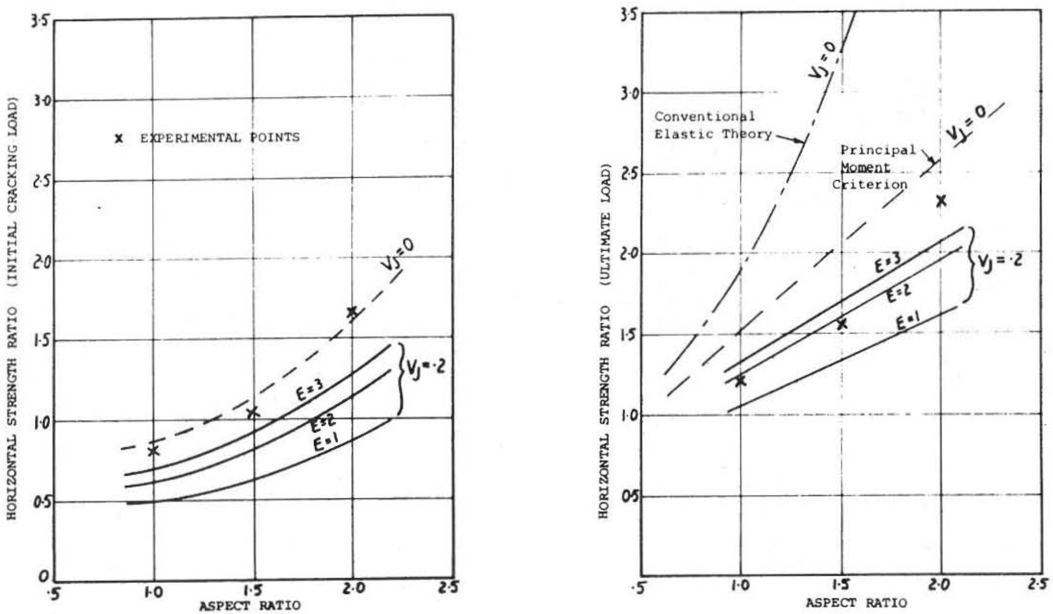

FIGURE 12: STRENGTHS OF ONE-THIRD SCALE PANELS SIMPLY SUPPORTED ON FOUR SIDES.

WALL NO:	SIZE: H x L (m)	ASPECT RATIO: H/L	NO OF SIDES SUPPORTED:	INITIAL CRACKING:		ULTIMATE FAILURE:	
				LOAD kN/m²	HORIZONTAL SPAN RATIO:	LOAD kN/m²	HORIZONTAL SPAN RATIO:
1A 1B 1C 1D 1E	1.05 x 1.05	1.19	3			14.05 11.81 12.48 13.56 13.26	Mean = 0.99 C.V. = .068
2A 2B 2C 2D 2E	1.05 x 2.49	2.37	3			3.45 4.16 3.85 4.45 4.43	Mean = 1.23 C.V. = .104
3A 3B 3C 3D 3E	1.05 x 4.35	4.14	3			1.73 1.43 1.67 1.41 1.79	Mean = 1.48 C.V. = .107
4A 4B 4C 4D 4E	1.87 x 1.25	0.67	4	Accidently Broken		12.62 16.60 13.95 13.40	Mean = 1.08 C.V. = .122
5A 5B 5C 5D 5E	1.87 x 2.49	1.33	4	2.81 2.33 1.94 2.40 3.37	Mean = 0.78 C.V. = .211	3.34 3.65 3.43 4.00 3.92	Mean = 1.11 C.V. = .080
6A 6B 6C 6D 6E	1.87 x 4.35	2.33	4	1.24 1.25 1.64 1.71 1.71	Mean = 1.39 C.V. = 0.161	1.85 1.96 1.74 1.96 2.04	Mean = 1.76 C.V. = .061

Mean Joint Strength F'_v = 0.87 MPa

Orthogonal Strength Ratio R = 3.50

TABLE 1: RESULTS OF TESTS ON FULL-SIZED PANELS WITH TWO-WAY ACTION.

WALL NO:	SIZE: HxL (m)	ASPECT RATIO H/L	NO OF SIDES SUPPORTED:	DERIVED JOINT STRENGTH: (MPa)	INITIAL CRACKING:		ULTIMATE FAILURE:	
					LOAD kN/m²	HORIZONTAL SPAN RATIO:	LOAD kN/m²	HORIZONTAL SPAN RATIO:
5 8 9	.686 x .686	1.0	4	0.82 0.74 1.17	6.56 6.80 11.67	0.73 0.84 0.91 Mean = 0.83	10.94 9.38 16.28	1.22 1.16 1.27 Mean = 1.22
6 10	.686 x 1.029	1.5	4	0.77 1.17	4.62 4.82	1.26 0.87 Mean = 1.07	5.97 8.26	1.63 1.48 Mean = 1.56
7 11	.686 x 1.372	2.0	4	0.69 1.17	3.65 4.48	1.93 1.40 Mean = 1.67	5.21 6.15	2.76 1.92 Mean = 2.34

Orthogonal Strength Ratio R = 3.44

TABLE 2: RESULTS OF TESTS ON ONE-THIRD SCALE PANELS WITH TWO-WAY ACTION.