Uniaxial and Biaxial Bending of Reinforced Brickwork Columns

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ABSTRACT

In a previous paper the authors described an iterative solution for the analysis of rectangular reinforced brickwork columns subjected to biaxial bending. In this paper the method is developed further and interaction diagrams included which enable the designer to consider the effect of different eccentricities of loading about both axes. The curves are suitable for solving for both uniaxial and biaxial bending.

A brief description of laboratory tests on columns is also included and a comparison made of experimental and theoretical values.

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INTRODUCTION

Columns of reinforced concrete loaded eccentrically about one axis have been studied extensively and some recommendations have also been put forward for uniaxial bending of reinforced masonry. In the case of biaxial bending of reinforced concrete columns it is usual to consider the two axes separately and to use some combination of two separate uniaxial cases.

Very little work appears to have been done for the case of biaxial bending in columns of reinforced brickwork and a procedure for the design of such columns is outlined in this paper. Only one type of rectangular section is considered and although the number of interaction diagrams is limited to three cases it will be apparent that the method can be extended to other sections and additional load cases.

Biaxial bending can arise directly from an applied load which is eccentric to both axes or indirectly from forces introduced by wind or earthquake and it is important that the columns are adequately designed to allow for these moments.

The problem is non-linear in that the bending produces a deformation which introduces secondary bending and the procedure used for obtaining a solution is based on an iterative method similar to that used by Farah and Huggins (1) for columns of reinforced concrete. The columns are first subdivided into sections and a triple iteration procedure used 1) across the section 2) between adjacent cross-sections 3) over the column length. (2).

BASIS OF THE METHOD

Since the eccentricities at any section will consist of the algebraic sum of the applied eccentricity and a component of the deflection it follows that a knowledge of the variation of the eccentricity implies a knowledge of the deflected form.

(a) Initially the following assumptions are made with respect to the central section.

1. The eccentricities
2. The corner strains

The forces and moments at the central section can now be calculated independently on the basis of the two assumptions and any differences corrected for, by changing the corner strains incrementally. The calculation of the forces and moments from the assumed corner strains is based on an integration of the stresses over the section.

(b) Once the two sets of forces and moments are within acceptable limits the analysis proceeds to the next section by first calculating the value of the eccentricity at the second section as a function of the previous value and the curvature. This requires a second iteration since the curvature is itself a function of the two eccentricities.

The corner strain at the second section are now assumed and the first iteration repeated at section two until the two sets of forces and moments are in agreement.

(c) Finally the process reaches the top of the column and the calculated eccentricity compared with the known applied eccentricity. If the difference is not within acceptable limits the whole process is repeated by incrementing the assumed values of the eccentricities at the central section.

This would be very tedious for hand calculation and a computer program has been developed in Fortran to carry out the iterative processes. A flow chart for this program is shown in Fig. 1. In this chart the second iteration between adjacent cross-sections has been omitted since it was found that, for the sections considered, the inclusion of this iteration did not greatly affect the results.

MATERIAL PROPERTIES AND GEOMETRICAL DATA

Stress-strain relationships must be defined for the three materials used in the construction of the columns and for this present work the relationships used are as shown in Figs. 2 and 3.

Although the computer programme is written in non-dimensional form it is based on columns of rectangular section similar to that shown in Fig. 4. The modification required to analyse other sectional forms would present no difficulties.

THEORETICAL RESULTS

Interaction diagrams are shown in Figs. 5, 6 and 7 for a typical rectangular section for which $d_1 = 0.9T$ and $d_2 = 0.85B$. The diagrams are based on values of the axial load P of 0.2, 0.3 and 0.4.

where $P = N/f_m BT$

For these interaction diagrams it has been assumed that the grout and the brickwork have identical stress-strain characteristics.

A set of these charts could be produced for other load cases and other rectangular sections with different values of d_1 and d_2. The charts shown have been derived using factors of safety of 2.5 for brickwork and 1.15 for steel so that ultimate values of the moments are indicated.

The charts can be used in two ways.

1. Knowing the applied axial load and the moments about both axes, it is first necessary to select the appropriate chart for the axial load (or interpolate between two charts). Then by locating the interaction point using the known values of moments, the required amount of steel can be ascertained.

2. Knowing the axial load and the area of steel used, suitable values of the ratio of M_x to M_y can be determined.

Example 1:

Consider a rectangular column with B = 100mm, T = 200mm, $d_1 = 0.9T$ and $d_2 = 0.85B$. Assuming that the axial load (N) is 300 kN and the moments are $M_x = 7.2$kNm and $M_y = 2.4$kNm. The required area of steel can be determined as shown below. The design stresses are taken as $f_m = 15\text{N/mm}^2$ and $F_y = 460\text{N/mm}^2$.

Solution

$$P = N/f_m BT = \frac{300 \times 100}{15 \times 100 \times 200} = 0.30$$

$$M_x/f_m BT^2 = \frac{7.2 \times 1000 \times 1000}{15 \times 100 \times 200 \times 200} = 0.12$$

$$M_y/f_m\ TB^2 = \frac{2.4 \times 1000 \times 1000}{15 \times 100 \times 100 \times 200} = 0.08$$

From chart (Fig. 6)

$$A_s/f_m\ BT = 13 \times 10^{-4}$$

i.e.
$$A_s = \frac{13 \times 15 \times 100 \times 200}{1000} = \underline{390}\ \text{mm}^2$$

Example 2:

Using the same section and design stresses as given in Example 1 and assuming that the steel area is known to be 390mm^2, then safe combination of M_x and M_y can be determined from the charts as shown below,

$$\text{Calculate}\ \frac{N}{f_m\ BT} = \frac{300}{15 \times 100 \times 200} = 0.3$$

Therefore use <u>Fig. 6</u>

$$\frac{A_s}{BT\ f_m} = \frac{390}{100 \times 200 \times 15} = 13 \times 10^{-4}$$

The broken line lying between $A_s/BT\ f_m$ values of 15×10^{-4} and 10×10^{-4} shown in Fig. 6 represents safe combinations of M_x and M_y.

EXPERIMENTAL INVESTIGATION

A test rig was constructed for application of axial loads and biaxial bending to columns constructed of half scale brickwork. Details of the test rig are shown in Fig. 8.

Initial tests indicated that there were inherent weaknesses in the rig construction which required modification. These changes were concerned mainly with the support conditions at either end and the final arrangement the axial load was applied through steel balls placed at both end of the column.

The results obtained from three subsequent tests are shown below.

Test No.	Ult. axial load kN	Measured Values (kNm) Mu_x	Mu_y	Theoretical Values (kNm) Mu_x	Mu_y
1	235.8	10.95	4.07	10.08	4.48
2	235.8	9.12	4.98	8.19	4.71
3	**294.8**	**7.08**	**3,01**	**8.65**	**4.27**

The deflection profiles determined using the computer programme, for constant axial load P and different combinations of M_x and M_y are shown in Figs. 9 and 10. The measured values obtained from the tests are also shown on the same diagrams.

CONCLUSIONS

The limited number of practical tests completed to date shows that the theoretical results are conservative. Additional testing is required before firm conclusion can be drawn but the results indicate that the theoretical approach gives results which are in close agreement with the measure values.

REFERENCES

1. FARAH, A. and HIGGINS, M.W. 'Analysis of Reinforced concrete columns subjected to Longitudinal Load and biaxial bending'. ACI Journal, July 1969, pp. 569 - 575.

2. DAVIES, S.R. and ELTRAIFY, E.A. 'Biaxial Bending of Reinforced Masonry Columns'. 5th International Masonry Conference, Washington, 1979.

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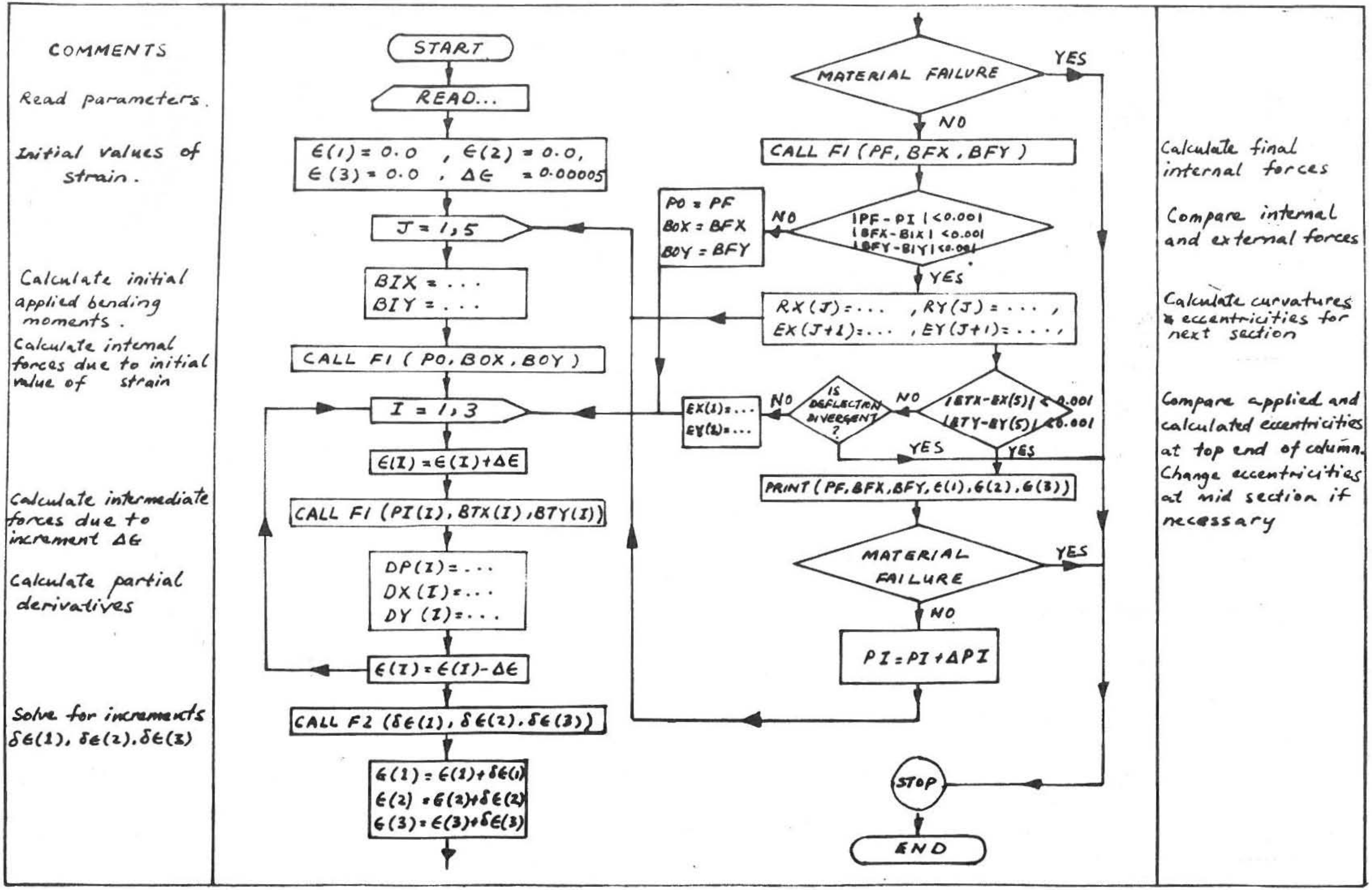

COMMENTS
Read parameters.
Initial values of strain.
Calculate initial applied bending moments.
Calculate internal forces due to initial value of strain
Calculate intermediate forces due to increment Δϵ
Calculate partial derivatives
Solve for increments δϵ(1), δϵ(2), δϵ(3)
START
READ...
ϵ(1) = 0.0 , ϵ(2) = 0.0,
ϵ(3) = 0.0 , Δϵ = 0.00005
J = 1, 5
BIX = . . .
BIY = . . .
CALL FI (PO, BOX, BOY)
I = 1, 3
ϵ(I) = ϵ(I) + Δϵ
CALL FI (PI(I), BTX(I), BTY(I))
DP(I) = . . .
DX (I) = . . .
DY (I) = . . .
ϵ(I) = ϵ(I) - Δϵ
CALL F2 (δϵ(1), δϵ(2), δϵ(3))
ϵ(1) = ϵ(1) + δϵ(1)
ϵ(2) = ϵ(2) + δϵ(2)
ϵ(3) = ϵ(3) + δϵ(3)
MATERIAL FAILURE
YES
NO
CALL FI (PF, BFX, BFY)
PO = PF
BOX = BFX
BOY = BFY
NO
\|PF - PI\| < 0.001
\|BFX - BIX\| < 0.001
\|BFY - BIY\| < 0.001
YES
RX(J) = . . . , RY(J) = . . . ,
EX(J+1) = . . . , EY(J+1) = . . . ,
EX(1) = . . .
EY(1) = . . .
NO
IS DEFLECTION DIVERGENT ?
NO
\|BTX - BX(5)\| < 0.001
\|BTY - BY(5)\| < 0.001
YES
YES
PRINT (PF, BFX, BFY, ϵ(1), ϵ(2), ϵ(3))
MATERIAL FAILURE
YES
NO
PI = PI + ΔPI
STOP
END
Calculate final internal forces
Compare internal and external forces
Calculate curvatures & eccentricities for next section
Compare applied and calculated eccentricities at top end of column. Change eccentricities at mid section if necessary

FIG. 1

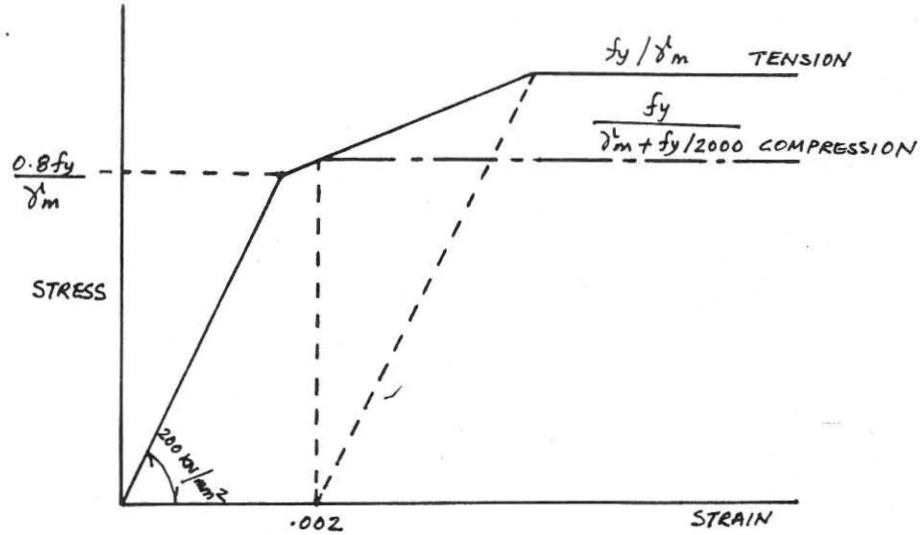


FIG 2. Short-term design stress-strain relation for reinforcement (fy in N/mm²)

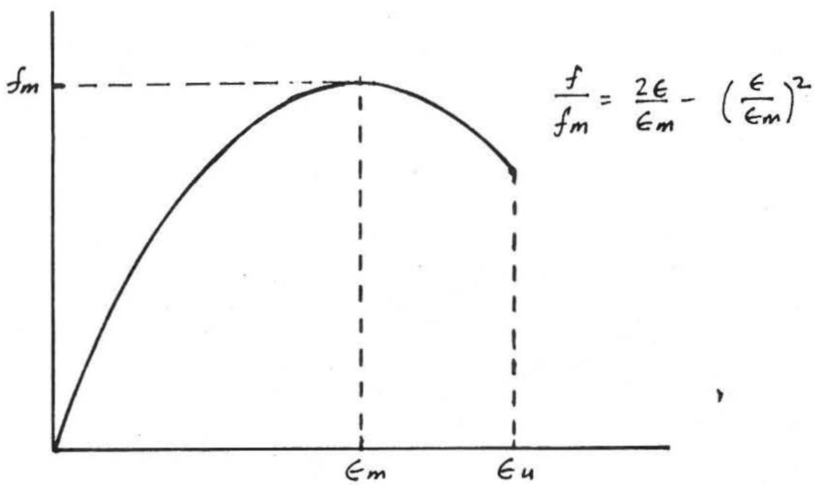


FIG. 3. Stress - strain relationship for brickwork (and grout).

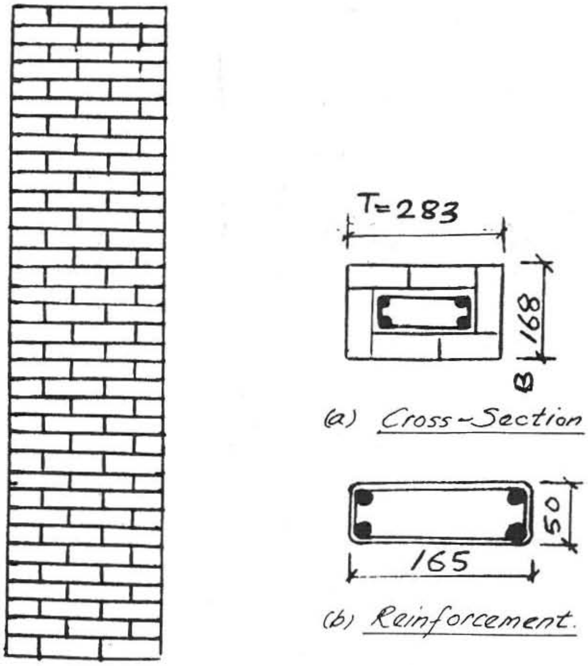


FIG 4.

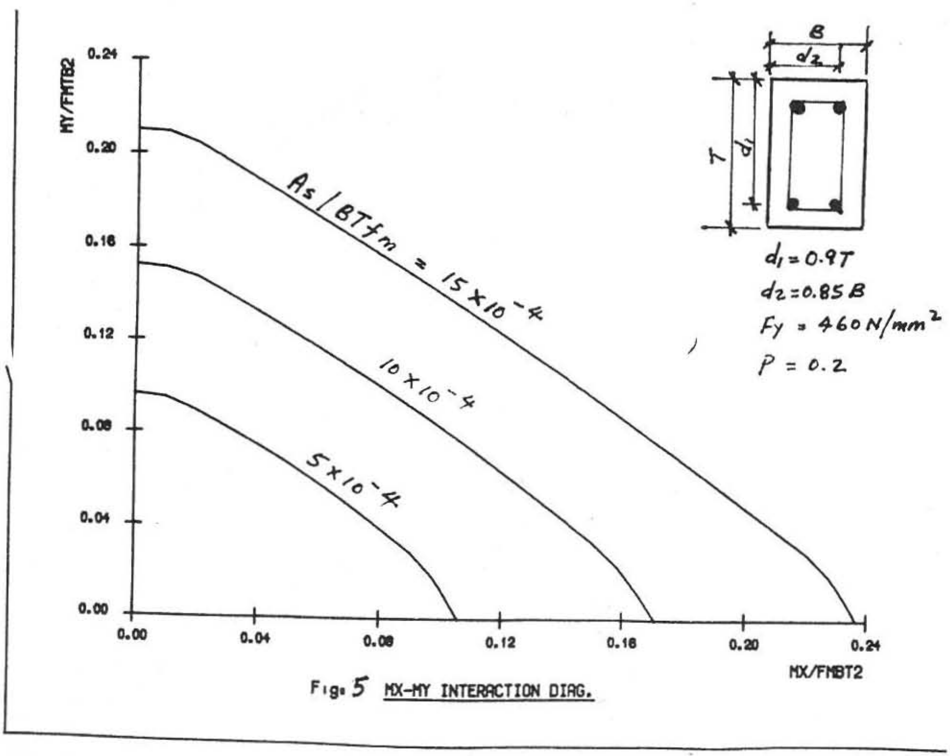


Fig. 5 MX-MY INTERACTION DIAG.

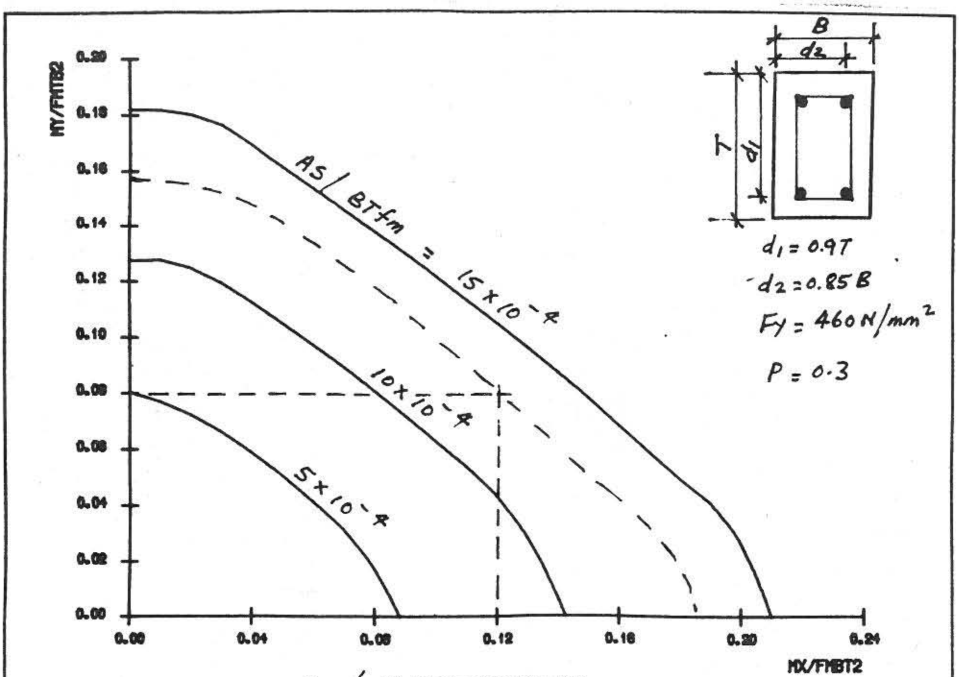


Fig. 6 MX-MY INTERACTION DIAG.

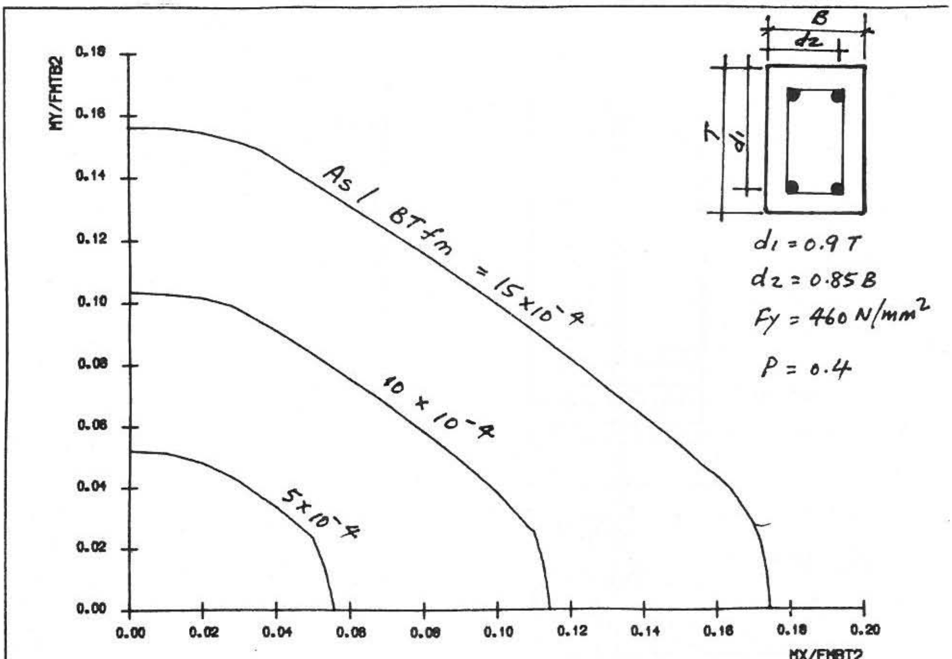
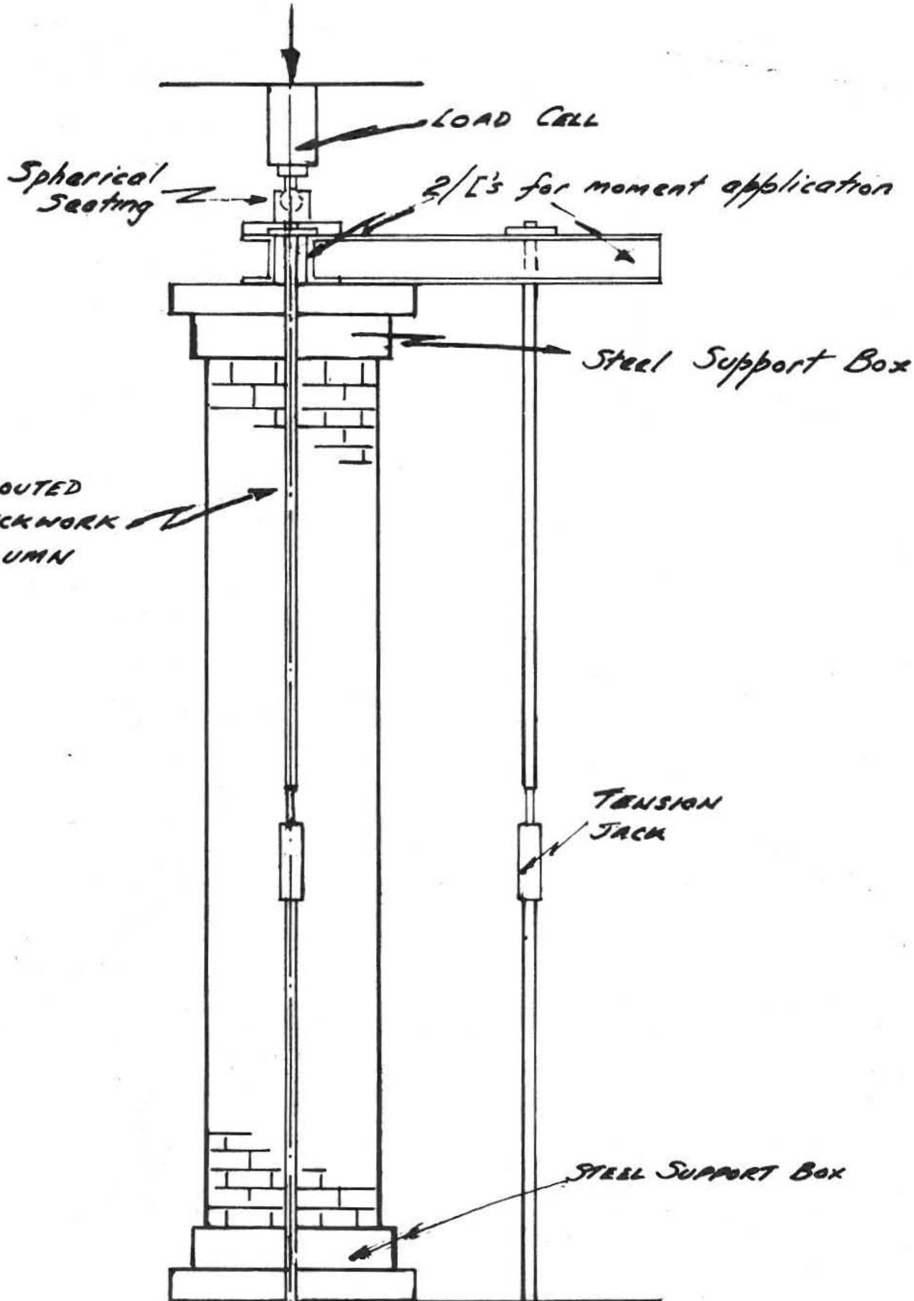
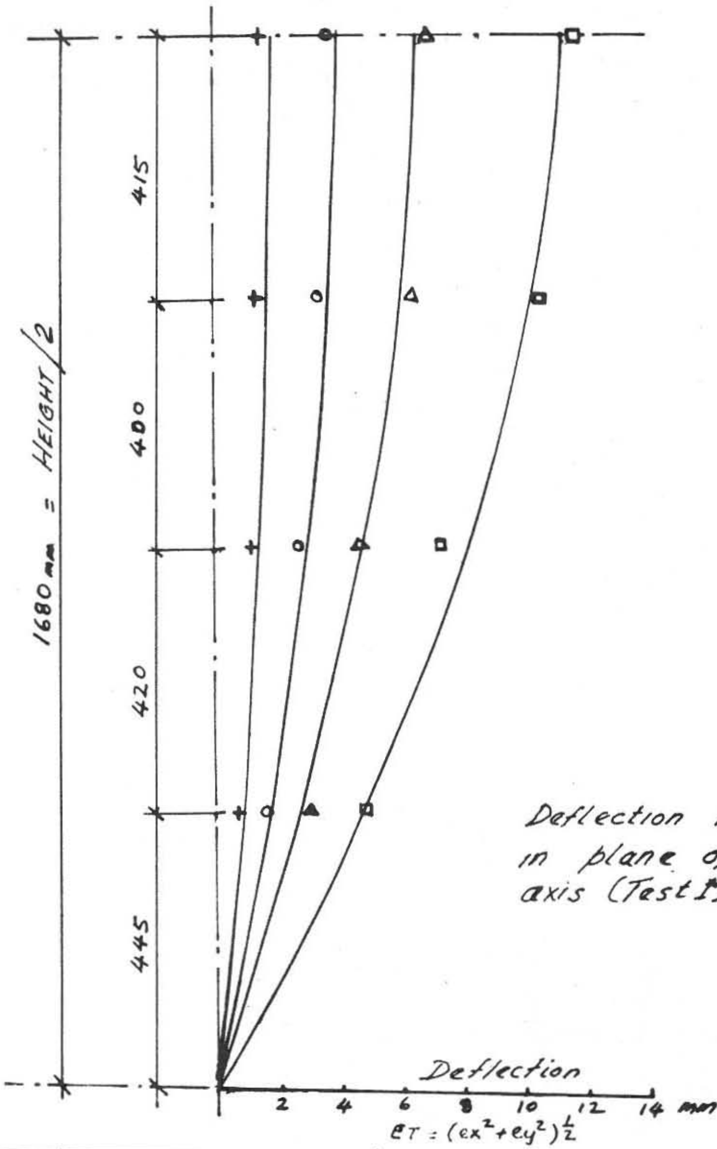


Fig. 7 MX-MY INTERACTION DIAG.



LOAD CELL
Spherical Seating
2/[s for moment application
Steel Support Box
GROUTED BRICKWORK COLUMN
TENSION JACK
STEEL SUPPORT BOX
Spherical Seating.

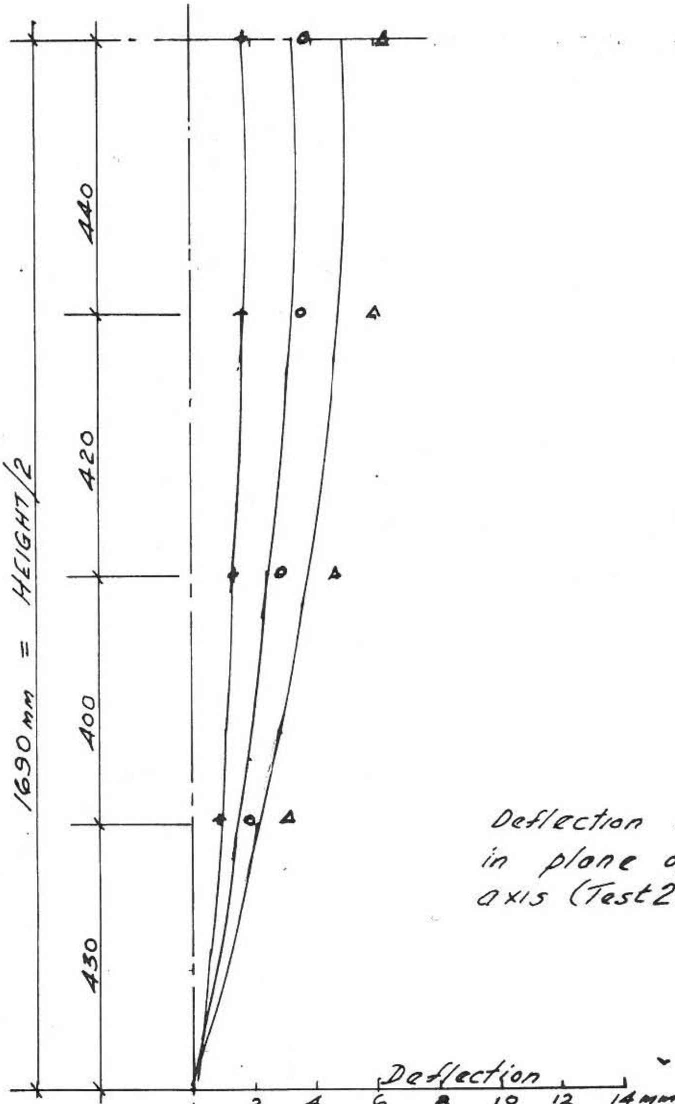
FIGURE 8



Deflection Profile in plane of neutral axis (Test I)

LOAD STAGE	N (KN)	M_x (KNm)	M_y (KNm)
1 +	235.8	2.01	1.00
2 o	235.8	4.00	2.00
3 △	235.8	6.90	3.00
4 □	235.8	9.00	3.97

FIGURE 9



Deflection Profile in plane of neutral axis (Test 2)

LOAD STAGE	N (kN)	Mx (kNm)	My (kNm)
1 +	294.8	2.03	1.01
2 o	294.8	3.04	2.03
3 Δ	294.8	4.02	3.02

FIGURE 10