Force Eccentricities in Load Bearing Masonry Walls

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Olika teorier för beräkning av väggars utböjning jämförs. Ändvinkeländringen hos väggen jämföres med ändvinkeländringen hos plattan som vilar på väggen och med vinkeländringen i knutpunkten. Ändvinkeländring och vinkeländring i knutpunkten är avgörande storheter för beräkning av kraftexcentriciteten i väggen. Försöksresultat är fåtaliga.

Abstract

Different theories of wall deflection are reviewed and compared. The magnitudes of end rotations are compared and related to the end rotation of the slab resting on the wall and to the joint yielding. End rotations and joint yielding are decisive factors for estimating the force eccentricities in the walls. Data from tests are scarce.

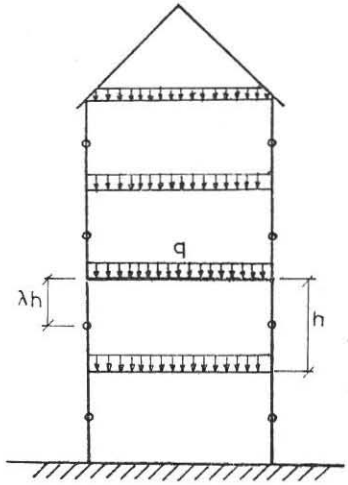


Fig. 1 - Simplified building structure with loadbearing walls.

In buildings with loadbearing masonry walls the external walls have the largest force eccentricities. The external walls are normally bent in S-shape with an inflexion point at about mid-height. Fig. 1. A real building has of course internal walls crosswalls and so on.

The eccentricity depends upon several parameters: Slab stiffness, wall stiffness, joint behaviour, axial load magnitude, slab span and wall height.

The condition that determines the eccentricity is the geometrical relation

$$\varphi_v + \theta = \varphi_h \tag{1}$$

where φ_v is the end rotation of the wall at the joint, θ is the joint deformation due to cracking and/or plastification in the joint, and φ_h is the slab end rotation.

Sometimes the joint rotation θ can be assumed to be

$$\left.\begin{array}{lll} \theta = 0 & \text{for} & M < M_{p\ell} \\ 0 < \theta < \theta_{ult} & \text{for} & M = M_{p\ell} \end{array}\right\} \tag{2}$$

See figure 2.

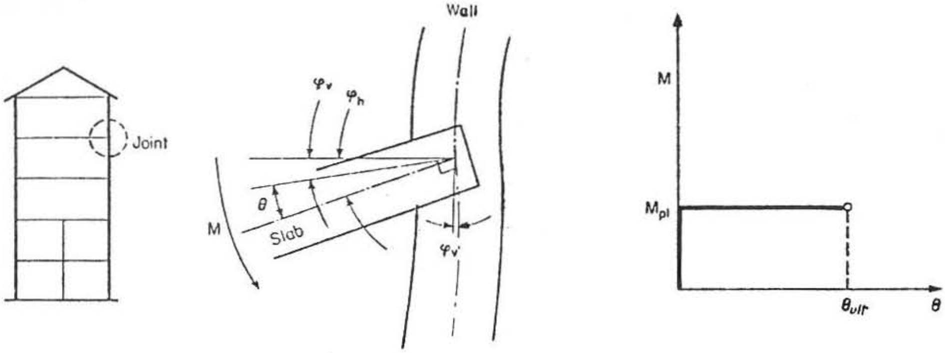


Fig. 2 - Type of joint and idealized ralationship between applied moment M and angle of rotation θ in a joint. θ_{ult} equals ultimate rotation. $M_{p\ell}$ equals "yielding" moment.

We will now turn to the determination of φ_h, φ_v and θ more in detail.

Deflection of Wall

We consider a load-carrying structure of the type shown in Fig. 1. The loads q produce bending moments and normal forces acting on the walls. A feature which is common to all storeys is the fact that the eccentricities of the normal force in a wall are opposite in direction at the top and bottom ends of the wall. The typical case of loading represented in Fig. 3, is obtained when the wall is one-storey high.

In order to calculate the moment distribution in the load-carrying structure shown in Fig. 1, it is necessary to compute the angle of rotation φ_v of the wall as a function of a load which is applied so that its eccentricities are opposite in direction at the top and bottom ends of the wall. This calculation is complicated by the circumstance that the wall has little or no tensile strength.

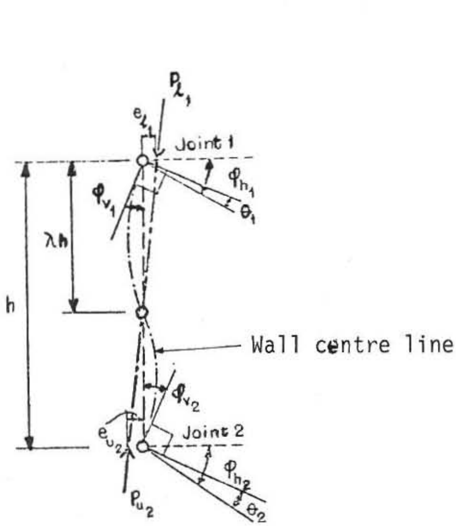


Fig. 3 - General representation of loads and deformations in the case of a wall is one-storey high.

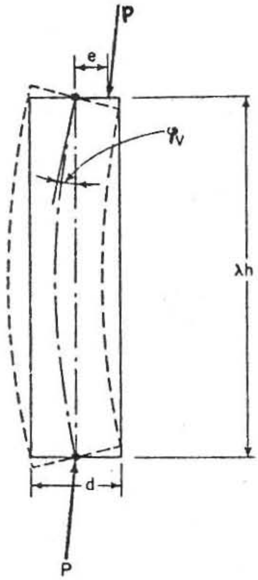
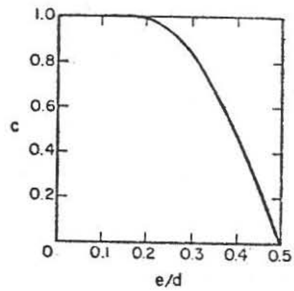


Fig. 3a - Wall subjected to eccentric compression; the part of the wall comprised between the floor slab and the point of inflection.



$$\varphi_V = \frac{P\,e \cdot \lambda h}{c\;3\;E_V I_V}$$

Fig. 3b - Effect produced by the eccentricity of the load on the real flexural rigidity of a masonry wall whose tensile strength is assumed to be equal to zero. [1] NYLANDER.

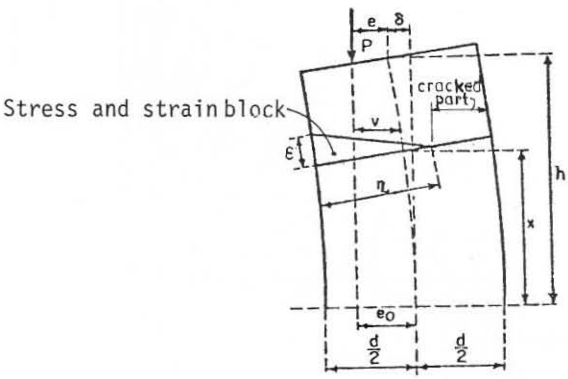


Fig. 4 - Elementary part of a masonry wall in a deflected state. Notations. [2] ANGERVO and PUTKONEN.

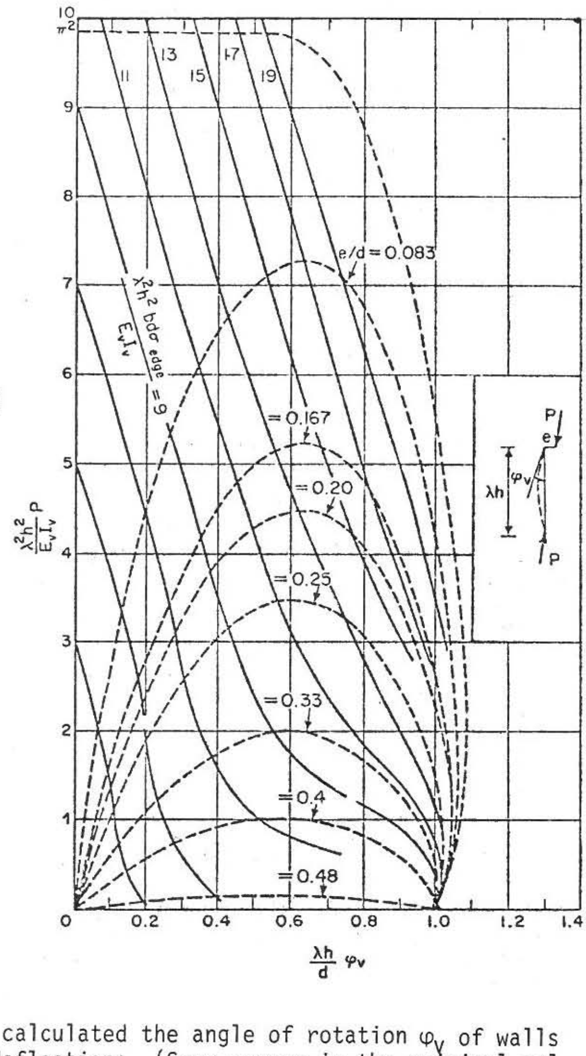
The problem under consideration has been dealt with in the literature by means of two methods of calculation which are different in principle. One of these methods takes no account of the additional deflections which give rise to changes in the effective cross-sectional area. In the other method, again, the additional deflections and the changes in the cross-sectional area resulting from them are taken into account, sometimes exactly and sometimes approximately.

A method of calculation in which the additional deflections are taken into account in the determination of the angles of rotation at the top and bottom ends of the wall has been devised by [4] ANGERVO and PUTKONEN.

The basic equations were solved by [2] ANGERVO as well as by [5] CHAPMAN and SLATFORD (with initial deflections) starting out from the deflected column part shown in Fig. 4.

An evaluation of the theoretical results give, for a column eccentrically loaded at one end (at the joint) and centrally loaded at the other end (at the inflexion point), relations shown in Fig. 5. Also curves for constant stresses are traced in the figure.

Fig. 5 - Angles of rotation of, and edge stresses in, a wall having no tensile strenght. One end of the wall is submitted to an eccentric load, while the other end is subjected to a central load. [6] SAHLIN

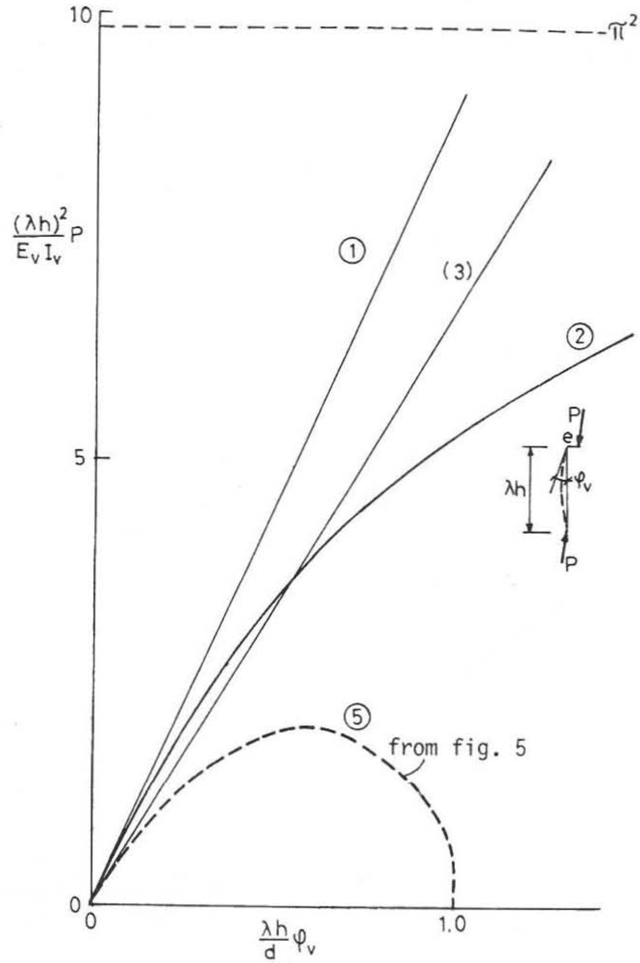


[3] KAZINCZY and [1] NYLANDER have calculated the angle of rotation φ_V of walls while disregarding the additional deflections. (Some errors in the original calculations have been corrected in Fig. 3 b). With the notations used in Fig. 3, we get

$$\varphi_V = \frac{P\ e \cdot \lambda h}{c\ 3\ E_V I_V} \tag{3}$$

where $E_V\ I_V$ is the flexural rigidity of the wall, and c is obtained from the diagram shown in Fig. 3b. For constant e this expression gives a straight line marked (3) in Fig. 6

Fig. 6 - Angles of rotation of wall for $e/d = \frac{1}{3}$ from different theories.



[7] FRISCH-FAY has further studied the relationship for masonry walls with some tensile strength, as well as [5] CHAPMAN and SLATFORD.

[8] PARLAND has studied the effect of incomplete cracking, i.e. cracks open up in bed joints but not in the masonry blocks themselves. This results in stiffer columns. For masonry units with the height-thickness ratio 1 the stiffness will increase by about 10 % for $\frac{e}{d} = \frac{1}{3}$ and up to 100 % for large eccentricities.

An totally uncracked column is of course even stiffer than ekv (3) indicates (for $c < 1$), see the solid line marked 1 in fig. 6.

If the second-order effects are taken into account the uncracked column will follow the line marked 2.

To sum up. If second-order effects are not taken into account the column behaviour should lie somewhere between lines 1 and (3) depending upon the extent of cracking. If the second-order effects are taken into account the column behaviour should lie somewhere between the curves 2 and 5 with curves based on Angervos, Parlands and Frisch-Fays equations in between.

Judged from Fig. 6 the differences seem to be tremendous. In the realistic load-range the differences are very small; what matters most is whether the column is assumed to crack or not (for large eccentricities).

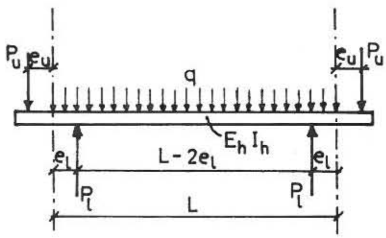
At $\frac{P \lambda^2 h^2}{E_v I_v} = 1$, the stress curves from Fig. 5 give $\frac{\lambda^2 h^2 b\ d\ \sigma}{E\ I} \approx 4$

For a rectangular column with $I = \frac{b\ d^3}{12}$ and $\frac{\lambda h}{d}$ set to 10 this means $\sigma = E/300$ which is a high edge stress for normal masonry walls. Thus in this case the differences between φ_v according to 3 and 5 is less than 20 %, and the absolute value about 0.02.

Deformation of Floor Slab

In the case where a floor slab forms part of a load-carrying structure of the type represented in Fig. 1, the slab is in principle submitted to the loads shown in Fig. 7.

Fig.7 - Loads acting on a floor slab which forms part of a loadcarrying structure of the type shown in Fig. 1



The angle of rotation at the support is

$$\varphi_h = \frac{q\ L^3}{24\ E_h I_h} - (P_u e_u + P_\ell e_\ell) \frac{L}{2\ E_h I_h} \tag{4}$$

if $e_\ell << L$

This method of calculation is applicable on the assumption that the flexural rigidity of the floor slab does not change under the action of the load. This assumption is normally not fulfilled when the slab is made of some common material, such as ordinary concrete, lightweight concrete, timber, etc. However, if the value of $E_h I_h$ is appropriately chosen, then Eq. (4) can be used for a study of the phenomena in question.

As can be seen from Eq. (4) the angular rotation of the slab-end depends upon a number of factors among other the geometry of the slab and the modulus of elasticity. Furthermore it depends upon the load and the force eccentricity above and below the slab. Let us now for comparison take an ordinary concrete slab with a thickness of about 18 cm and a span of 4 meters and apply the formula (4). The assumption that the eccentricities in the wall are 0 gives a maximum value for φ_h. Furthermore let us assume that $\frac{\lambda h}{d}$ is 10 before. Under these assumptions we calculate $\frac{\lambda h}{d} \cdot \varphi_h$ to 0.016 which is of the same order of magnitude as the angular rotation of the wall end.

For a very rigid wall with high vertical axial loads equation (4) will give end rotations of the slab approching 0 which enables us to calculate e directly from (4).

The calculation of the slab end rotations will not be carried further since this is a rather simple problem. Instead we turn our interest to the deformation and strength of the joint between the slab and the wall.

Deformation and Strength of Joints

[9] EMPERGER has investigated concrete beams, and [3] KAZINCZY has examined steel beams, which were built in at the supports in brick masonry walls. These by now old investigations corroborate the assumption that the behaviour of the joint between a concrete slab and a brick masonry wall shall in some measure be plastic. The details of this behaviour must be determined by tests.

The difference in the angle of rotation between the slab and the wall is influenced not only by the effect of splitting, but also by the plastification of the mortar, the bricks, and the slab at the joint between the slab and the wall. This effect of plastification can scarcely be calculated, and must therefore be determined by means of tests. The main difficulty met with in this connection is to separate the effects of plastification and splitting. Consequently, it seems justifiable to take account of the above-mentioned two effects at the same time, and to investigate experimentally the total difference in the angle of totation between the wall and the floor slab. This total difference is denoted by θ.

The only possibility to investigate the values of θ is to perform full scale tests. Such tests have been reported by [10, 6] SAHLIN and by [11] GERMANINO and MACCHI, [12] HENDRY and others. Since this discussion only is on a qualitative basis we here constrain ourselves to briefly recure the 1959 tests.

In the theoretical studies, we assume the following expressions for the angle of rotation θ of the joint:

$$\begin{aligned} &\theta = 0 \text{ for } 0 \leqq M < M_{p\ell} \\ &0 \leqq \theta \leqq \theta_{ult} \text{ for } M = M_{p\ell} \end{aligned} \qquad (5)$$

The ultimate value of the angle of rotation, θ_{ult}, is supposed to be dependent on the normal force transmitted in the joint.

Experimental evidence

Full scale tests, [6, 10] SAHLIN show that a considerably plasticity in the joints occur above a certain moment. In Fig. 8 the test set-up (1959) is exemplified and in Fig. 9, some typical test results are shown.

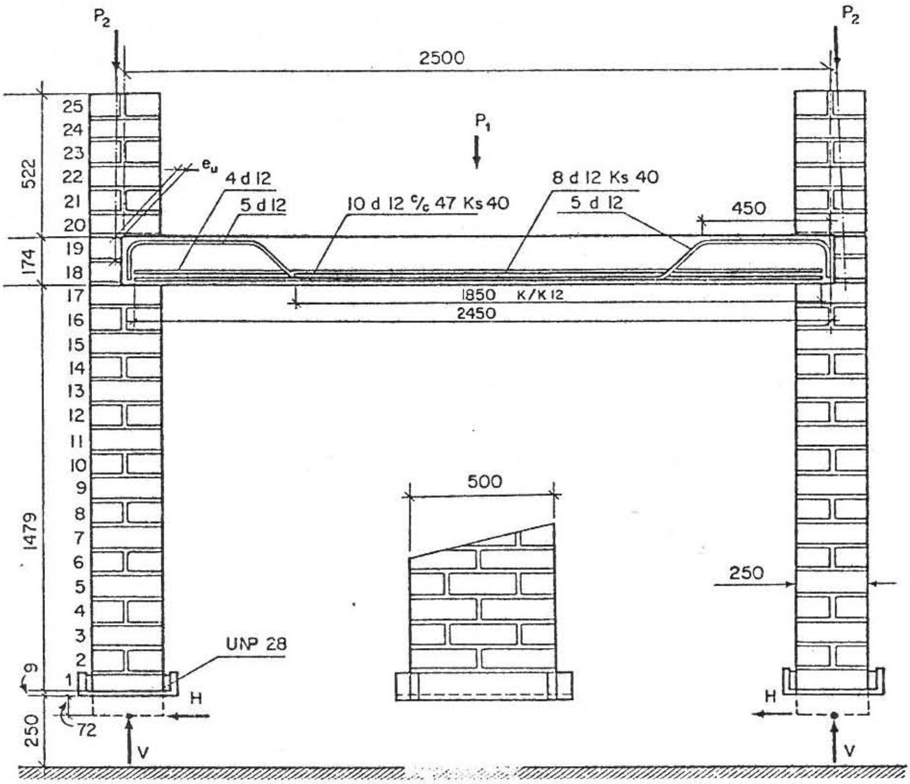


Fig. 8 - Full scale test specimen simulating external walls in an intermediate story in a multistory building. All distances are in millimeters. SAHLIN 1959.

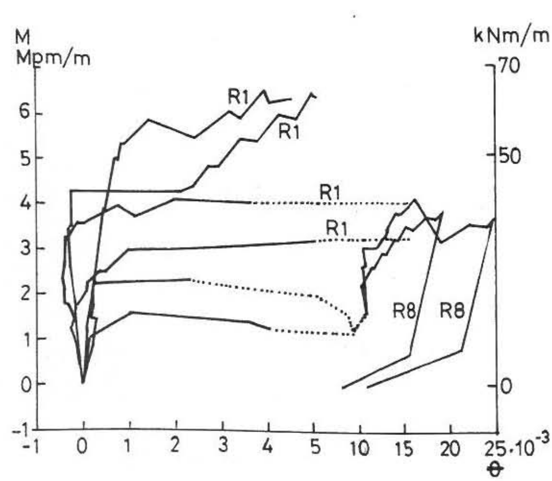


Fig. 9 - Relation between the restraining moment acting on the slab and the angle of rotation of the joint. Walls built with Type KC 2:1:15 lime and cement mortar.

Furthermore the tests showed that the ultimate rotation in the joints decreased with increased axial load. P. For a certain type of test the following relation was obtained

$$\theta_{ult} = \theta_0 \, (1-\kappa) \tag{6}$$

where

$$\theta_0 \approx 0.03, \quad \kappa = P/P_{ult},$$

and

$$P_{ult} = \text{failure load for concentric loading}$$

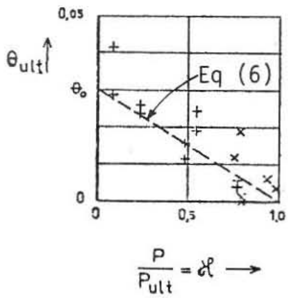


Fig. 10 - Observed values of the angle of rotation of the joint at the joint at the ultimate load.

[13] SINHA and HENDRY report tests on frames and found good agreements between theory and test, that gave no yielding in the joints in some cases. Similar results are reported by [11] GERMANINO and MACCHI. Further references may be found in the books of [10] SAHLIN and [12] HENDRY.

From fig 10 it can be observed that for low axial loads the value of the rotation in the joint is of the same order of magnitude as the maximum rotation in the wall and also of the slab end. For extremely low axial loads it is even twice as high as for a normal wall end rotation at failure.

It is therefore obvious that for the calculation of the eccentricity in the wall at high loads the rotation in the joint will play a very important roll. Sometimes the joint may not yield at all and in such circumstances the rotation is 0 and the calculation is simplified. The writer wants to stress the need for more experimental data to faciliate the calculation of load eccentricities in masonry walls. Many different types of joints are used and far from all types have been experimentally investigated. Also all wall testing should produce data for the angle of rotation of the wall end rather then the deflection. The deflection curves can easily be fitted whether they are sinusoidal, parabloic or of almost any shape. Since the angle of rotation at the end is the derivative of the deflection it is much more sensitive to the choice of the formula for the deflection curve. The deflection curve should therefore be tested for fit to the experimental angle of rotation rather than to the midheight deflection for a column.

Unintended eccentricities

In the foregoing we have assumed that all structural members are perfect in material and geometri. The conditions in practice are obviously not so. Therefore one has to take a number of imperfections into account. For the determination of the force eccentricity in the walls two different kinds of unintended eccentricities must be accounted for: member imperfections and system imperfections.

Member imperfections

The imperfections of a wall stem from two different sources.

The first one is related to the geometri of a wall. Inadvertently the wall is built in a bow, C-shaped, so that it deviates from a straight line from the bottom to the top.

The second type of imperfection is connected with the imperfections of the material. Even if the wall were built to a perfect geometri, upon testing it would start to bent in one or another direction due to the uneven distribution of the modulus of elasticity over the cross section. Also some bricks and some portions of the mortar may have lower strenght and stiffness than the rest and therefore the testing machine sees the wall as if it were imperfect or bent.

The both imperfections mentioned interact together, and in practice one often collects both these in one single term for practical reasons. Very often these two imperfections are combined into one, which is assumed to have its maximum at mid-height of the wall, when buckling curves are established for masonry walls. Often the figure h/300 is assumed.

System imperfections

In practice one wall is built and then a slab is placed on top of it and thereafter the next wall is beeing built on top of the joint between the slab and the wall. It often happens that one wall is misplaced in relation to the other. This misplacement of walls results in load eccentricities in the wall. These eccentricities can be taken into account either as a misplacement of the actual force or by an extra moment at the joint.

It is impossible to build a wall to exact plumb and therefore the wall has a lean inwards or outwards. This inclination must be taken into account since it creates a horizontal force that has to be taken up via the slabs and the shear walls perpendicular to the inclined wall.

Design procedures for masonry walls

It is obvious that an exact calculation of the eccentricity as a function of the applied load as shown in beforegoing would be too complicated in many cases. In the daily design work it is therefore advisible to use some simplifying assumptions and other design aids. For example, in the recently published [14] International Recommendations for Masonry Structures (DIB-report publication 58) it is recommended that one of three different schemes is assumed for the calculation of the force eccentricity in a wall).

1. The general approch, as described, with semirigid joints in which rotations are possible in the joint at a reasonable constant value of the end moment of the slab and wall can sometimes be modified. This scheme is only valid within the rotation capacity of the joint and this must be checked for the limit state under examination.

2. It is also possible to use a rigid joint scheme. The plastification which takes place in the joint is neglected which of course increases the load force eccentricities.

3. A further simplification is to assume hinged joints. In this scheme the eccentricities are assumed.

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