Safety Index Analysis of Brick Masonry

by

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SUMMARY

This paper outlines the principles of development of a new Limit States Design code for load bearing masonry structures in Canada. By means of a particular case, the general safety index analysis is presented together with numerical results for the existing code and a proposed new code formulation for the design of eccentrically loaded walls.

1. INTRODUCTION

Canadian code writing authorities have initiated a program to develop new limit states design procedures with common load factors. The basis for the new steel design code and for both the concrete and masonry codes which are under development is a safety index analysis (1,4,6). Such analysis involves development of limit state functions which include random variables to account for all uncertainties.

One major case in masonry design concerns vertical walls loaded by eccentric vertical forces as well as lateral loads due to wind or earthquake. In this paper, the general approach is demonstrated for a particular class of wall designs involving relatively small eccentricities.

2. BASIC MATERIAL PROPERTIES

To establish a design procedure, several basic mechanical properties of masonry must be established. The shape of masonry units, mortar type and mortar bedding arrangements significantly effect wall capacity as measured by the basic ultimate prism strength f'_m (5). For ultimate limit states, a characteristic value of the conventional short, axially loaded prism capacity corresponding to approximately a 5% fractile can be used.

Based on an extensive study of published test data (2), the average initial elastic moduli, E, can be estimated as a constant times the characteristic unit or prism strength with correction factors for long term effects. For example,

$$E = 220.6\ f'_{br} + 145{,}000\ \text{psi} \tag{1}$$

with f'_{br} = unit strength, provides an acceptable estimate of the average initial modulus for clay brick masonry.

All design calculations are based on net area.

3. WALL CAPACITY

3.1 Theoretical Short Section Capacity

For vertically loaded masonry walls, a basic situation involves a short section subjected to an axial force P and bending moments M. Alternatively, the moment can be expressed as an end load eccentricity e=M/P. For such walls, lateral wall deflections are so small that applied static moments are not increased by a so-called P-Δ effect.

For ultimate limit states, experimental studies suggest that design can be based on a set of general assumptions including: plane sections remain plane; maximum stresses and strains; and linear stress-strain behaviour. With these assumptions, the stress distribution across an uncracked section, for example, is given by:

$$\sigma = \frac{P}{A_n} + \frac{M}{S_n} \tag{2}$$

where A_n is the net sectional area and S_n is the section modulus. The cross section can crack leading to a reduced area and section modulus.

Assuming linear stress-strain behaviour until the masonry crushes in compression at a stress of f'_m, the interaction diagram shown in Fig. 1 is obtained (7). Also shown in Fig. 1 are selected test results.

If the load eccentricity is known, the ultimate capacity interaction equations can be solved to obtain the force capacity P. For example, for a short section with load eccentricity less than the kern, the theoretical load capacity per unit length, P, is from Eq. (2), the solution to

$$Pe - (t\ f_p - P)\ \frac{t}{6} = 0 \qquad (3)$$

where t is the wall thickness and f_p is the basic prism failure stress.

3.2 Slenderness Effects

When a wall has a significant height h or slenderness ratio h/r, external applied bending moments cause lateral deflections which in turn lead to additional internal moments - the P-Δ effects. For very slender walls buckling can occur, but for walls admitted by the code, failure occurs by material crushing with or without tensile cracking along the height of portions of the wall. Behaviour depends on the ratio of the two wall end eccentricities e_1 and e_2.

The problem with masonry, as with concrete columns, is to devise a simple procedure for predicting lateral deflections. The flexural rigidity, EI, of the wall varies along the height due to cracking and also depends on the load level, i.e., increased load leads to increased deflections, leading to increased bending moments, leading to an expansion of the cracked region through the section and along the height.

Description of such complete behaviour by an "equivalent" linear analysis is inherently approximate. Several approaches are possible but a simple mechanist approach has been adopted since it tends to give a relatively unbiased estimate of capacity reduction with slenderness.

To calculate lateral displacements it is assumed that the flexural rigidity EI is given by the initial tangent modulus of elasticity and an effective moment of inertia I_{eff} given by

$$I_{eff} = (I_{end\ 1} + I_{end\ 2})/4 \qquad 0 < e_1/e_2 < +1 \qquad (4)$$

$$I_{eff} = \min.\begin{Bmatrix} (I_{end\ 1} + I)/4 \\ (I_{end\ 2} + I)/4 \end{Bmatrix} \qquad -1 < e_1/e_2 < 0 \qquad (5)$$

where $I_{end\ 1}$ and $I_{end\ 2}$ are the end moments of inertia assuming cracked sections, if necessary, and I is the uncracked section inertia. Other symbols are defined in Fig. 2 where the rationale for these assumptions is suggested (3).

In analysis, the bending moment diagram due to applied moments and P-Δ moments are added. The section axial load capacity is obtained from the short section interaction diagram of Fig. 1 either at one end or along the height of wall.

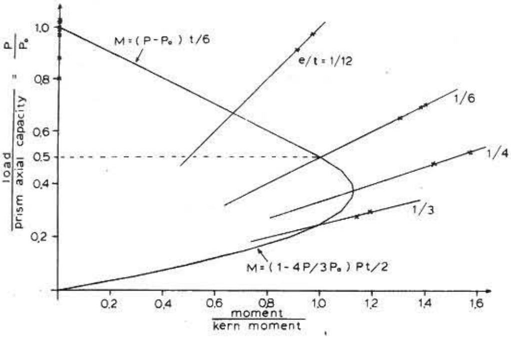

FIG.1 INTERACTION EQUATIONS – SOLID SECTION – from yokel, mathey, and dikkers

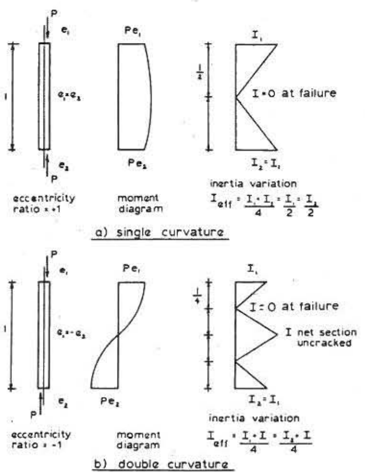

FIGURE 2 REASONING BEHIND THE EFFECTIVE INERTIA ASSUMPTION

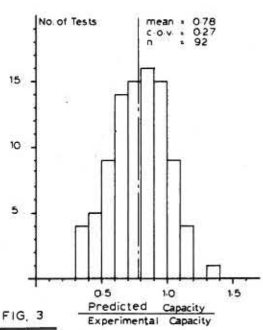

FIG. 3

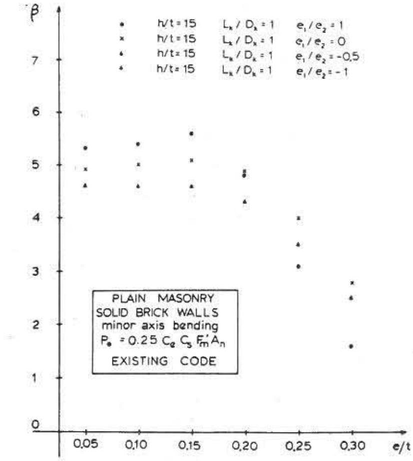

FIG.4a SAFETY INDEX Vs. LOAD ECCENTRICITY RATIO

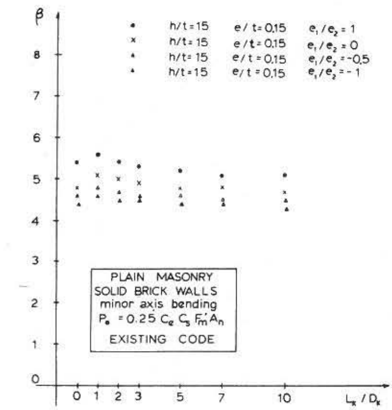

FIG.4b SAFETY INDEX Vs. LIVE - DEAD LOAD RATIO

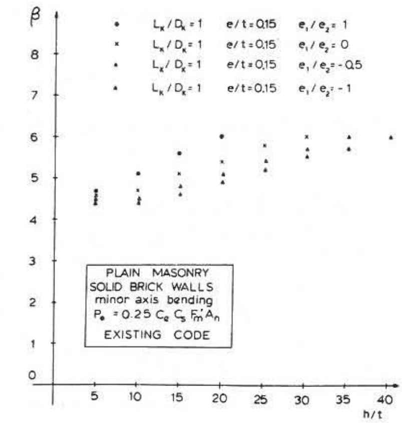

FIG.4c SAFETY INDEX Vs. SLENDERNESS RATIO

Based on the short section theoretical capacities with corrected applied moments derived from a conventional linear analysis and "equivalent" elastic stiffness EI_{eff}, wall capacity can be estimated for experimental conditions. Shown in Fig. 3 is a histogram of the ratio of predicted axial load capacity for test walls with a large variety of thicknesses, eccentricities and slenderness. These results suggest a Guassian distribution about the mean.

3.3 Modelling Errors

As suggested by the data in Fig. 1, the simple theoretical short section capacity based on linear stress-strain behaviour does not provide an unbiased estimate of experimental short section capacity. Similarly, a biased estimate of wall capacity is obtained as shown in Fig. 3.

Based on an analysis of available test data, it was found that an unbiased estimate of experimental results could be obtained if the theoretical capacity is divided by a modelling variable M with the statistical parameters shown in Table 1 and a Guassian probability distribution.

TABLE 1 GLOBAL MODELLING ERROR M - SOLID BRICK

Eccentricity Conditions	Mean Value $\overline{M}$	Coefficient of Variation V_M
$e/t \leq \frac{1}{6}$	$0.95 - 1.5\ e/t$	0.14
$e/t > \frac{1}{6}$	$0.70 - 0.9(\frac{e}{t} - \frac{1}{6})$	$0.14 + 0.96\ (\frac{e}{t} - \frac{1}{6})$

4. SAFETY ANALYSIS

The new code proposals will be based on a level II safety index analysis (4,6) with explicit consideration of uncertainties due to material variability, workmanship, dead and live load magnitudes, load analysis, modelling errors etc. A first step in analysis is to define a limit state or G function for a particular design condition with appropriate random variables.

As an example, consider the simple case of Eq. 3, modified to include the random modelling error M as follows:

$$P = \frac{1}{M} \left[t\ f_p/(1 + \frac{6e}{t})\right] \tag{6}$$

where P and M are random variables while f_p, e and t are known.

To introduce workmanship and prism strength uncertainties, f_p can be replaced by a random variable WFf'_m where f'_m is the deterministic code specified prism strength, and W and F are random variables to account for uncertainties in workmanship and prism strength.

To account for uncertainties in load eccentricities, e can be replaced by a variable $EC.e_k$ where EC is a random variable and e_k is the calculated nominal load eccentricity.

The final expression for the true wall capacity is then

$$\text{Wall Capacity} = \frac{1}{M} \cdot t \cdot W.F.f'_m/(1 + \frac{6EC \cdot e_K}{t}) \tag{7}$$

Uncertainties in net area and t have been ignored or imbedded in, for example, the workmanship factor W. M,W,F and EC are random variables while f'_m, e_k and t are the conventional fixed specified values. Note that increased moments due to P-Δ effects can lead to a cracked section although a wall is uncracked at its ends.

In design calculations specific deterministic total loads P_k are obtained as the sum of the conventional values of design dead loads D_K and live load L_K. In reality, load effects are random time dependent variables.

To introduce relevant uncertainties, the true applied axial load using the Canadian ultimate state format can be written as

$$\text{Applied Load} = (1.25.D.D_k + 1.5.L.L_k).LA \tag{8}$$

where LA, D and L are appropriate random variables to account for uncertainties in analysis and load variations while 1.25 and 1.5 are load factors.

Finally, the G function defining failure is given in this example as:

$$\text{Applied Load} - \text{Wall Capacity} = 0, \text{ or} \tag{9}$$

$$M(1.25.D.D_k+1.50.L.L_k).LA - t.W.F.f'_m/(1 + 6.\frac{EC.e_k}{t}) = 0 \tag{10}$$

The structure will fail if a realisation of the random variables D, L, LA, M, W, F, and E fall into the failure region.

Estimated parameters for the random variables involved are shown in Tables 1 and 2.

TABLE 2 Random Variables and Distribution Functions for Safety Analysis

UNCERTAIN VARIABLES (1)	(2)	Mean/Specified (3)	C.O.V. (4)	Distribution Functions (5)	Specified Values* (6)
D	Dead Load	1.00	0.07	normal	D_k
L	Live Load	0.70	0.30	Extreme Type I largest	L_K
LA	Load Analysis Factor	1.00	0.20	normal	1.0
M	Global Modelling Error Factor	Details see Table 1		normal	1.0
W	Rigorous Work Inspection	1.00	0.10	normal	1.0
	Moderate Work Inspection	0.80	0.15		
	Uninspected	0.70	0.20		
F	Masonry Ultimate Compressive Strength Factor	1.25	0.08	normal	f'_m
EC	Load Eccentricity Factor	1.00	0.20	normal	e_k

* (characteristic)

5. CALIBRATION OF THE PRESENT CANADIAN CODE

For a given set of design parameters D_K, L_K, e_K, h, f'_m, and masonry type, a design code effectively specifies a minimum wall thickness t. In level II Safety Analysis G functions such as Eq. (10) are transformed to a reduced variable space through changes of variables of the type $x = (x-\overline{x})/\sigma_x$. By means of programming techniques, the shortest distance β from the new "g" function to the origin can be calculated (4,6). The safety index β is a measure of the safety of a structural element.

This has been done for a wide spectrum of design cases using the equations given in the present Canadian Masonry Code. Results are shown in Fig. 4 for brick masonry and a variety of dead to live load ratios, eccentricites, curvatures and slenderness ratios. Moderate inspection in Table 2 was assumed.

These results suggest that the present code is more conservative for single curvature than for double curvature with safety generally increasing with wall slenderness. There is a significant decrease in safety level for large eccentricities due to the fact that wall cracking is very sensitive to eccentricity and involves a great deal of uncertainty.

By comparison to codes for other materials, β values in the order of 5 are relatively high. Target values in the order of 4 are often discussed which suggests that safety levels in a new code might be decreased somewhat.

6. A NEW CODE PROPOSAL

A simple code procedure can be based on the following basic principles:

(1) linear stress-strain behaviour in compression up to a crushing stress equal to f'_m.

(2) Neglect of tension capacity except in the region of almost pure bending.

The basic design equation for an uncracked section, for example, then becomes

$$\frac{P_D}{A_n} + \frac{M_D}{S_n} \leqslant \phi \ f'_m \tag{11}$$

where P_D and M_D are calculated from the factored design load, $1.25\ D_k + 1.50\ L_k$

(3) A minimum eccentricity of t/12 in single curvature.

(4) Effective moment of inertia based on Eq. (4) and (5) with elastic modulus given as a linear function of f'_m or f'_{br}.

(5) Calculation of the lateral deflections caused by end moments and transverse design loads followed by addition of the P-Δ moments to the elementary statical bending moments.

(6) Use of the interaction diagrams from steps (1) and (2) with the total moments from step (5) to ensure that Eq. (11) is satisfied.

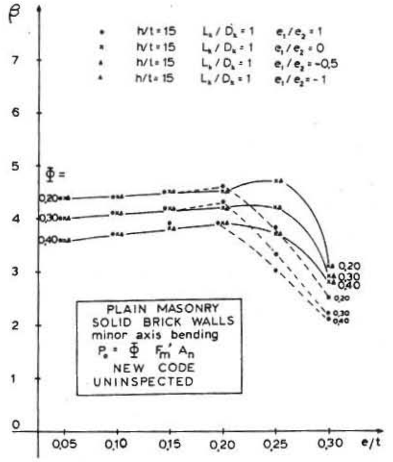

FIG.5a SAFETY INDEX Vs. LOAD ECCENTRICITY RATIO

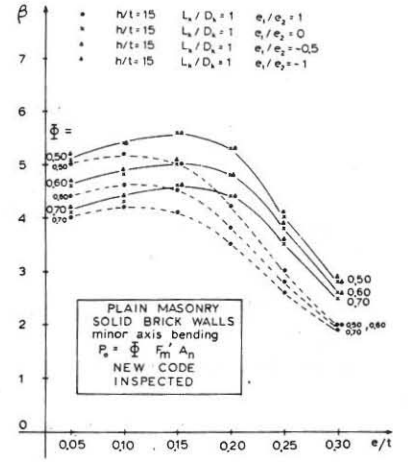

FIG.6a SAFETY INDEX Vs. LOAD ECCENTRICITY RATIO

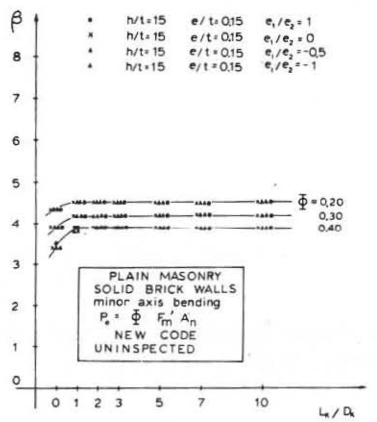

FIG.5b SAFETY INDEX Vs. LIVE-DEAD LOAD RATIO

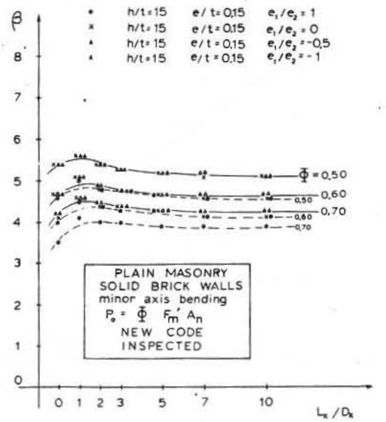

FIG.6b SAFETY INDEX Vs. LIVE-DEAD LOAD RATIO

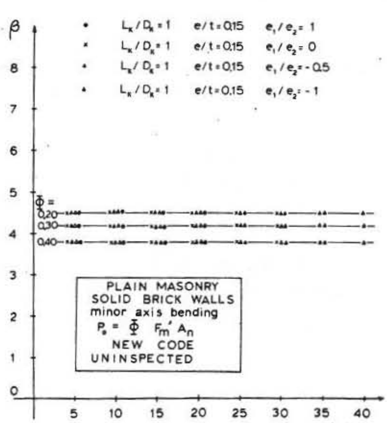

FIG.5c SAFETY INDEX Vs. SLENDERNESS RATIO

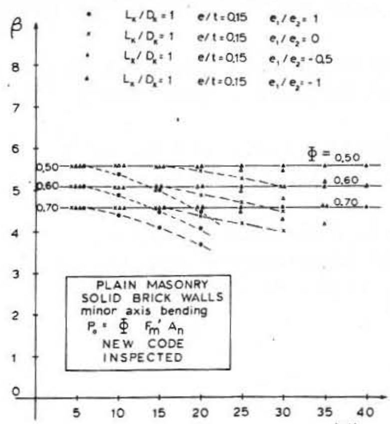

FIG.6c SAFETY INDEX Vs. SLENDERNESS RATIO

For the majority of practical cases, calculation of P-Δ effects can be avoided by means of a reduced ϕ factor.

The results of a safety index analysis for such a design procedure are shown in Figs. 5,6. The ϕ factors shown were used to obtain β values in the order of 4 or 5. For many cases, nearly constant values of the safety index are obtained over a range of conditions. However, some sensitivity to wall curvature is shown.

These results suggest that ϕ factors of approximately 0.60 and 0.20 for inspected and uninspected masonry would yield average β values of around 4.5. In addition limitations on maximum eccentricity and maximum slenderness are essential.

7. CONCLUSIONS

The approach to development of a new limit states design code in Canada has been discussed by means of an example. Results suggest that current practice leads to levels of safety for wall design that depend on load eccentricities and slenderness. Safety levels are high relative to other materials.

A relatively simple code approach based on conventional engineering analysis has been proposed and shown to provide more consistent levels of safety under certain conditions.

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