LOAD CARRYING CAPACITY OF MASONRY PIERS WITH OPENINGS.

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ABSTRACT To find out the extent to which some masonry bridges must be adapted for the new loading conditions, the most important step is concerned with the safety of tall holed piers. This paper describes a mathematical model that succeeds in explaining the causes of the systematic cracking around the openings. Furthermore, suitable methods to study the overall stability of the piers and their dynamic behaviour are presented. Finally, guidelines for restoration are given.

1. INTRODUCTION

The regional railway network serving the district to the north and west of Milan was first built at the end of the last century. Depending on their locations, the bridges were made either entirely of bricks or else, when stone masonry was used for the piers, with brickwork only for the arches.
Heavier and faster trains made it necessary to modernize and strengthen these lines, and this in turn implied and extensive survey into the needs of improving the bridges so that they could safely cope with the changed service loads.
The taller and, for that period, more advanced bridges (i.e. for these purposes with piers of at least 30 m in height) all contained a platform halfway up as an inspection catwalk (mainly for military purposes, see fig. 1). This means that these piers contain openings that involve stress concentrations.
To find out the extent to which these bridges would adequate for the new loading situation, and to see how much restoration and/or improvement may be needed, certain steps have to be taken:

- data must be obtained on the mechanical characteristics and state of degradation of the masonry (whether bricks or stonework [1], [2]), and on the properties of the foundation soils;
- the causes of systematic cracking around the openings of the piers must be established, in order to prevent this occurring in the future, after restoration;
- structural analysis must be carried out on the various component members (arches, piers, abutments, foundations) to determine their safety level and to assess the need for any additional measures to increase the strength of the masonry or the cross-sections of the piers.

It is above all the structural analysis of the holed piers that requires the most attention. This has to be developed in three steps:

- evaluation of the stress concentration around the openings (as an aspect of local safety to prevent cracking, see fig. 2);
- assessment of overall stability;
- dynamic response of the system to hunting or earthquake induced excitation.

Work on each of these steps can be greatly facilitated by the use of proper idealizations of the stress-strain relationship and by the choice of suitable techniques to convert the actual 3-D reality of the piers into 2-D computer codes.
This leads to considerable saving in computer time, and seems worth further explanation to underline its operational simplicity.

2. STRESS CONCENTRATIONS

The systematic presence of cracking around the openings makes it reasonable to presume that there is a strict causal relationship between the cracks, the openings in the piers and the constraints exerted by the catwalk.
A first clue on the cause of cracking is to be found in the well known studies on stress concentrations around a hole due to plane states of stress in the linear range [3], [4], [5]. To be precise, the results of these studies cannot be directly extended to own particular case, because of the form of the opening and the particular nature of the material (practically strengthless in tension).
However, the limitations are not as decisive as they might seem to be at first sight, since the main purpose of this preliminary investigation is to observe the creation of tensile stresses which the material cannot cope with and that lead to cracking. So it seems reasonable to make use of a model of material with linear behaviour provided that, as in this case, the ratio between the maximum compressive stresses and the stress at collapse is not greater than 1/10.
As a consequence, it seems that the maximum horizontal tensile stresses, great enough to induce vertical cracking, are confined to the areas of masonry immediately above on below the opening.
In reality, however, this does not happen. This may be due to two factors: a) the width and thickness of the piers are comparable to each other, and this already constitutes a limitation on the plane approach; b) the presence of flat bracing arches imposes, in the area under the opening, boundary conditions that in fact make the problem three dimensional.
Three dimensional solutions in closed form [6] are not suitable for this particular case, for two reasons. First, because they introduce analytical difficulties that are practically insuperable; second, because the effects of the third dimension only become noticeable for high values of ν (Poisson's ratio), and this does not arise for stone masonry, for which $0.05 < \nu < 0.1$.
For all the reasons put forward earlier, but also to avoid the excessive computations involved in a 3-D analysis that would not be justifiable for present purposes, it seemed suitable, and sufficiently representative of the structural complexity in question, to carry out a numerical analysis by finite elements of a 2-D model able to simulate locally the constraints due to the flat bracing arches.
The numerical model refers to the length of pier that includes the opening, as illustrated in fig. 2. This choice does not imply any thickening of the mesh in individual zones close to the opening, because there was no particular reason in this case to investigate the tensile stress gradients and their consequent maximum values, given the already mentioned characteristics of stone masonry. Each processing took 60" in terms of CPU time.
Only a uniformly distributed vertical load was considered, for the following reasons.
Dead loads prevail over live loads in this case.
Horizontal forces due to wind and huntinig, even though in combination and present up to their maximim level, do not initiate vertical tensile stresses. Besides this, they are relatively small compared to the vertical loads, in percentage, and so do not cause significant variations from the uniform distribution.
In agreement with the De St.Venant postulate, the loaded cross-section is sufficiently far from the top of the pier, and its distance from the top of the opening is greater than the width of the opening itself.
The numerical analysis used the SAP V2 computer code for a plane state of stress. This is only exactly satisfied when the material shows $\nu \to 0$. However, it is reasonable to assume that for $\nu \simeq 0.05$ the approximation is still acceptable. In fact, as shown in [6] for a parallelepiped crossed by a circular hole, the maximum stress in the third direction (z) are very modest, and anyway $\sigma_{z\ max}/p \leq 0.1$ for $\nu = 0.05$.
All that is required, and it is quite reasonable in the present case, is to

interpret the stone masonry as material with a Poisson ratio $\nu_z \to 0$ in the third dimension.
The flat bracing arches, which provide the constraints against displacements in direction y, is simulated by attributing only to the material in the area of the springers, orthotropic behaviour characterised by an ideal elastic modulus E_y much greater than E_x, the latter being the value assumed for the masonry of the pier. In fact the presence of the arch gives a high degree of extensional stiffness to the masonry elements around the springers.
No precise experimental data is available on the value to be attributed the E_y modulus of ideal material with greater stiffness. So a series of calculations were carried out with 10, 20, 40, 100 as the assumed values for the ratio E_y/E_x, in order to have some basis for judging what influence variations in this ratio might have on the stress state, and particularly on the distribution and size of the tensile stresses.
In each case the numerical analysis confirms the presence of horizontal tensile stresses along a good part of the length which is in fact damaged in the pier.
Variations in the ratio exert only a modest influence on the extent of the tensile zone and the size of the maximum tensile stresses.
Fig. 3, illustrates two of the σ_y (x) stress distributions that lead to damage in the masonry. The amount of this damage, even in the part which in the model is subjected to compressive stresses, should be no surprise. In fact, even a relatively short crack will change the model. So even without any specific analysis it is clear how the crack, once it has started, can propagate as in the pier.
Fig. 3 also shows the variation in direction y of stresses σ_x at the height of the springing line. They are practically independent of ratio E_y/E_x. As was to be expected, the ratio between the values of the stresses σ_x on the fibres at the two ends is very close to three.
The results overall show that the mathematical model proposed here is sufficiently representative of the real complex structure and is able to show the causes of cracking as soon as the ratio E_y/E_x rises above 10.
It may be of some confort to note that above this limit the solution is little influenced by variations in E_y/E_x. This means that the indeterminacy of this ratio, which can never be eliminate, does not invalidate the model in question.

3. STABILITY OF PIERS

Any decision about the structural safety of the piers must also be based on a study of their stability. In that sense the catwalks at the top and half way up the piers constitute very stiff longitudinal constraints. Because of the effective slenderness values only transversal dispacements are to be feared. The catwalks are in fact rather compliant constraints against transversal displacement, and as such are difficult to define. However, they do offer some modest contribution which, as it is on the side of safety, will here be neglected.
The presence of the holes is of no great importance for present purposes, given their position (fig. 1, 2). When the entire cross-section reacting, the ratio between the span of the opening and the larger side of the cross-section is never greater than 0.2, so that the reduction in inertia is less than 1%.
To sum up, given these conditions the analysis can be performed on a pier model of variable cross-section, with no openings and subject to its dead weight and a load P acting with known eccentricity $\bar{e}$ at the top. When no specific dynamic analysis is required (see the following Sect. 3), the real constitutive law of the material ($\sigma - \varepsilon$), which in fact is non linear, must be substituted with the ideal law of elastic-brittle material. As Sahlin suggests, the E modulus is given the average value between the tangent at the origin and the secant at failure [7].
As the dead weight must also be taken into account, the indefinite equilibrium equation has to be expressed in terms of shear [8]. The resulting differential

equation has the form:

$$y''' + [P+qx(1+r.x)]y'/[EJ(x)] = 0 \qquad (1)$$

in which the superscript indicates the order of derivation of the displacement function $y(x)$, $J(x)$ is the moment of inertia of the variable section, $\bar{q}$ is the weight per unit of length referring to the top section and $r=(h_0-\bar{h})/(2lh)$ with l the height of the pier. The depth of the cross-section at the top of the pier is $\bar{h}$, and at the foot h_0.
Equation (1) must satisfy the following boundary conditions:

$y(0) = k$; $y'(0) = 0$; $y''(l) = Pe/(EJ)$, where $k = 0$ for undamaged cross-section and $k = (6\, e_0 - h_0)/4$ for cracked cross-section.

The differential formulation of the problem is entirely general. The value of $J(x)$ must obviously refer to the effective reacting section.
To integrate (1) it is convenient to use the method of finite differences. The resulting system of algebraic equations can be summarised in the form

$$[A]\ \{y\} = \{b\}. \qquad (2)$$

Here [A] and {b} are, respectively, the square matrix of order n (with n the number of elements, of length a, into which the structure has been sub-divided), and the column-vector of the n known terms:

$$\begin{vmatrix}
1 & c_1 & -1 & 0 & 0 & 0 & 0 & \cdots & & & & & \\
-c_2 & 0 & c_2 & -1 & 0 & 0 & 0 & \cdots & & & & & \\
1 & -c_3 & 0 & c_3 & -1 & 0 & 0 & \cdots & & & & & \\
0 & 1 & -c_4 & 0 & c_4 & -1 & 0 & \cdots & & & & & \\
\cdots & & & & & & & & & & & & \\
\cdots & 0 & 1 & -c_{n-3} & 0 & c_{n-3} & -1 & & & & & & \\
\cdots & 0 & 0 & 1 & c_{n-2} & 1 & c_{n-2} & -2 & & & & & \\
\cdots & 0 & 0 & d_{n-3} & d_{n-2} & d_{n-1} & d_n & & & & & &
\end{vmatrix}$$

in which $c_i = a^2\ [P + q.i.a.(1+r.i.a.)]/(EJ\ ) + 2$ and also

$d_{n-3} = 2/3 - (1/12)\ c_n$; $d_{n-2} = (1/2)\ c_n -4$; $d_{n-1} = 8 - (7/4)\ c_n$;
$d_n = (4/3)\ c_n - 14/3$.

The elements of the column-vector {b} are:

$b_1 = c\ k$; $b_2 = -k$; $b_3 = 0$ .... $b_{n-2} = 0$;

$b_{n-1} = Pe/EJ$; $b_n = -Pe(c_n + 4)/(4EJ\ )$.

The coefficient of the last row of matrix [A] were obtained by applying the method of "backward differences", which facilitates the solution of the particularly difficult problem of odd order differential equations. In processing the problem all differences higher than the fourth order are

neglected.
There are simple modifications to be introduced should it be necessary to know the response of the system to horizontal forces which, in compliance with the codes, translate wind and hunting effects in static terms. In that case equation (1) would become non homogeneous, with obvious variations in defining the elements of vector {b}. In any case, the differential problem is brought back to a non homogeneous algebraic system of linear equations.
It is worth noting that with the conditions $\bar{e}=0$ and $k=0$ this system becomes homogeneous, and its solution depends on determining eigen-value, which is typical of classical instability problems.
The following process was adopted to establish the value of the critical load.
For an assigned eccentricity value e at the top, the load P is increased, and for cross-section length the eccentricity of the vertical forces is determined through the ratio $EJ(x)y''/N(x)$. In this way it is possible to redefine the section which is actually reacting and correct, as necessary, its geometrical characteristics. Iteration of the process can be continued up to the desired degree of approximation.
The stationary point on the load-displacement curve supplies P .
For piers of modest slenderness, it may be the condition $\sigma = \sigma_c$ (reached at the point under greatest stress) which signals collapse. This is in fact, the way that the bridge piers of fig. 1 behave. However, their service load is very confortably on the safe side.

3. DYNAMIC BEHAVIOUR

Once again, for the same sort of reasons as in the preceding section on stability, most importance will be given to the study of piers when the displacements occur out of the mean plane of the bridge, and in the absence of live loads.
First of all a modal analysis of the piers was carried out, with the help of a suitable F.E. computer code, in order to find the periods and modal shapes. For this purpose it is essential to know the value of the E_0 modulus corresponding to the tangent at the origin of the stress-strain curve ($\sigma - \varepsilon$) of the masonry.
For old constructions it is difficult make reliable estimates of this value. However, useful information on the possible interval of variations in the period T as a function of E_0 can be obtained from the well known formula [9]

$$T \equiv [ml^4/(E_0 J)]^{\frac{1}{2}} \tag{3}$$

which can also be satisfactorily applied to piers of slightly varying cross-section.
A study of the results concerning the central bridge piers of fig. 1 and the relative openings (fig. 2), with or without the horizontal constraints provided by the catwalks, leads to the following conclusions:

- the presence of the openings has no great influence on the definition of the periods and modal shapes;
- the presence of the horizontal constraints, by adding considerably to the mass of the system, leads to a significant increase in the natural period, which rises from 0.64 to 0.72 secs.;
- expression (3) is a reliable instrument for establishing the $T(E_0)$ relationship.

Still today the Italian codes specify no particular methods for the seismic checking of masonry structures.
Assuming a damping coefficient of 5% for the masonry, a survey of the response spectra of accelerograms from some of the classical earthquakes in seismic engineering shows that the seismic coefficient envisaged by Italian regulations may be insufficient in the interval $0.3 \leq T \leq 0.6$ secs.

The attention of engineers is directed towards this result when having to deal with the seismic checking of piers through the method based on defining a seismic coefficient.
The next step was the dynamic analysis, non linear in geometrical and constitutive laws, by means of the computer code ADINA [10]. It was assumed that the stress-strain curve ($\sigma - \varepsilon$) of the masonry was qualitatively similar to the so-called concrete model [11]. The tensile strength was taken to be 0.05 MPa, a reasonable value for decaying masonry. The failure domain of the masonry is based on the work of Kupfer [11] and Page [12]. Of course, when the ADINA code is used, the study of pier stability should ignore the method proposed here in sect. 2. The dynamic analysis used a step-by-step method, and applied a sinusoidal type of accelerogram to the soil, with an amplitude that also varied sinusoidally. The maximum acceleration was about 0.2 g.
The model of the pier has 2-D solid elements with 4 nodal points of suitable thickness. The effect of the catwalks is simulated by "beam elements" able to represent the actual stiffness of the catwalks.
Comparisons were made to decide what influence the openings had on the dynamic response of the piers with horizontal constraints. The results showed that:

- the local stress state around the opening (fig. 2) undergoes significant variations that must be carefully taken into account for practical purposes;
- the horizontal displacements show no significant variations everywhere.

A comparison between situations with and without constraints brings to light a considerable difference in the response. In particular, it is worth underlining that the maximum difference in terms of displacements is about 50%. This shows how important it is to include the catwalks in the analysis. Another effective method to do this in through a model based on the so-called "boundary elements".

4. CONCLUSIONS

This study has clearly shown the need to repair the damaged areas of the piers. It is essential to restore the structural continuity endangered by the vertical cracks (fig. 2) by injecting them with cement mortar, which would satisfy a number of requirements. Greater durability is conferred on the structure, undesirable stress states due to environmental conditions may be prevented, greater strength is given to the material and the original cross-sections are restored.
However, it is also reasonable to be expected that the damage may arise again, under normal service conditions. So it is evidently necessary to protect the threatened areas from tensile stresses by applying an equilibrated system of horizontal forces above and below the openings strong enough to replace to tensile stresses with a limited level of compressive ones.
This would serve three purposes:

- it would offer precautions against the inevitably approximate results from the numerical analysis which depend on the model used and on the notoriously uncertain mechanical characteristics of masonry;
- it would give local improvement to the strength of the material in the presence of compressive stresses in the two main directions of the plane model, in the same way as for concrete [11];
- it would ensure conformity between the real structure and the model used to study overall stability and dynamic behaviour.

These additional horizontal compressive stresses are achieved by a two-by-two system of steel beams tightened with an impact spanner to provide the correct prestressing in the masonry. This too was simulated in the numerical analysis fig. 4 shows the distribution of stress σ_y , which are everywhere compressive, in the previously damaged position. The position and number of the horizontal

forces applied depend on the need to spread the effect of precompression and limit the maximum values of σ_y .
The numerical analysis also demonstrated that, as was to be expected, the values of σ_x changed hardly at all.
There are also evident computational advantages to be obtained by translating, with suitable expedients, the 3-D reality of the construction into simpler computer codes that make use of 2-D elements. When necessary (if the particular aspect of the problem permits it) the real stress-strain curve (σ - ε) of the old masonry, which is generally so unreliable, can be interpreted as a straight line.

5. REFERENCES

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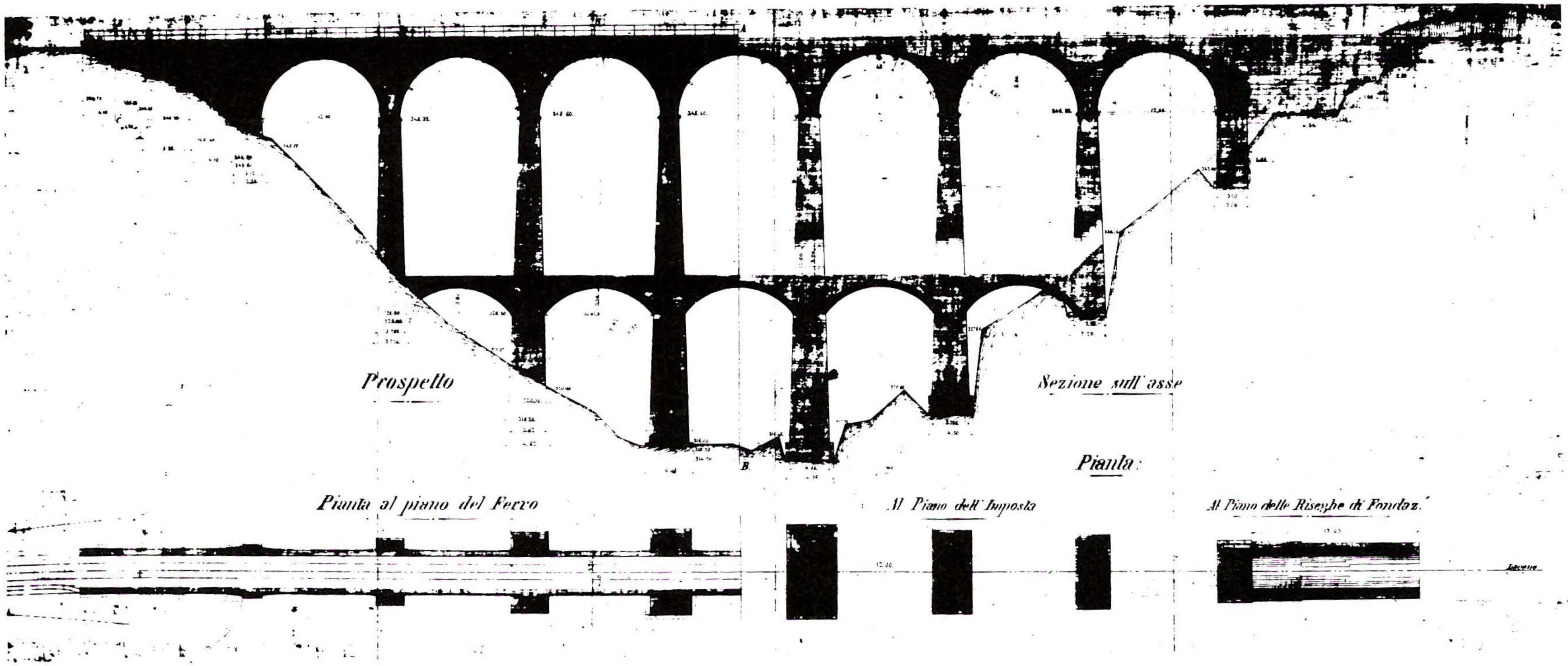

Fig. 1 A typical masonry bridge with catwalks

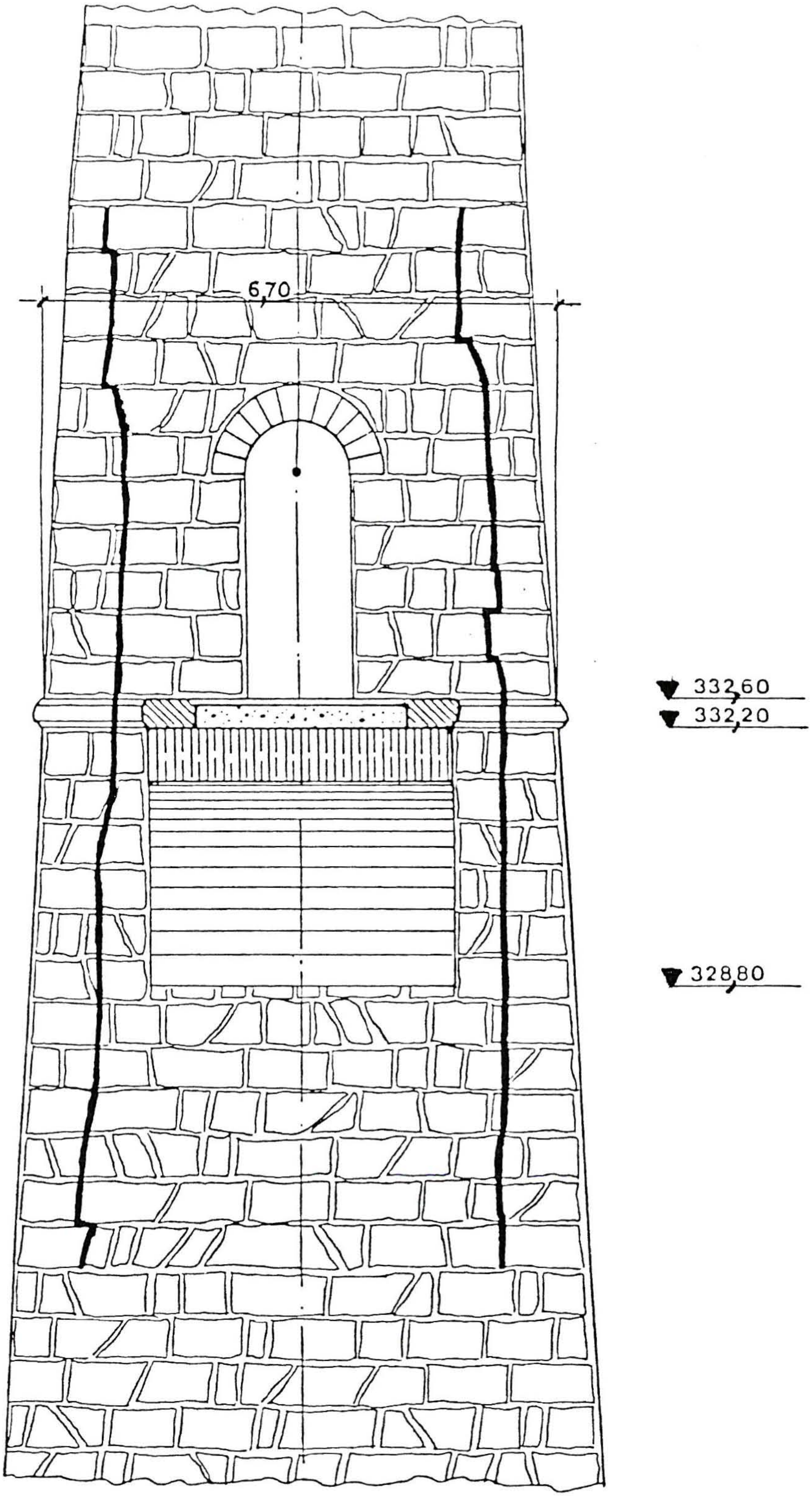

Fig. 2 Cracking along the piers

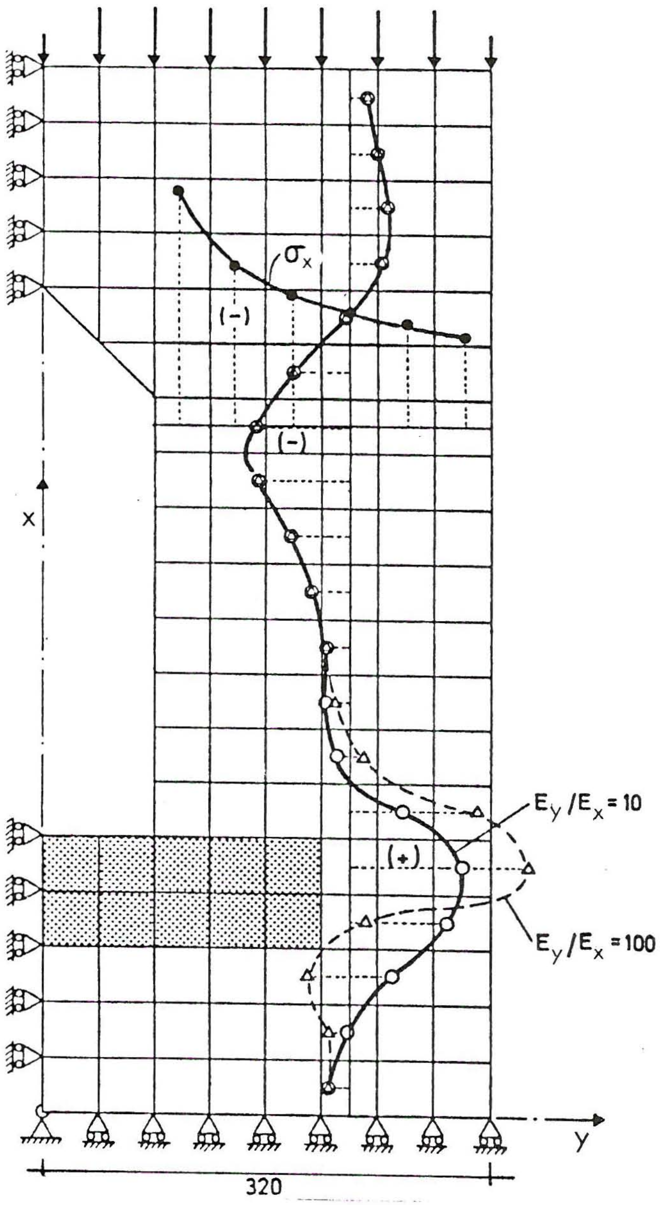

Fig. 3 Stress distributions depending on E_y/E_x ratio

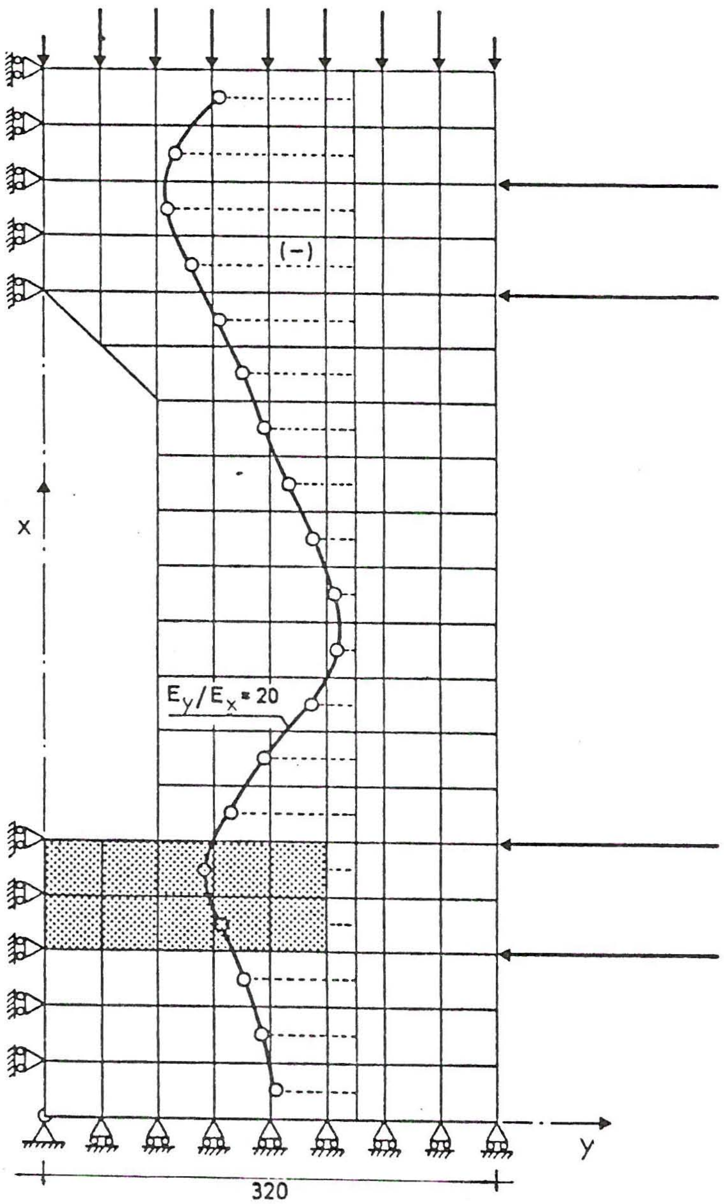

Fig. 4 Distribution of horizontal stresses after prestressing