

PREDICTED RESPONSE OF REINFORCED BRICKWORK TO IN-PLANE TENSILE LOADING

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ABSTRACT A panel of reinforced brickwork is considered, having dimensions much larger than the bricks and the reinforcement spacing. The panel is subject to uniform, in-plane loading stresses, with one of the principal stresses always being tensile. The possible failure modes include bed, perpend-bed, and perpend-brick. An elasto-visco/plastic algorithm forms the basis of a mathematical model that incorporates these modes. This mathematical model has been used to examine the effects of the principal stress orientation, the sign and relative magnitude of the minor principal stress, and the reinforcement and its prestress level.

1. INTRODUCTION

When brickwork is subject to in-plane loading its response is determined by the magnitudes and orientation of the two principal stresses, σ_1 and σ_2 . This orientation can be conveniently measured as the clockwise inclination, θ , of σ_1 with respect to the vertical. A typical panel of brickwork with vertically oriented reinforcement is shown in Figure 1. The loading stresses illustrated in this figure are uniformly distributed and, as can be seen, the major principal stress is always tensile while the minor principal stress may be tensile or compressive.

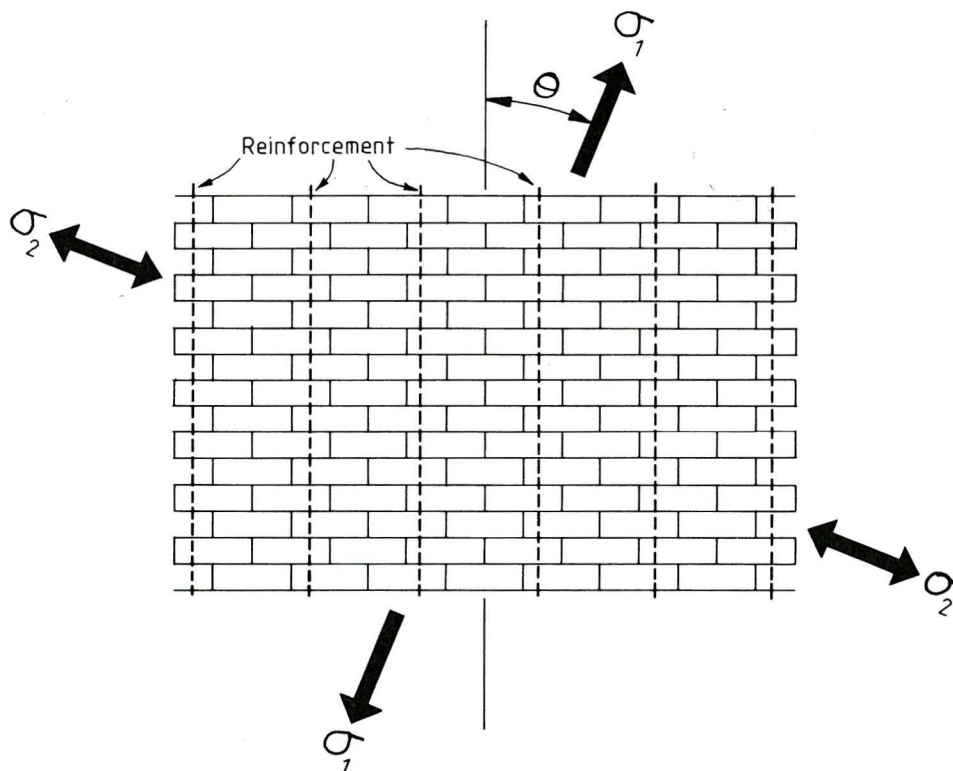


Figure 1 Uniform loading stresses applied to reinforced brickwork panel

Previous workers have made considerable progress in establishing, both experimentally and theoretically, a general failure criterion for unreinforced brickwork in terms of σ_1 , σ_2 , and θ . For example, Page (1) has used the finite element method to derive the biaxial failure criterion in the tension-tension range. The shape of the failure surface was found to be greatly influenced by the ratio of the shear strength of the mortar to its tensile strength. In further work, Page (2) has used a biaxial test rig, with brush platens, to examine the failure criterion in the compression-compression range and the compression-tension range. .

For the compression-compression range, failure often occurred by splitting parallel to the plane of the brickwork panel. Variations in the orientation of the principal stresses, i.e. variations of θ , did not exert a great influence. The exceptions to this pattern of behaviour were for cases approaching uniaxial conditions when splitting failures did not occur and variations in θ were highly significant. In marked contrast, the behaviour observed in the compression-tension range showed that failure never occurred by splitting parallel to the panel and variations in θ exerted a critical influence over the complete range.

The work reported in this paper forms part of a research project aimed at deriving and implementing constitutive equations for reinforced brickwork subject to in-plane loading. The detailed development of the theory will be described elsewhere (3), while this paper is devoted to the presentation of aspects of the predicted behaviour of a panel of reinforced stretcher bond brickwork. The panel is subject to uniformly distributed loading stresses in the compression-tension and tension-tension ranges. These aspects of behaviour include the interaction between σ_1 , σ_2 , and θ and the various relevant modes of brickwork failure, and the modification of response caused by the reinforcement and its prestress level. The height and lateral extent of the brickwork panel are assumed to be much greater than the dimensions of the bricks or the spacing of the reinforcement. The reinforcement is placed vertically and, after placement, may be prestressed before being fully grouted to the brickwork panel.

In future research it is planned to incorporate the constitutive equations for the brickwork into finite element codes thereby allowing the solution of problems involving stress gradients and complex boundary conditions.

2. MODES OF BRICKWORK FAILURE

Failure of the brickwork is defined as the formation of persistent discontinuities which, because of the uniform loading conditions, will occur in sets of uniformly spaced members. The five possible modes of brickwork failure are shown in Figure 2, where one member of each set of discontinuities is illustrated. Since the mortar is weak compared with the brick, each mode is initiated by failure of either the horizontal mortar (bed) or the vertical mortar (perpend). For the first mode, failure is confined to the bed and this case arises whenever the bed, and not the perpend, is first to reach its failure conditions. It is therefore referred to as the 'bed mode'. The other four failure modes are all initiated by failure of the perpend, with subsequent shedding of its load until failure also occurs, either in the bed, (Modes 2,3,4) or through the brick (Mode 5).

Modes 2 and 3 involve bed and perpend in a stepped fashion to form what are effectively diagonal failure planes. The normal to the diagonal failure plane for Mode 2 has a positive gradient, so that this mode is referred to as the 'positive diagonal mode'. Mode 3 is the mirror image of Mode 2 and is hence known as the 'negative diagonal mode'. Because of this mirror image effect, the occurrence of one of the diagonal modes, rather than the other, will depend on

the sign of θ , this being the same as the sign of the applied vertical-horizontal shear stress. Positive values of θ , and this shear stress, are associated with failure in the 'positive diagonal mode', and vice versa.

For the fourth mode, the failure follows a uni-directional trend in the perpendicular but is alternating with regard to the bedding. As can be seen in Figure 2 this produces an effective vertical failure plane so that this mode is referred to as the 'vertical mortar mode'. The alternating sequence of bedding planes involved in the failure is reflected in an alternating pattern of upward and downward normals to these planes. Hence, the 'vertical mortar mode' can be considered as intermediate between the 'positive diagonal mode' and the 'negative diagonal mode', e.g. for general loading states the 'vertical mortar mode' will always be less critically stressed than one of the 'diagonal' modes, and more critically stressed than the other. This is because the 'diagonal' mode that is most critically stressed will have every bed at incipient failure conditions while for the 'vertical mortar mode' only alternative beds will be in this state. It follows that if θ is positive, failure occurs first in the 'positive diagonal mode', while if θ is negative, failure occurs first in the 'negative diagonal mode'. When $\theta = 0^\circ$ or 90° , i.e. the applied vertical-horizontal shear stress is zero, the failure loads will be identical for the 'vertical mortar mode' and the two 'diagonal' modes. Since the 'vertical mortar mode' of failure can never occur at a lower load than the critical 'diagonal' mode, it does not feature in subsequent discussion and analyses.

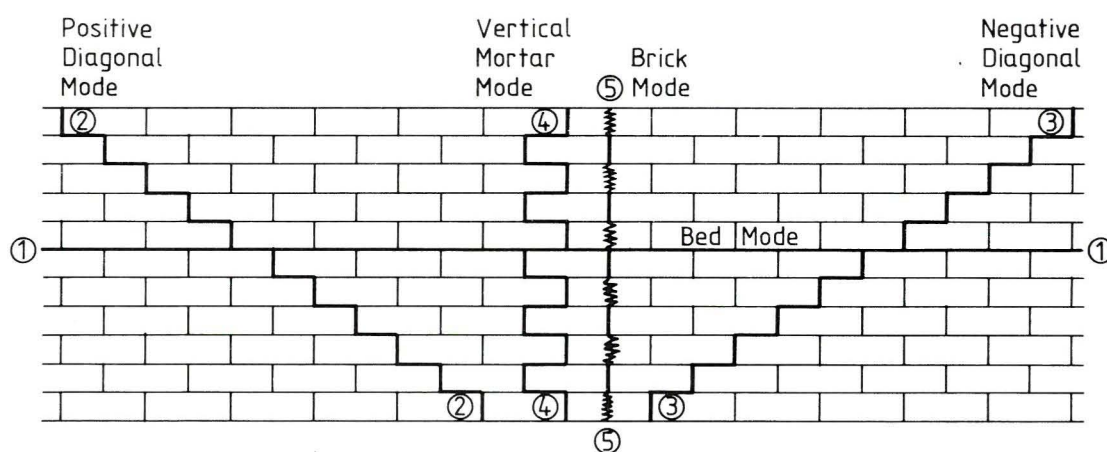


Figure 2 Modes of brickwork failure

The fifth, and last, mode of failure consists of a vertical plane initiated in the perpendicular and eventually resulting in fracture through the brick. Hence, it is known as the 'brick mode'. This mode can occur as an alternative to either of the 'diagonal' modes and is most likely to develop under the combined conditions of;

- (a) relatively high strength of the bed mortar compared with the brick, and
- (b) the combination of loading stresses being more conducive to brick failure rather than bed failure.

The possibility of brickwork failure due to splitting parallel to the plane of the panel is not considered because the presence of tensile loading stresses has been shown to preclude it (2).

3. MATHEMATICAL MODEL

The mathematical model used in this study is based on recognizing that observed failures in brickwork are effectively planar in nature, even when these effective failure planes are composed of separate components that may be coplanar (Mode 5) or non-coplanar (Modes 2,3,4). Under conditions of uniformly distributed applied stresses, these failure planes will tend to occur in sets of parallel, evenly spaced members.

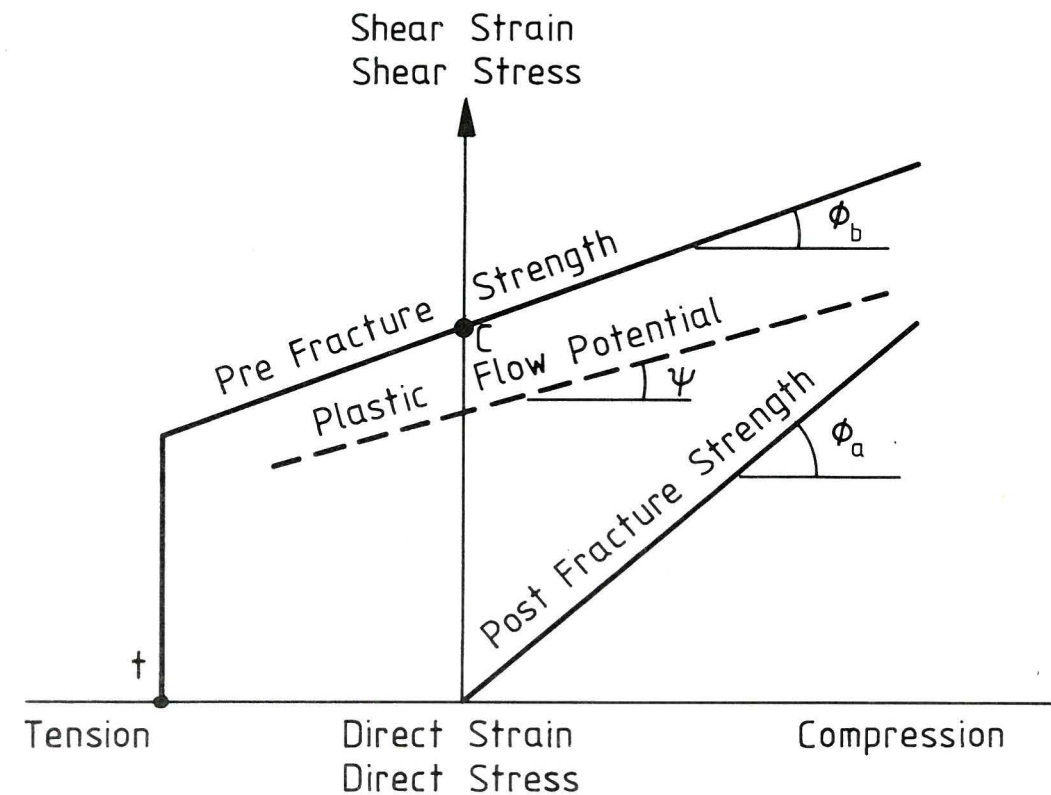
Constitutive equations for the behaviour of a rock mass intersected by sets of discontinuity planes have been presented by Zienkiewicz and Pande (4). These equations are based on an elasto-visco/plastic algorithm, in which time stepping can be used as a computational convenience to solve problems where strain softening and non-associated flow are relevant. This work has been extended to apply to reinforced, discontinuous masses (5,6). In a further development, of particular relevance to brickwork, Gerrard (3) has introduced the concept of reinforced 'composite' planes. These 'composite' planes are formed as a repeated sequence of 'component' planes, each of which can have different properties and orientation. A 'composite' plane is assumed to behave elastically until one of its 'component' planes reaches failure conditions. Thereafter this 'component' plane is constrained to remain on its failure envelope so that any further loading is effectively shed onto the 'component' planes that have remained within their elastic range. Eventually, with increase in load, the last 'component' plane will reach incipient failure and the 'composite' plane is then considered to have failed. A form of 'brittle' failure can be simply included in the constitutive equations. For each 'component' plane a pre-fracture strength envelope is defined to include aspects of tensile, cohesive and frictional strength, with a post-fracture strength envelope based only on frictional strength (e.g. see Figure 3). This means that when a 'component' plane reaches its pre-fracture strength, and fractures, it is forced to retreat to its post-fracture strength envelope. This process of retreat, in itself, will shed considerable load onto other 'component' planes.

In the context of brickwork, 'composite' planes could be considered to be formed by perpend and bed 'components' (Modes 2,3,4) or perpend and brick 'components' (Mode 5). Bed failure (Mode 1) involves continuous planes of weakness and hence requires no special treatment. The solution of practical brickwork problems involves the following sequence,

- (a) The elastic properties of the brickwork are computed from those of the brick, bed, and perpend using techniques described by Gerrard (7).
- (b) For an increment of applied load, the elastic properties in the brickwork are used to determine the stresses in the brick, bed, and perpend. This determination is repeated under conditions of increasing load until failure is reached in either the bed or the perpend. Failure of the bed, by definition, represents failure of the brickwork panel, but the same does not apply to failure of the perpend. When the perpend fails, it sheds load to both the bed and the brick until ultimately failure occurs in one of the 'diagonal' modes or in the 'brick' mode. The brick, bed, and perpend are all assumed to be 'brittle', i.e. to have pre-fracture strengths and post-fracture strengths as illustrated and specified in Figure 3.

The pre-fracture strength for brick, bed, and perpend are all assumed to follow the simple Mohr-Coulomb theory with a tensile cut-off. However, it is recognized that there is some contention of the validity of this theory when applied to masonry materials experiencing tension, e.g. Khoo and Hendry (8) have measured a concave failure envelope for brick material while a convex failure envelope for mortar has been suggested by Page (1). Improved descriptions of the failure

envelopes for brick and mortar can be readily incorporated into the mathematical model used in this study.



	t MPa	c MPa	ϕ_b Deg.	ϕ_a Deg.	ψ Deg.
Brick	14	14	15	40	8
Bed Mortar	1.5	3	30	40	12
Perpend Mortar	0.75	1.5	20	30	10

Figure 3 Strength and flow properties of brick, bed, and perpend

4. PREDICTED RESPONSE OF BRICKWORK

The results obtained are reported for two distinct cases,

- the effect of different stress regimes on unreinforced brickwork panels, and
- the effect of reinforcement in the case of loading by uniaxial tension.

For the first case the panel was subject to three different stress regimes; viz., $\sigma_2 = -\sigma_1$, $\sigma_2 = 0$, and $\sigma_2 = \sigma_1$. The first represents the maximum shear condition in the compression-tension range, the third represents the extreme case of zero shear in the tension-tension range, and the second represents the

boundary between the two ranges. Bearing in mind that brickwork strength has been defined as the point at which continuous discontinuities are developed, the results obtained for each stress regime are shown in Figure 4, plotted against the inclination of the principal stresses, θ . It can be seen that, in general, the less tensile the minor principal stress, σ_2 , the stronger the brickwork. However, the minor principal stress is predicted to have no influence for the extreme case when $\theta = 0^\circ$, i.e. the major principal stress is normal to the mortar bed. At the other end of the spectrum, i.e. when $\theta = 90^\circ$, the minor principal stress has maximal effect to the extent that for $\sigma_2 = -\sigma_1$, the 'brick mode' of failure provides lower strength than that for the 'positive diagonal mode'. For $\sigma_2 = 0$, the strength at $\theta = 90^\circ$ ('positive diagonal mode') is almost three times that when $\theta = 0^\circ$ ('bed mode'). On the other hand, when $\sigma_2 = \sigma_1$ the brickwork strength is minimal and there is no variation in strength with θ .

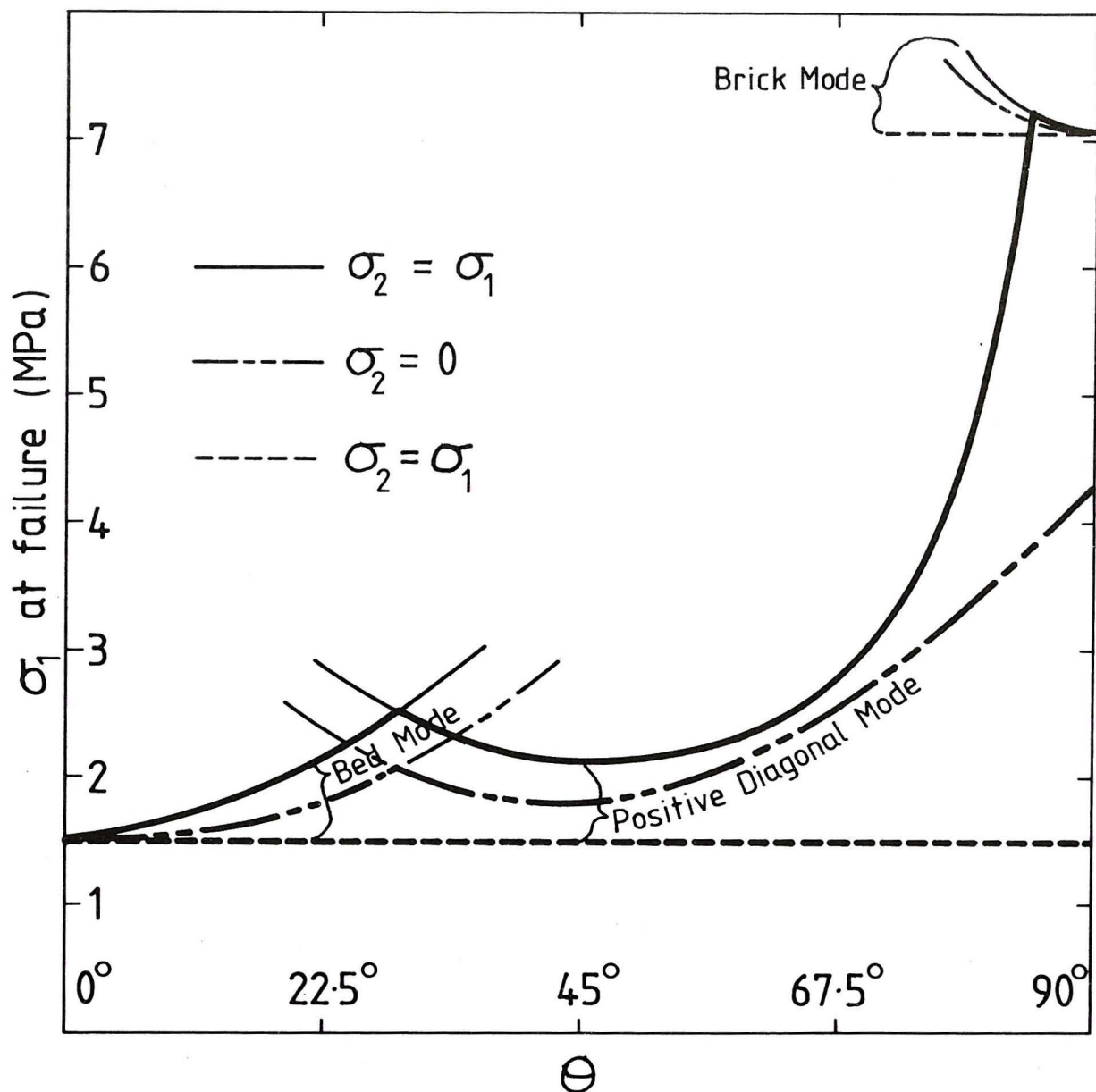


Figure 4 Strength of unreinforced brickwork

The results summarized above diverge in some important aspects with those reported previously by Page (1), e.g.,

- (a) minimum strength is predicted for $\theta = 0^\circ$, as against minimum strength predicted for intermediate values of θ (1),
- (b) the same minimum strength is predicted for all values of σ_2 when $\theta = 0$, as against a variety of values (1),
- (c) for $\sigma_1 = \sigma_2$, the same values of strength are predicted for all values of θ , as against minimal values for intermediate values of θ and maximal values for $\theta = 0^\circ, 90^\circ$ (1), and
- (d) the failure surface has discontinuities where there is a change from one failure mode to another, as against a smooth failure surface (1).

The above divergencies may be due to different assumptions with regard to the failure behaviour for brick and mortar and require further research to be adequately resolved.

For the study of reinforced brickwork loaded in uniaxial tension, the reinforcement was placed vertically and the volumetric proportion of reinforcement was taken as 0.1%. The reinforcement modulus was 200 GPa, the yield stress (σ_y) was 750 MPa, and the prestress levels (σ_o/σ_y) were taken as 0%, 20%, and 70%. As shown in Figure 5, at low angles of θ failure by the 'bed mode' controlled, while for higher angles of θ failure by the 'positive diagonal mode' controlled.

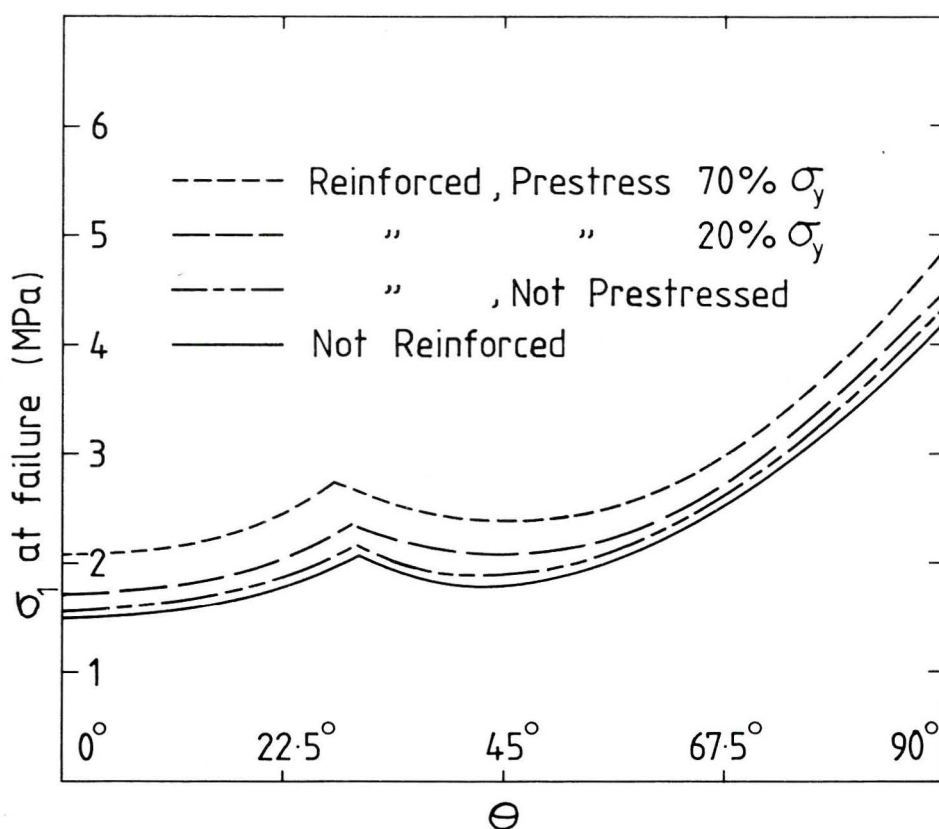


Figure 5 Strength of reinforced brickwork

The placement of vertical reinforcement provides a small improvement in strength across the full range of θ . This improvement would be expected for small values of θ , when the bed is in tension, but also occurs at high values of θ , when the relevant failure mode requires the bed to be in shear. This mode calls into play the shear resistance of the reinforcement, thereby resulting in a strengthening effect. Intuitively, the advantage of placing reinforcement vertically, rather than horizontally, can be seen, since horizontally placed reinforcement would only provide an advantage for high values of θ .

The results shown in Figure 5 indicate that the relatively small strengthening effect of the reinforcement is considerably enhanced if the reinforcement is prestressed. This applies for all values of θ . However, it should be borne in mind that, in this paper, failure has been defined as the formation of continuous discontinuities throughout the brickwork. In terms of ultimate strength, the relative advantage of prestressing would be minimal.

5. DISCUSSION

- (a) The mathematical model based on the concept of reinforced 'composite' planes, and using an elasto-visco/plastic algorithm, has been applied to the various modes of failure involved in brickwork problems subject to uniform, in-plane loading. Predictions of behaviour have been provided for the following two cases,
 - (i) unreinforced brickwork in the compression-tension and tension-tension ranges, and
 - (ii) prestressed brickwork subject to uniaxial tension with the reinforcement oriented vertically.
- (b) The results show the effect of different failure modes in producing discontinuities in the failure surface for brickwork, and the relative importance of prestressing the reinforcement in order to minimise brickwork cracking.
- (c) Some aspects of the predicted behaviour of unreinforced brickwork show significant differences when compared with previously reported results. These divergencies appear to be due to different assumptions with regard to the mortar properties. Further investigation of this point, together with more general studies, will require much more detailed information, than appears to be currently available, to indicate the nature of the deformational and strength response of the components of brickwork when subject to combined tension and compression.
- (d) The mathematical model developed for reinforced brickwork, subject to in-plane loadings, has been applied to simple problems, without stress gradients. It has the potential to be used together with the finite element method to solve in-plane problems involving stress gradients and complex boundary conditions.

6. REFERENCES

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