PROPOSAL FOR MASONRY DESIGN IN COMMON ACTION WITH CONCRETE AND REINFORCEMENT

C. ZELGER, Prof. Dr.-Ing.
Leitender akademischer Direktor am Materialprüfungsamt für das Bauwesen der Technischen Universität München

ABSTRACT This paper recommends a simplified method for the design of masonry acting in combination with reinforced concrete in pockets or cavities. It compares the results with strong solutions taking into account the real stress strain relationship of the three materials in common action.
An example will show, wether the recommended simplified design method may be acceptable in the general case of an eccentrically loaded slender column consisting of masonry, concrete and reinforcement, all acting together.

Dieser Beitrag empfiehlt eine vereinfachte Bemessungsmethode für Mauerwerk, das mit Stahlbeton zusammenwirkt. Er vergleicht die Ergebnisse einer vereinfachten Rechnung mit strengen Lösungen, welche das wirkliche Spannungs-Dehnungs-Verhalten der drei zusammenwirkenden Materialien berücksichtigt.
Anhand eines Beispieles wird geprüft, ob das empfohlene Bemessungsverfahren auf den allgemeinen Fall eines exzentrisch belasteten schlanken Pfeilers anwendbar ist.

1. PROBLEM FORMULATION

Masonry is even less able to sustain tensile forces than concrete and it therefore follows that components subjected to bending should be reinforced on the tension side. Compaction in the formwork minimizes air content in concrete but unfortunately it is impractical to completely embed the reinforcement in the thin mortar joints thus minimizing mortar air content in the same way. Inferior protection against corrosion and imperfect bond are the consequences. In order to avoid these disadvantages the reinforcement should be accomodated in hollow blocks or large cavities which can be completely filled with relatively non-porous fine aggregate infill concrete. Examples are pocket walls or cavity walls. In this way composite cross-sections are formed from masonry and reinforced concrete. Problems do not arise provided the concrete lies in the tensile zone because tensile strength is not taken into account. However, columns and walls are, as a rule, primarily subjected to compression and undergo comparatively little bending. The following design options are available in such cases:

a) The masonry content is ignored, the residual cross-section is designed according to reinforced-concrete codes.

b) The compound cross-section is treated as though it were composed solely of masonry.

c) The compound cross-section is modified and then treated as reinforced concrete.

Method a) is only applied if cross-section content and strength of the masonry are minimal with respect to the concrete or if there is doubt concerning the common action of both materials. This method ignores the load capacity of the masonry.
Method b) is common in countries where solid or only slighty perforated masonry units of medium and high strength are standard and therefore the

strengths of masonry and concrete or high-strength mortar show no appreciable difference.
Method c) comes into consideration if significant differences in strength exist between masonry and concrete and a considerable portion of the concrete cross-section is subjected to compression.

This paper is only concerned with the latter case. The example of a column encased in masonry (outer low-strength masonry and inner medium-strength reinforced concrete) will be examined to ascertain whether the generally accepted method of design for reinforced concrete construction applied to a modified cross-section reproduces the actual loadbearing behaviour of the composite cross-section with sufficient accuracy.

2. MODIFICATION OF THE CROSS-SECTION

Most national codes recommend the same shape of stress-strain diagram for masonry and concrete. It is therefore possible to take, for example, concrete as reference material and reduce the cross-section area of the masonry by the ratio of the design strengths f_m/f_c (Index m for masonry, c for concrete). It may then be easily perceived that the same calculated bending moments and normal forces in the ultimate limit state (diagram of interaction) are obtained, regardless of whether one takes as a basis the real compound cross-section with different design strengths for masonry and concrete or the modified cross-section with the design strength of the reference material being uniform.

3. STRESS DISTRIBUTION IN THE COMPRESSION ZONE

The assumptions for stress distribution in the compression zone extend from rectangular (e.g. BS 5628), triangular, bilinear, parabolic to the parable-rectangle diagram, as in Fig. 1 (CIB International Recommendations, CEB/FIP Model Code).

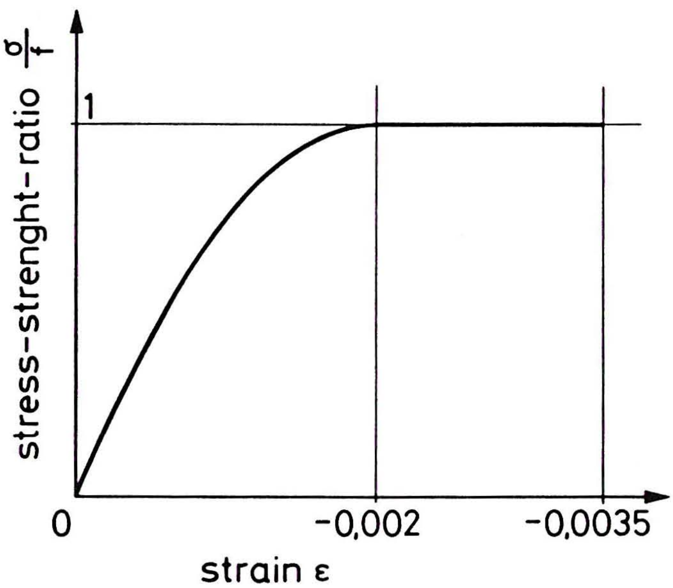

Fig 1

Parable-rectangle diagram

The shape of the stress distribution is of secondary importance provided that bending is the principal factor involved and not stability. A stress-strain relationship which is as realistic as possible is necessary if, for example, the loadbearing capacity of a slender wall or a slender column is to be calculated according to second-order theory.

4. INFLUENCE OF DUCTILITY ON THE COMMON ACTION OF DIFFERENT MATERIALS

It is well known that brick masonry is subject to brittle failure, particularly if high-strength bricks are used. It therefore appears doubtful whether the ultimate strain of - 0.0035 adopted in most countries and a non-linear stress-strain relationship deviating from Hooke's Law are justified. As long as components of uniform material are considered, overestimation of the ductility scarcely affects the calculated load capacity. Only deformability and advance notice of collapse are less than expected. If, however, two materials with different ductilities are in common action, the more brittle one can collapse under unfavourable conditions long before the strength of the other has been exceeded. Before postulating a simplified method of design as in Method c) (see above), a comparison with exact theory on the basis of the real stress-strain behaviour would appear expedient.

5. COMPARISON BETWEEN EXACT AND SIMLIFIED CALCULATION

The comparison between exact theory and simplified calculation is carried out using a concrete example so as to afford improved clarity.

5.1 Encased column as design example (see fig 2)

Square cross-section, side length t = 49 cm
Masonry casing, thickness 11.5 cm
Design strength $f_m = \alpha \cdot f_c$
Core of fine aggregate infill concrete, thickness 26 cm
Design strength f_c = 10.5 N/mm^2
Reinforcement in each corner 1 Ø 24 with a total
cross-section area A = 18 cm^2
Distance from the axis 11 cm
Characteristic strength f_s = 420 N/mm^2

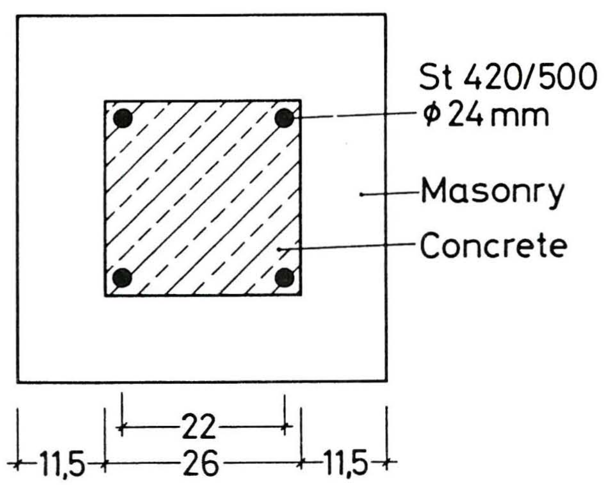

Fig 2 Cross-section for the design example

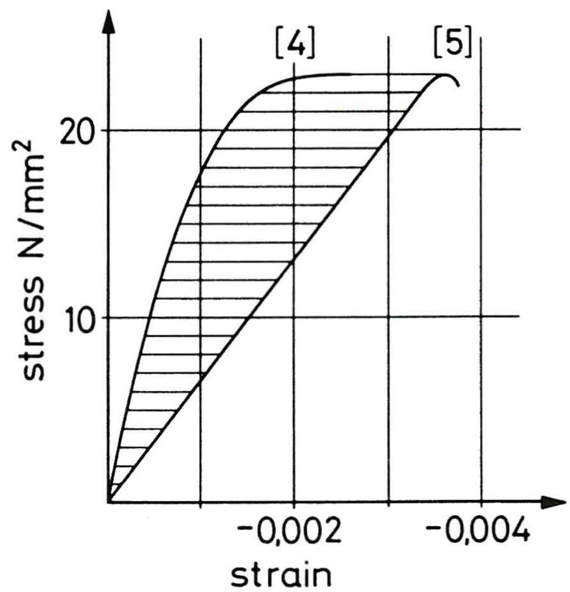

Fig 3 Measured stress-strain curves from the Literature /4/ and /5/. Most cases of practical importance should lie with the shaded area

5.2 True stress-strain curve for the masonry

Unfortunately complete stress-strain curves which include the downward slope after the maximum stress has been exceded are seldom to be found in the specialist literature. Some complete curves are given in /5/. They show only short downward slopes which indicate that high-strength masonry with approx. 23 N/mm^2 strength is subject to brittle failure with an almost linear stress-strain curve (See Fig. 3).

In masonry with the same compression strength of 23 N/mm^2 a non-linear stress-strain curve was observed at another point /4/. This was, however, only pursued as far as 88 % of the maximum stress. If extrapolation is carried out using a cubical parabola the steeper slope shown in the stress-strain curve in Fig. 3 is obtained, and this may also be very well approximated by a parabolic-rectangle diagram with a maximum at - 0.002 and ultimate strain of - 0.0026.

As shown in Fig. 3, very different deformation behaviour was observed in brick masonry. It may be assumed that most real stress-strain curves lie between the two curves shown.

5.3 Diagram of interaction of the modified cross-section as a basis for design

Fig. 4 shows interaction diagrams for the cross-section in Fig. 2. The concrete in the core is taken as reference material. The ratio α of the masonry strength and concrete strength is set at 0.5, 0.75 and 1.0.

The content of the masonry was weighted with these α values. Apart from that the same stress-strain curve as in Fig. 1 was taken as a basis for both materials for the sake of simplicity.

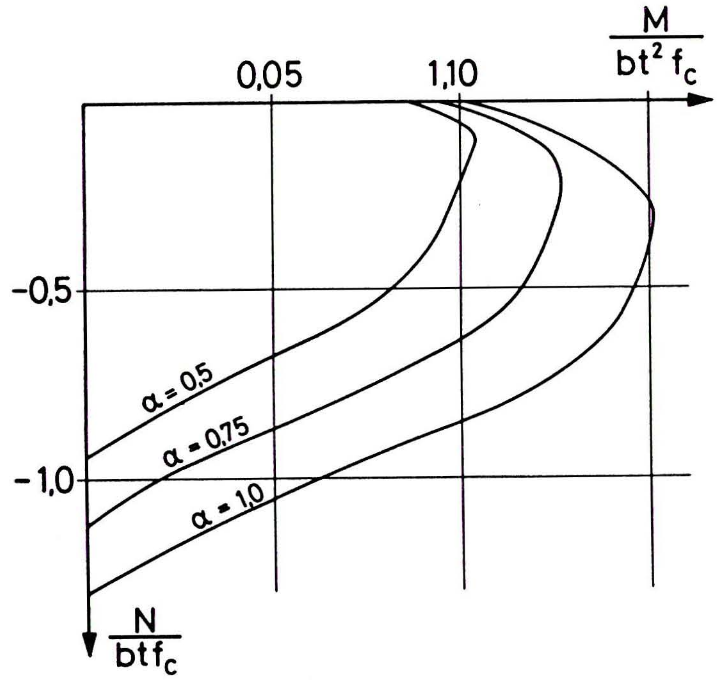

Fig 4 Interaction diagrams for the modified cross-section. Stress-strain relationship as in Fig.1 for both materials. b = width; t = thickness; $\alpha = f_m/f_c$

5.4 Exact calculation

It is extremely difficult to work out the solution to the differential equation for the bending curve according to second order theory if it must be taken into account that the real material behaviour does not obey Hooke's Law and a sinusoidal imperfection of h/300 such as occurred here has to be allowed for. The fact that the cross-section is composed of elements with different stress-strain relationships complicates matters further. It is necessary to fall back on numerical methods (e.g. after Runge-Kutta /6/)

requiring time-consuming iteration to fulfill prescribed boundary conditions. A number of calculations have been carried out for the previous example using this method.

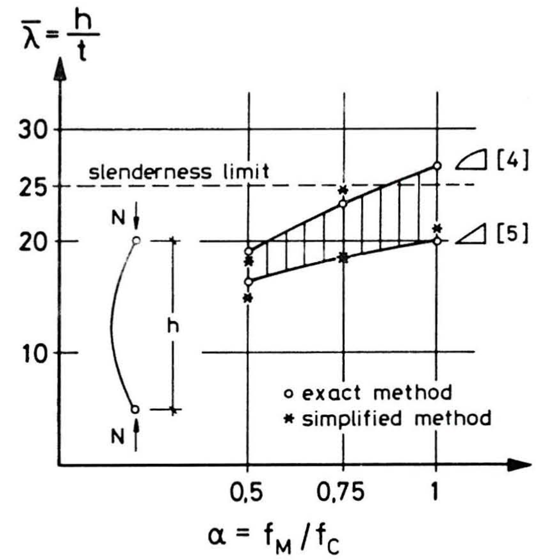

Fig 5 Ultimate slenderness leading to collapse dependent on strength ratio α.
N = Normal force
bt = Cross-section area
$N/btf_c = -0.5$

Fig. 5 shows the influence of different masonry strengths (x-axis) on the ultimate slenderness (y-axis). The significant difference (shaded area) between the two curves clearly demonstrates that the shape of the stress-strain curve must be taken into account. The upper curve is based on parabolic stress-strain relationship and the lower curve on triangular stress-strain relationship. The designations /4/ and /5/ correspond to Fig.3 and the bibliography.

For the case α = 75, parabolic stress distribution /4/ and varying centre normal force, the ultimate slenderness is shown in Fig.6. The influence of an offcentre load is demonstrated in Fig.7.

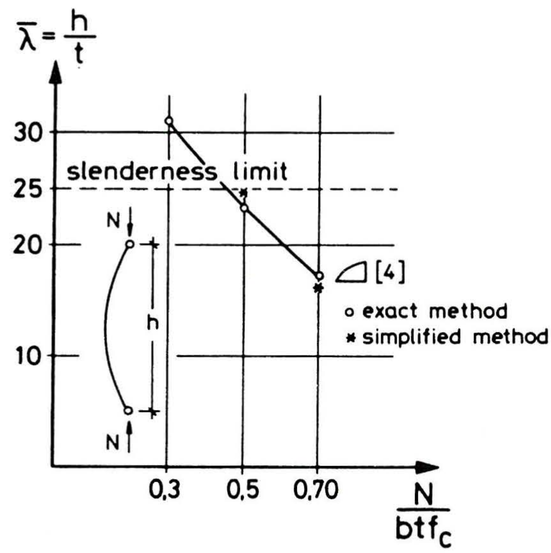

Fig 6 Ultimate slenderness dependent on normal force. α = 0,75

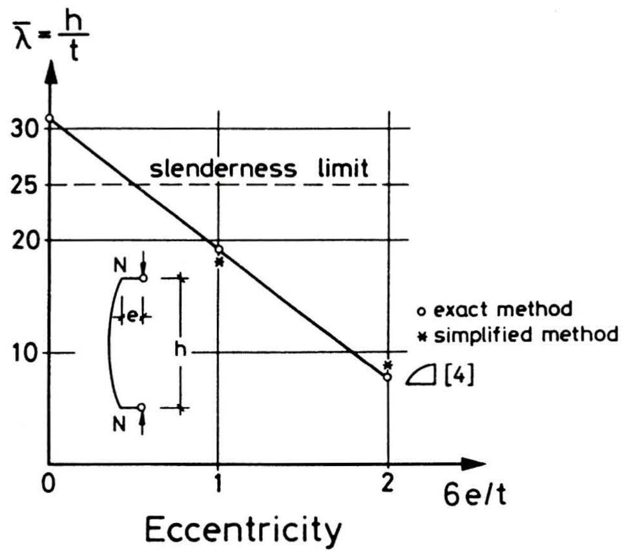

Fig 7 Ultimate slenderness dependent on eccentricity. α = 0.75
N/btf_c = - 0.3.
Parabolic stress-strain relationship /4/.

5.5 Simplified calculation

Practical application requires simple design methods. If possible, existing codes and design guides (Tables) should be used. Therefore a method which is common in reinforced concrete is proposed here and examined with respect to the exact solutions given.

In this method an approximate value v which is dependent on slenderness and eccentricity is given for the deflection according to second order theory. This value already contains an imperfection of h/300. It is added to the planned eccentricity e. It then remains to prove using normal methods that the cross-section is designed satisfactorily for the moment N (e + v) and the normal force N. The interaction diagram serves for this purpose (see Fig. 4). For slendernesses $\lambda \leq 70$ and eccentricities $e/t < 0.3$ the German codes for reinforced concrete /7/ give the following approximation for v/t:

$$\frac{v}{t} = \frac{\lambda - 20}{100} \cdot \sqrt{0.10 + \frac{e}{t}}\ ; \qquad \frac{v}{t} \geqq 0 \qquad (1)$$

Where $\lambda = \frac{h}{i}$

h = effective length

i = radius of inertia

Checking with respect to the design example used here confirms the practicality of this rule-of-thumb if slendernesses $\bar{\lambda} > 25$ ($\lambda > 87$) are excluded and a further correction factor is employed to take into account the influence of unfavourable stress-strain curves. It is suggested that the formula (1) be modified as follows:

$$\frac{v}{t} = \frac{\lambda - 20}{100} \cdot \sqrt{0.10 + \frac{e}{t}} \cdot \frac{1}{1 - (1 - \gamma) \cdot J_m/J} \qquad (2)$$

γ = "Degree of common action" of the masonry

J_m = Content of the masonry cross-section at the total moment of inertia

J = Total moment of inertia

The ratio of stress, at a strain of -0.002, and design strength of the masonry is introduced as the degree of common action of the masonry. This definition is based on the idea that the strength of the concrete is fully exceded at a centre strain of -.002, while the masonry only reaches a stress of $\gamma \cdot f_m \leqq f_m$ at the same strain.

The stress-strain curves displayed in Fig. 3 yield degrees of common action of 1.0 and 0.56 for near parabolic /4/ and linear /5/ stress distribution.

At $\gamma = 1$ correction is unnecessary (eqn. (2) is identical to equ. (1)). When $\gamma < 1$ a reduction in the loadbearing capacity is necessary and this occurs indirectly due to an increase in the additional eccentricity v/t. An unfavourable γ value can only have an effect equivalent to the extent of masonry involvement in the total cross-section. The factor J_m/J takes approximate account of this condition. The cross-section of the design example in Fig. 2 and the stress-strain curves in /4/ and /5/ (see Fig.3) yield the numerical values in Table 1.

Table 1 Cross-section values and correction factors for the design example

Ratio α f_m/f_c	J_m	J	J_m/J	i	$\frac{1}{1-(1-\gamma)J_m/J}$	
					$\gamma = 1.0$	$\gamma = 0.56$
-	dm^4	dm^4	-	cm	-	-
1.00	44.2	56.8	0.78	13.5	1.00	1.52
0.75	33.2	45.7	0.73	13.0	1.00	1.47
0.50	22.1	34.6	0.64	12.4	1.00	1.39

Table 2 Comparison between exact theory and simplified calculation

Ratio α f_m/f_c	γ	$\frac{h}{t}$	$\frac{h}{i}$	$\frac{e}{t}$	$\frac{v}{t}$ (Gl.2)	N/btf_c exact calculation	N/btf_c simplified calculation	deviation %
1.00	1.00	(26,7)	(96.9)	0	(0.243)	(0.50)	(0.58)	(+16)
1.00	0.56	20.1	73.0	0	0.257	0.50	0.56	+12
0.75	1.00	23.3	87.6	0	0.214	0.50	0.53	+ 6
0.75	0.56	18.5	69.6	0	0.232	0.50	0.50	0
0.50	1.00	19.0	75.2	0	0.175	0.50	0.48	- 4
0.50	0.56	16.2	64.2	0	0.195	0.50	0.45	-10
0.75	1.00	(31.0)	(116.5)	0	(0.305)	(0.30)	(0.40)	(+33)
0.75	1.00	17.1	64.3	0	0.140	0.70	0.67	- 4
0.75	1.00	19.4	72.9	0.17	0.273	0.30	0.29	- 3
0.75	1.00	7.9	29.7	0.33	0.064	0.30	0.32	+ 5

Table 2 contains for all exactly calculated cases (see Figs. 5, 6 and 7) the ultimate slenderness, the eccentricity, additional eccentricity determined according to eqn. 2 and the loadbearing capacities determined therefrom according to the exact and simplified method described above. The values lying outside the restriction $\lambda \leq 25$ aregiven in brackets. On average the deviations are below 1 % but in individual cases they lie between - 10 and + 12 %.

The preciseness of the approximation is not ideal. However, as compared to the gross simplification with respect to the exact calculation it is acceptable. It should be remembered that the approximation also includes the deviation between real and idealised shape of the stress-strain relationship (compare Figs. 1 and 3), which is usually ignored. Maximum deviations still lie within the normal standard deviation of the strength and may easily be compensated for by the safety factor. In practice any disadvantages stemming from the restriction $\lambda \leq 25$ are negligible because very slender columns and walls are in any case not pleasing to the eye and should be avoided if at all possible.

The results obtained by the simplified method are shown by an asterisk in Figures 5, 6 and 7.

6. SUMMARY

A simple approximation method is proposed for determining the loadbearing capacity under primarily normal-force loading in columns and walls with a reinforced concrete core and masonry case. An additional eccentricity is determined from the slenderness and planned eccentricity which reproduces the approximate deflection of the column due to second order theory including initial imperfection. The different characteristics of masonry and concrete are taken into account by modifying the cross-section and introducing a degree of common action. Instead of a complicated calculation according to second order theory it need only be proved that the cross-section has been satisfactorily designed for the normal force and the moment resulting from the planned and additional eccentricities. An encased column was taken as an example and the approximation method was compared with the exact method to show that correlation is sufficiently good for practical application.

Bibliography:

/1/ British Standard Code of Practice for Structural use of Masonry, Draft Part 2, Reinforced and Prestressed Masonry, BS5628

/2/ CIB (Conceil International du Bâtiment) International Recommendations for Reinforced and Unreinforced Masonry Structures, Draft 1

/3/ CEB/FIP - Model Code for Concrete Structures

/4/ Schellbach, Jung: "Tragfähigkeit und Tragverhalten von hochfestem Mauerwerk", Institut für Ziegelforschung Essen E.V.

/5/ H.R. Hotgkinson, S. Davis: "The Stress Strain Relationships of Brickwork when stressed in Directions other than Normal to the Bed Face", 6. International Brick Masonry Conference, Rome, 1982

/6/ R. Zurmühl: "Praktische Mathematik für Ingenieure und Physiker", Springer-Verlag, 1957

/7/ DIN 1045, Beton und Stahlbeton, Bemessung und Ausführung, Beuth Verlag GmbH, Berlin, Köln, 1978