APPLICATION OF THE FINITE ELEMENT METHOD TO THE DESIGN OF WALL BEAM

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ABSTRACT It is a usual practice that many brick masonry walls in buildings are supported by reinforced concrete wall beams, Recent investigation and experiments demonstrate that brick walls and supporting beams work together in the form of composite deep beams. So it is advisable to apply the finite element method in the evaluation of such beams. On the basis of computer analyses, we have derived a series of practical formulas for design purposes. Application of these formulas in design practices has achieved good technical-economic results.

1. INTRODUCTION

Recent experiments show that reinforced concrete wall beams work together with the brick walls which they support, thus forming composite deep beams. Since 1975, we have made use of the finite element method and prepared a computer programme, in order to carry out an elastic analysis of such composite beams. On the basis of the data from computer calculations, we have derived by means of regression analysis a series of formulas available for use in practical design. Application of these formulas in designs of single-storey mill buildings

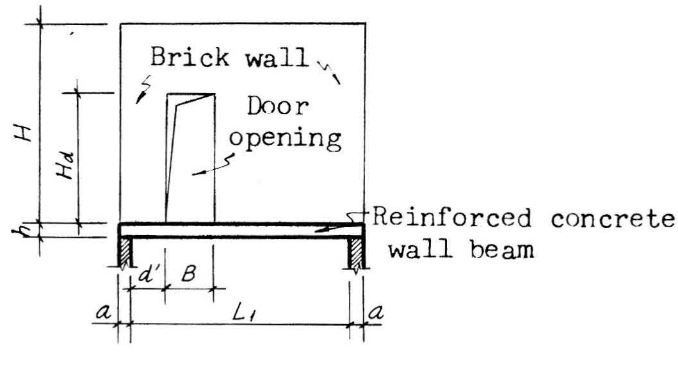


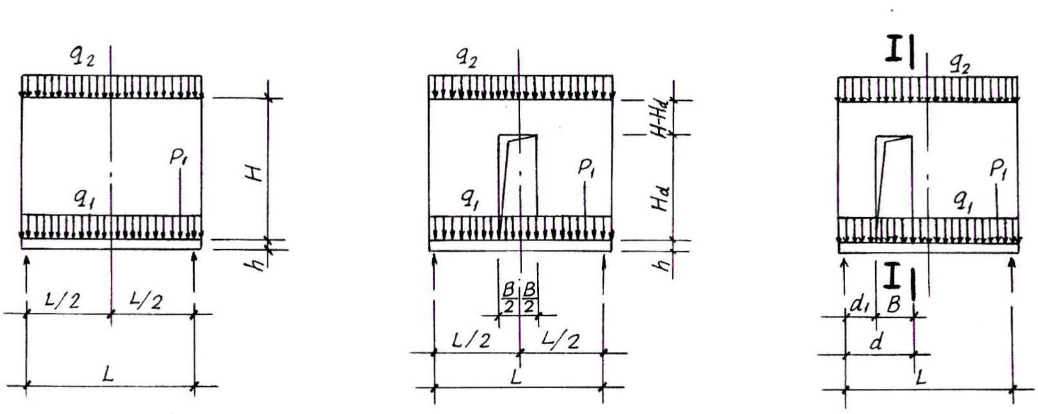
Fig. 1 Brick Wall and Wall Beam

since 1979 has resulted in good technical-economic effects. In comparison with the design based on the theory of beams on elastic foundation as proposed by Russian Professor B.N. Zhemochkin, wall beams designed by the method presented in this paper can save 10 percent of concrete and 15 percent of steel reinforcement. More recently, we have completed the work of modification and improvement on the formulas first developed in 1979, so that their application can be extended to multi-storey buildings. Described in this paper is the design method after improvement.

2. Tensile Strength of Wall Beam

2.1 Design Formulas for Bending Moment and Tension

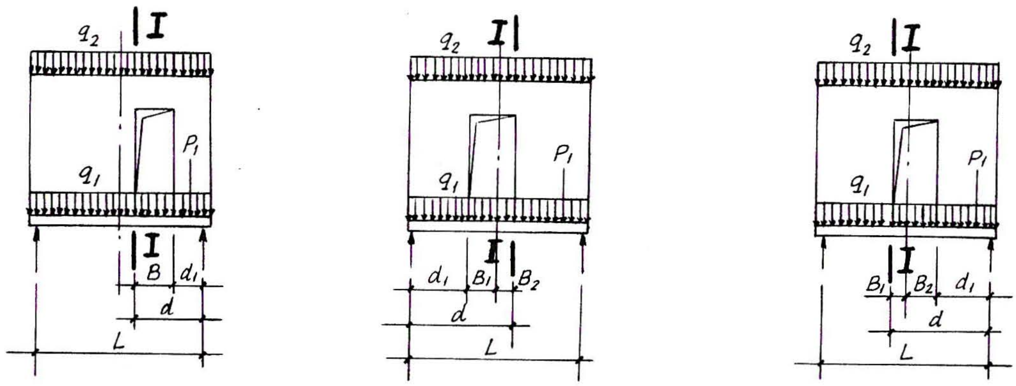
2.1.1 Determination of the Most Unfavourable Cross-section



(a)Wall without opening

(b)Wall with opening, which is symmetrical about the centre-line of beam span

(c)Wall with opening in the left half-span



(d)Wall with opening in the right half-span

(e) $B_1 > B_2$

(f) $B_1 < B_2$

Fig. 2 Design Sketch of Composite Beam

According to computer analyses and experimental results, the most unfavourable cross-section for tensile strength can be determined as follows: 1. In the case of walls without opening or walls with symmetrical opening about centre-line of beam span, use the section at midsapn (Fig. 2(a) and (b)); 2. In the case of walls with off-centred openings, use the section at that vertical side of the opening which is nearer to the centre-line of beam span (Fig. 2(c) to (f)).

2.1.2 Bending Moment M in Wall Beam

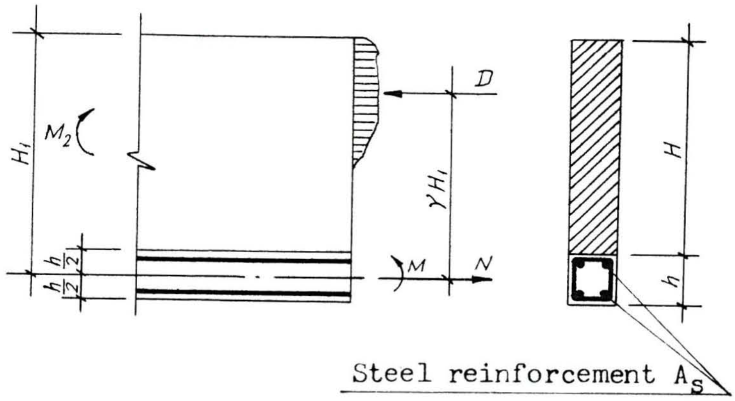


Fig. 3 Design Sketch of Normal Cross-section

Both computer analyses and recent experiments have shown, that a wall beam is a reinforced concrete member subjected to tension and bending. Computer analyses yielded maximum tensile stress in the bottom extreme fibres of the beam and minimum tensile stress or compressive stress in its top extreme fibres, whereas experimental results indicated that the total tension in the bottom longitudinal reinforcement is larger than that in the top longitudinal reinforcement or compression is produced in the top reinforcement. Therefore, in addition to tension N, there is a bending moment M in a normal cross-section of wall beam (Fig. 3). Computer analyses also made clear that the bending moment in a wall beam under the action of external load q_2 is a function of the bending moment M_2 of the composite beam at the section under consideration, and thus can be expressed as,

$$M = \alpha M_2 \tag{a}$$

where, α - moment distribution factor determined by the method described in Sect. 2.2

In multi-storey buildings, the bending moment M_1 due to loads q_1 and P_1 from the floor directly supported by the wall beam can conservatively be assumed to be taken solely by the wall beam. Hence, the bending moment M in a wall beam under the combined action of loads q_1, P_1 and q_2 can be computed using the following equation,

$$M = M_1 + \alpha M_2 \tag{1}$$

2.1.3 Axial Tension N in Wall Beam

According to Fig. 3, the axial tension N in a beam which supports brick wall without opening is evaluated by the following equation,

$$N = \frac{M_2 - \alpha M_2}{\gamma H_1} = \frac{(1-\alpha)M_2}{\gamma H_1} \tag{b}$$

in which, γ is the moment arm factor, to be obtained from Eq.(5). It is

learned from computer calculations, that the height of moment arm γH will be changed when the wall has an opening, and that bending moment M' may exist in the part of brick masonry above the opening (Fig. 4). These will necessarily have influence on the tension N obtained from Eq. (b). With a view to taking a many-sided consideration of such effects and making the equation simple as well as practical, an influence factor of opening, ψ, is introduced. Thereby, the axial tension N in a wall beam can be expressed as,

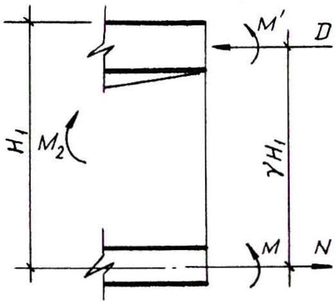


Fig. 4 Design Sketch of Normal Cross-section

$$N = \psi \frac{(1-\alpha)M_2}{\gamma H_1} \tag{2}$$

where, ψ - influence factor of opening, for off-centred opening use Eq.(6) to determine ψ, for opening symmetrical about the centre-line of beam span and for walls without opening, $\psi = 1$;

H_1 - calculated height of moment arm, $H_1=H+0.5h$, in which h is height of beam; when $H > L$, take $H_1=L+0.5h$ (for explanation, see p. 6).

2.2 Moment Distribution Factor α

2.2.1 Wall Without Opening

As a result of regression analyses performed on the basis of the results from computer calculations of 59 wall beams, the following expression for the moment distribution factor α is obtained,

$$\alpha = \frac{h}{H_2}(0.82+0.035\, E_c/E_b - 0.068\, L/h) \tag{3}$$

where, E_c - modulus of elasticity of concrete;

E_b - modulus of elasticity of brick masonry;

L - span length of beam, equal to the smaller value of $1.05L_1$ and L_1+a, where L_1 is the clear span length and a the length of bearing on support;

H_2 - calculated height of cross-section, to be used for determining the moment distribution factor. $H_2=0.75L+h$ when $H > 0.75L$ and $H_2=H+h$ for other cases, (for explanation, see p. 5).

The 59 wall beams which were used in statistical analyses have the following parameters: $E_c/E_b=10\text{-}20$ (but, $E_c/E_b=15\text{-}20$ for beams with $h/L=0.065$, $E_c/E_b = 10\text{-}15$ for beams with $h/L=0.121$), $h/L=0.065\text{-}0.121$, and $H/L=0.34\text{-}0.97$. Referring to the regression equation (Eq. (3)), the correlation coefficient $r=0.956$, the standard deviation $s=0.013$, the mean value is 1.017 and the coefficient of variation $C_v=0.109$.

Results of computer calculations have shown, that for a given h/L and E_c/E_b, the relation between α and H/L is as shown in Fig. 5 (the data from beams with $H/L < 0.34$ and $H/L > 0.97$ were not used in the fore-mentioned regression analyses). It is noted in the figure that the value of α tends to become constant when H/L is equal to or greater than 0.75. For beams with $H/L > 0.75$, results from beams with $H/L=0.75$ may be used, giving solutions slightly

on the safe side. Therefore,

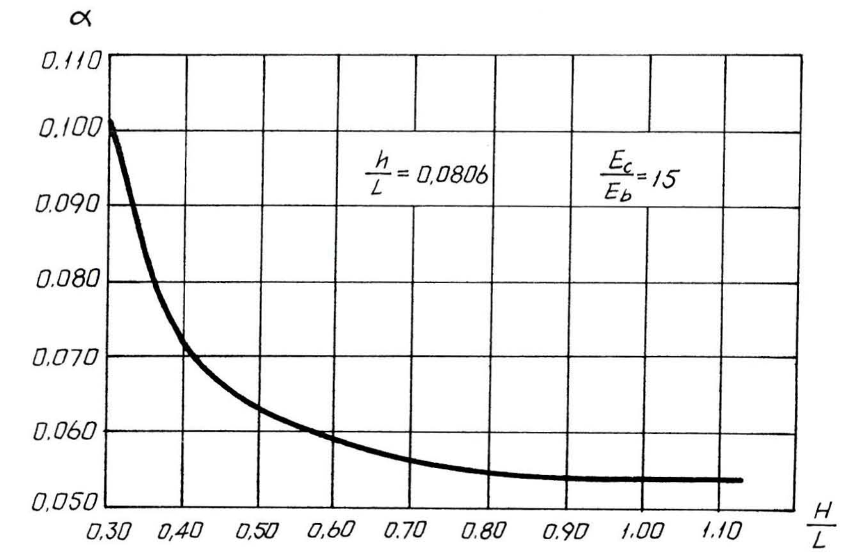


Fig. 5 α - H/L curve

to determine the value of α for beams with $H/L > 0.75$ (including those with $H/L > 1.0$), Eq. (3) can be used by substituting into it $0.75L+h$ for H_2. For beams with $H/L < 0.34$ and those with value of E_c/E_b and h/L not in conformity with the beams covered in statistical analyses, Eq.(3) should be modified to suit such cases.

The beams involved in statistical analyses have values of α varying from 0.02 to 0.24. This means that the bending moment distributed to the wall beam at its midspan is 2-24 percent of the total moment M_2 in the cross-section at midspan. For wall beams in single-storey mill buildings and for those in multi-storey buildings carrying light loads from the floor which the beams directly support, the bending moment M may affect the amount of beam reinforcement and, therefore, has to be taken into account.

2.2.2 Wall With Opening

Both computer calculations and experiments testify that the factor α will increase if the wall has an opening located off the centre-line of beam span. The regression analysis of results from computer calculations of 152 wall beams gives expressions for α as Eqs.(4a) and (4b), with $r=0.957$, $s=0.051$, mean value=1.073 and $C_v=0.168$.

$$\alpha = 0.33 + 2.8\ h/L - 1.56\ d_1/L \tag{4a}$$

when $d_1/L \leqslant 0.25$,

and

$$\alpha = 0.33 + 2.8\ h/L - 0.78\sqrt{\frac{d_1}{L}} \tag{4b}$$

when $d_1/L > 0.25$,

where, d_1 - distance from support to that vertical side of opening nearer to the support.

The computer-calculated beams included in the statistical analysis have the following parameters: h/L=0.065-0.121, d_1/L=0.016-0.419, B/L=0.161-0.581 and H/L=0.435-0.726. Within the above ranges, the calculated value of α may reach a maximum of 0.65, that is, 65 percent of the total moment in the cross-section under consideration is taken by wall beam.

It is observed in Eqs.(4a) and (4b) that h/L and d_1/L are the main parameters which have influence on α. The effect of the ratio of opening width to span length, B/L, on α is insignificant and finds no expression in Eqs.(4a) and (4b). In fact, however, B/L certainly has significant effect on the bending moment in wall beam, and this effect has been allowed for in the evaluation of M_2. The effect of H/L is not obvious either and so is not expressed in Eqs.(4a) and (4b). This phenomenon may be related to the fact that the ratio of height of brick masonry above opening to span length, $(H-Hd)/L$, is taken as a constant in computer calculations (the ratio is 0.08 in this paper). The reason for such a treatment is the adaptation to all unfavourable circumstances that may possibly occur and the attempt to obtain results on the safe side. Actually, if the brick wall is very high, $(H-Hd)/L$ will increase, whereas the value of α will decrease. For instance, when $(H-Hd)/L$=0.323, a wall beam with H/L=0.97 has a value of α which is 10-30 percent less than that computed according to Eqs.(4a) and (4b).

There is no obvious regularity in connection with the influence of E_c/E_b on α, so E_c/E_b does not appear in Eqs.(4a) and (4b). This problem requires further study.

It is noted in Eqs.(4a) and (4b), that under the condition of a given h/L, the value of α increases as d_1/L decreases, that is, the nearer to the support is the door opening located, the larger becomes the moment distribution factor. In this condition, the bending moment distributed to wall beam still tends to increase though M_2 is somewhat reduced.

2.3 Moment Arm Factor γ

By analysing the results from computer calculations of 59 beams supporting brick wall without opening, Eq.(5) is deduced for the moment arm factor γ. For this regression equation, r=0.992, s=0.016, mean value=1.008 and C_v=0.026.

$$\gamma = 1-0.54\ H_1/L \tag{5}$$

Results of computer calculations have shown, that the height of moment arm γH_1 and the value of N become constant when $H/L \approx 1.0$(Cf.Fig.6.). Hence, for wall beams with $H/L > 1.0$, use L+0.5h instead of H+0.5h for H_1, the calculated height of cross-section, in both Eq.(2) and Eq.(5).

Shown in Fig. 7 are the calculated correlation curve of γ versus H_1/L and the results of computer calculations, for the range where H_1/L is equal to or less than 1.0.

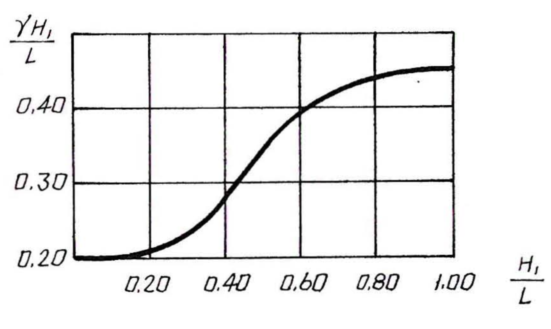


Fig. 6 $\dfrac{\gamma H_1}{L} - \dfrac{H_1}{L}$

From the standpoint of engineering practice, we can still use Eq.(5) to determine the value of γ for beams which support brick wall with opening, and meantime, introduce a factor ψ to account for the effect of opening on γ .

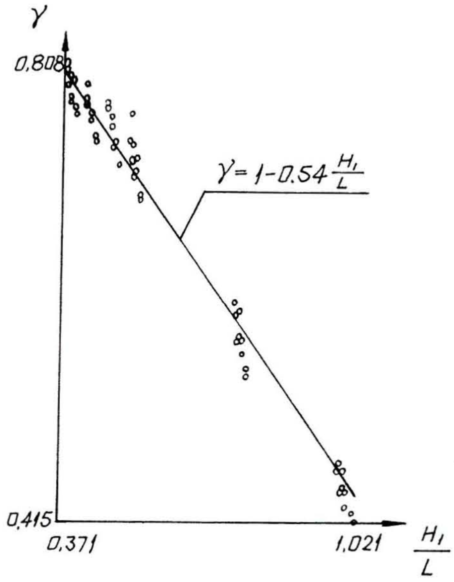


Fig. 7 γ - H_1/L curve

2.4 Influence Factor of Opening, ψ

Using the data from computer calculations of 120 beams supporting brick wall with off-centred opening, the regression analysis results in an expression for determining ψ as Eq.(6), for which, r=0.567, s=0.106, mean value= 0.984 and C_v=0.152

$$\psi = 0.4(1+ \frac{2d-B}{L}) \qquad (6)$$

in which, d - distance from support to that vertical side of opening which is nearer to the centre-line of beam span, $d=d_1+B$ (Fig. 2);

B - width of opening (Fig. 2).

2.5 Calculation for Longitudinal Reinforcement of Beam

Following the determination of the bending moment M and axial force N in a wall beam by use of the above method, the longitudinal reinforcement of the beam can be calculated in accordance with a reinforced concrete beam subjected to tension and bending.

3. Shear Strength of Wall Beam

3.1 Maximum Shear Q

On the basis of the results from computer analyses, the maximum shear Q in a wall beam is expressed by the following equation,

$$Q = \zeta Q_2 + Q_1 \qquad (7)$$

where, ζ - shear distribution factor, to be determined by the method described in Sect. 3.2;

Q_2 - shear in the cross-section at support due to load q_2;

Q_1 - shear in the cross-section at support due to load from the floor directly supported by wall beam.

Generally the maximum shear in a wall beam takes place in the section at the support. Nevertheless, computer calculations show that the maximum shear in a wall beam may occur within the limits of the opening width, when $d'/L \leq 0.125$ ($d'=d_1 -0.5a$, a being the length of bearing, Cf. Fig. 1) and $B/L \leq 0.5$.

3.2 Shear Distribution Factor ζ

3.2.1 Wall Without Opening

In the process of analysing the results from computer calculations of 59 wall beams mentioned above, it was found that the factor ζ varies only in a relatively small interval, i.e. ζ =0.36-0.48. For the sake of simplicity, ζ =0.42 is recommended.

3.2.2 Wall With Opening

According to the regression analyses based on computer calculations of 140 beams supporting brick wall with off-centred opening (with same ranges of variation for parameters h/L, d_1/L, B/L and H/L as the beams described in Sect. 2.2.2), Eq. (8) is derived for ζ , for which, r=0.845, s=0.169, mean value= 1.01 and C_v=0.097.

$$\zeta = 0.4\left[1.86 - 2.83\frac{B}{d} + 3\left(\frac{B}{d}\right)^2\right] \qquad (8)$$

In Eq.(8), ζ has taken consideration of the maximum shear in the section at support as well as that which will appear within the limits of the opening width under the fore-mentioned conditions.

3.3 Calculation for Stirrups in Wall Beam

Following the determination of the maximum shear in a wall beam, the required amount of stirrups can be calculated in accordance with a reinforced concrete member. In general conditions, it is permissible to gradually reduce the amount of stirrups, in successive segments, from support to midspan. For wall beams with $d'/L \leq 0.125$ and $B/L \leq 0.5$, only in the limits beyond the distance from support, d, can the amount of stirrups be properly reduced. However, when $d > \frac{L}{2}$, no reduction of stirrup is allowed in the whole span.