# NUMERICAL MODELLING OF THE STRUCTURAL BEHAVIOUR OF MASONRY BUILDINGS

F. MOTTA , E. D'AMORE
Istituto di Scienza delle costruzioni
Università di Catania, Italy

ABSTRACT This paper discusses the problems of non-linear numerical modelling - through finite elements and following the smeared approach - of the plane masonry panels subject to monotonic varying loads, using different material constitutive laws.

Numerical results are compared with experimental curves available in literature.

## 1. INTRODUCTION

Micro-modelling techniques - through finite element method- for the numerical simulation of mechanical behaviours of a non-linear type of masonry structures, which have already been set up during the recent years, have allowed us to obtain such results as to constitute a useful standard of calibration and comparison among parallel experimental studies.

Nevertheless the modelling of big masonry structures,in terms of acceptable computational efforts, can be achieved only if one accepts the drastic simplification by which the structure itself is considered as an assembling of masonry panels, and each panel as a planar continuum medium subject to membrane efforts.

It is then obvious that a successful modelling is strictly linked to the given options as for discretisation, constitutive laws and failure criteria.

This paper, within the frame of a wider research programme which is under way, intends to study an isolated wall, no matter with what openings, subject to monotonical loads, by means of modelling with isoparametric F.E. and carrying out the analysis through an incremental way.

Materials will be defined by constitutive laws for biaxial stress states, both of elastic-brittle type and of non-linear type, isotropic and orthotropic, and such that their identification parameters can be appreciated using the results of experimental tests of monoaxial stresses.

The Mohr-Coulomb failure criterion with tension cutoffs will be adopted, as well as another criterion which was originally formulated for plain concrete.

The anisotropy arising from cracking will be taken into account by modifying at any step of loading the material stiffness matrix.

Finally, there are some numerical applications, so as to allow a comparison of the different results obtained through the dif

ferent modellings wich have been proposed.

## 2. CONSTITUTIVE MODELS OF THE MATERIAL

### 2.1. Isotropic elastic-linear model.

For uncracked material, considered as homogeneous, the simplest model is a model which foresees an isotropic elastic linear behaviour. $E_o$, $\nu_o$ parameters, which are necessary to describe the constitutive law during the biaxial loading state, can be easily drawn from experimental monoaxial tests, suitably modifying the results according to the different loading directions, so as to take into account the existing anisotropy of the material. This law is combined with the Mohr-Coulomb failure criterion with tension cutoffs, even though the failure mechanism associated with this model is not verified in general by the test results.

The behaviour of the material, following the formation of the first set of cracks and/or after the first local crushings, will be described later on.

### 2.2. Isotropic non-linear model.

This non-linear model is based on the isotropic formulation, which was originally developed by Kupfer et al. for concrete, [1], introducing variable tangential bulk moduli $K_t$ and shear moduli $G_t$. In this case, stress and strain increments are decomposed into hydrostatic and deviatoric components and the tangent incremental constitutive law takes the following form:

$$d\varepsilon_{ij} = \frac{\delta_{ij}}{3} d\varepsilon_{ii} + de_{ij} = \delta_{ij} \frac{d\sigma_{ii}}{9\,K_t} + \frac{ds_{ij}}{2\,G_t} . \qquad (1)$$

The elastic moduli which appear in the previous equation can be related to the initial moduli ($K_o$, $G_o$), as evaluated experimentally by monoaxial tests, by means of the relationships, [2]

$$K_t = K_o \left( 1 - C_2 \frac{\sigma_{oct}}{\sigma_{octp}} \right) \; ; \quad G_t = G_o \left( 1 - \frac{\tau_{oct}}{\tau_{octp}} \right) . \qquad (2)$$

Symbols are explained in the appendix.

Octahedral shear and hydrostatic strengths can be calculated using the following schedule :

| Case | $\alpha$ | $\tau_{octp}$ | $\sigma_{octp}$ | |
|---|---|---|---|---|
| $\lvert\sigma_1\rvert > \lvert\sigma_2\rvert$ | $\sigma_2/\sigma_1$ | $A\,\lvert\sigma_{1p}\rvert$ | $A'\sigma_{1p}$ | (3) |
| $\lvert\sigma_2\rvert > \lvert\sigma_1\rvert$ | $\sigma_1/\sigma_2$ | $A\,\lvert\sigma_{2p}\rvert$ | $A'\sigma_{2p}$ | |

where

$$A = \frac{1}{3} \sqrt{2(1-\alpha+\alpha^2)} \quad ; \quad A' = \frac{1}{3} (1+\alpha) \tag{4}$$

and having indicated by $\sigma_{ip}$ the maximum stresses in the two principal directions, like as, for example, in Fig.1/a.

The $C_2$ constant can be evaluated experimentally by optimizing the results of biaxial compression tests. An approximate value can be also obtained from a monoaxial test by means of the relationship

$$C_2 = \frac{1}{3K_o} \frac{\sigma'_{oct}}{\varepsilon'_{oct}} \tag{5}$$

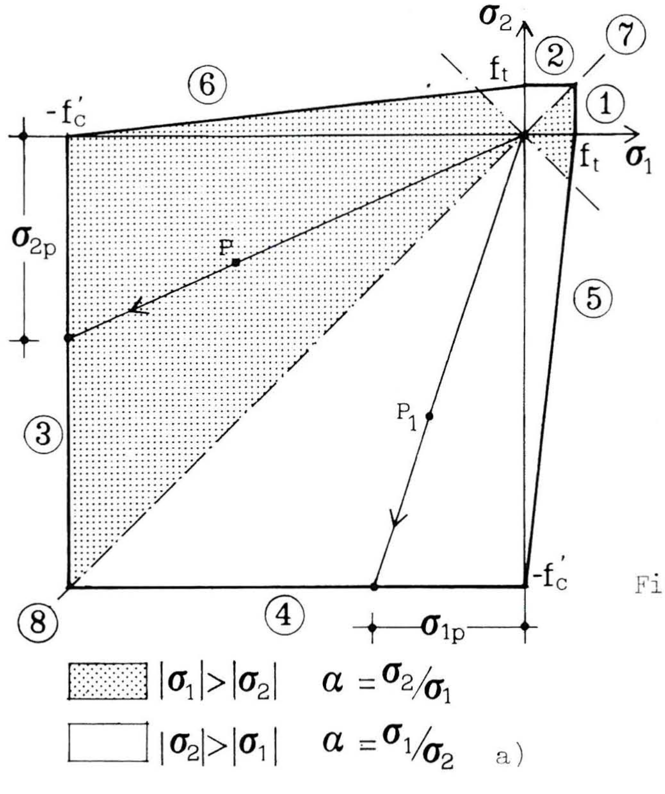
in which there are the co-ordinates of the S' (Fig.1/b) point.

Finally we adopt as biaxial failure criterion the simple Mohr-Coulomb criterion or the criterion proposed in [1] ,which also depends on uniaxial compressive and tensile ($f'_c$, $f'_t$) strengths of the material.

The description of the material is completed by defining the secant shear modulus through the following relation, of experimental origin , [3] ,

$$G_s = \frac{\tau_{octp}}{\gamma_{oct}} \left[ 1 - \exp\left( - \frac{\gamma_{oct}}{\tau_{octp}} G_o \right) \right] \tag{6}$$

whereas, for want of better information, it is advisable to consider $K_s \cong K_t$ .



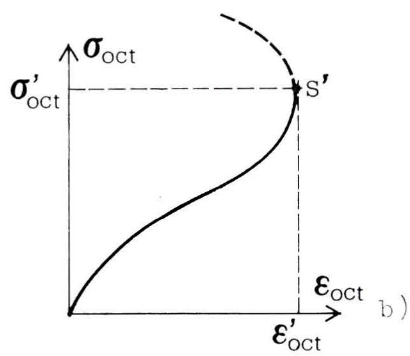




Fig.1.-(a) Failure criterion,

(b) Tipical volumetric σ/ε curve

## 2.3. Orthotropic non-linear model.

A constitutive model based on an orthotropic behaviour of the material, capable of representing both monotonic proportional loading and biaxial cyclic loading histories, has been initially proposed by Darwin and Pecknold , [4] .

The constitutive law for plane stress analysis, in tangential incremental form, results:

$$\begin{bmatrix} d\sigma_1 \\ d\sigma_2 \\ d\tau_{12} \end{bmatrix} = \frac{1}{1-\nu^2} \begin{bmatrix} E_1 & \nu\sqrt{E_1\ E_2} & o \\ & E_2 & o \\ sym & & G' \end{bmatrix} \begin{bmatrix} d\varepsilon_1 \\ d\varepsilon_2 \\ d\gamma_{12} \end{bmatrix} \tag{7}$$

in which $E_1$ , $E_2$ , $\nu^2 = \nu_1\ \nu_2$ , $G' = \frac{1}{4}\ (E_1 + E_2 - 2\nu\sqrt{E_1\ E_2}\ )$ ,

are stress-dependent material properties and $d\sigma/d\varepsilon$ are respectively the differential stress/strain vector in material co-ordinates.

It is convenient, for a formal separation of Poisson effect, to recall the concept of equivalent uniaxial strain, (EUS), i.e., for any stress, the strain corresponding to the true stress on the uniaxial loading curve. For a non-linear material it results

$$\varepsilon_{iu} = \Sigma \frac{\Delta\sigma_i}{E_i} \tag{8}$$

where $\Delta\sigma_i$ is the incremental change in stress and $E_i$ the varying tangent stiffness for a load increment corresponding to $\Delta\sigma_i$.

The sum is extended to all load increments.

The EUS $\varepsilon_{iu}$ represents that portion of the strain (i.e.without the Poisson effect) in the i-th direction of the principal axes, that controls the behaviour of the material, including softening and failure. In this case, to define the proporties of the material, it is necessary to introduce a family of uniaxial stress-strain relationships, each one for a different principal-stress ratio.

The curves assumed here for compressive loading, based on the Saenz formulation, result

$$\sigma_i = \frac{E_o\ \varepsilon_{iu}}{1 + \left(\frac{E_o}{E_s} - 2\right)\frac{\varepsilon_{iu}}{\varepsilon_{ic}} + \left(\frac{\varepsilon_{iu}}{\varepsilon_{ic}}\right)^2} \tag{9}$$

in which $E_s = \sigma_{ic}/\varepsilon_{ic}$ is the secant modulus at the point of maximum compressive stress and $\varepsilon_{ic}$ is the EUS at the same point.

The stress $\sigma_{ic}$, for every principal-stress ratio, is found by exploiting the strength failure envelope, for instance the Mohr-Coulomb one.

The value of $E_1$ and $E_2$ are found as the slope of the $\sigma_1-\varepsilon_{1u}$ and $\sigma_2-\varepsilon_{2u}$ curves at the current values of the EUS, which are accumulated during the load history using Eq.(8).

In the tension section we assume on the contrary an elastic linear law with $E_o$ modulus.

To evaluate the structural response of the masonry panel under post-peak stresses, the constitutive law is extended by a softening branch, as shown in Fig.2, having a linear form.

The unloading curve is a parabola and the reloading one is again linear, such as described in [5] .

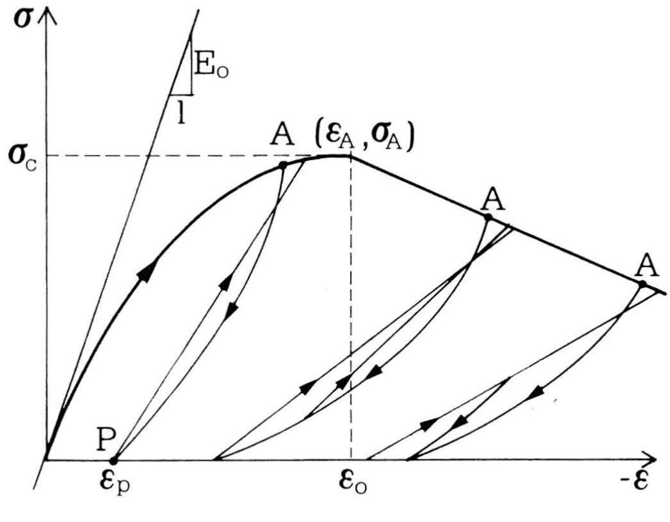



Fig.2.- Uniaxial σ-ε curve

Unloading curve equation

$$\eta = \sigma/\sigma_c \quad ; \quad \xi = \varepsilon/\varepsilon_o$$

$$\eta = (\xi-\xi_p)\left[A(\xi-\xi_A)+\eta/(\xi_A-\xi_p)\right]$$

$$A = 0.225\,\xi_A^2 - 1.225\,\xi_A + 1.85$$

$$\xi = \xi_A - \eta_A\,(\xi_A+\alpha)/(\eta_A+E\alpha)$$

$$\alpha = 0{,}1175\sqrt{\xi_A} \quad ; \quad E = E_o\,\varepsilon_o/\sigma_c$$

Reloading curve equation

$$\eta = E\beta\,(\xi-\xi_p)/(\xi_p+\beta)$$

$$\beta = 0{,}1002\,\sqrt{\xi_A}$$

3. ANISOTROPIC BEHAVIOUR ARISING FROM CRACKING OR CRUSHING

To introduce into the model the anisotropy arising from the presence of local cracking and/or crushing we take into account what follows.

i. In the tensile zones there is a smeared set of cracks when one of the principal tensions exceeds the value of the tensile strength $f'_t$ . The general direction of the cracks is orthogonal to the direction of the said principal tension;

ii. The behaviour following the opening of a crack, as it has to be such as to express zero tensions in the direction orthogonal to the crack, will be modelled assuming that the stiffness corresponds to zero in the said direction;

iii. In the case of compression strains acting orthogonally to the

crack, we assume that the stiffness of the cracked element still corresponds to the stiffness of the original uncracked element and the crack is assumed to be closed ;

iv. The reopening of a cracking system, which had developed before and had been reshut, takes place without any further work: in fact, in these elements the direction of the new cracks is no more linked to the actual principal tensions but stays identical to the one which was previously determined;

v. For the crushed material the stiffness is assumed to be zero for further loading and the current state of stress, at the point just before crushed, is assumed to be completely released;

vi. The roughness of the surfaces surrounding the generic crack is such that the incremental shear stress can be considered a linear function of the incremental strain, i.e. in the material co-ordinate system

$$\frac{1}{G}\ \frac{\Delta\tau}{\Delta\gamma} = \left\langle \begin{matrix} \eta' \\ \eta'' \end{matrix} \right. \quad ;\ 0 \leqslant \eta' \leqslant \eta'' \leqslant 1 \tag{10}$$

where $\eta'$ $\eta''$ are two opportune constants to be chosen in the said field, respectively in the case of a single cracking system or of two systems present at the same time, and G the shear modulus of uncracked material.

## 4. SOLUTION PROCEDURE

The solution procedure which has been adopted, through F.E.M., is shown elsewhere, [6] . In the present study we have used isoparametric elements with four nodes presenting linear shape functions. Integrations have been developed in four points chosen within the element itself.

## 5. NUMERICAL APPLICATIONS

The numerical modelling of masonry walls as proposed in this paper has been introduced into a special computer code which is also capable of accepting various options as for the constitutive laws and the failure criteria. The said code has already allowed us to obtain good results in the tests which have been carried out until now, as compared with other cases quoted in the technical literature.

Just as an example we have studied the case of a wall with several openings, already studied in [7] .

The mechanical properties and the other data of the problem are indicated later on. Thickness 0,4 m. Weight of the structure itself, 280 N/m$^3$. Distributed loads: vertical 140, lateral $\beta$x6875 N/m . Young modulus 2000 MPa. Compressive and tensile strength : 4 and 0,07 MPa. Poisson coefficient = 0,2. $\varepsilon_{cu} = 9.10^{-3}$ .

In Fig.3 we indicate the lateral force/displacement curves for two different points of the wall which have been obtained using different constitutive laws for the material.

The said curves are compared to the ones obtained in [7] through the ADINA code.

Vertical loads have been considered as being present with their first intensity starting from the first load step; the lateral loads, uniformely distributed in all integration points, have been applied according to a monotonically increasing law until collapse. In Fig.4 we show the evolution of the cracking state as the load increases, by using the models of which we have spoken in 2.2. and 2.3.

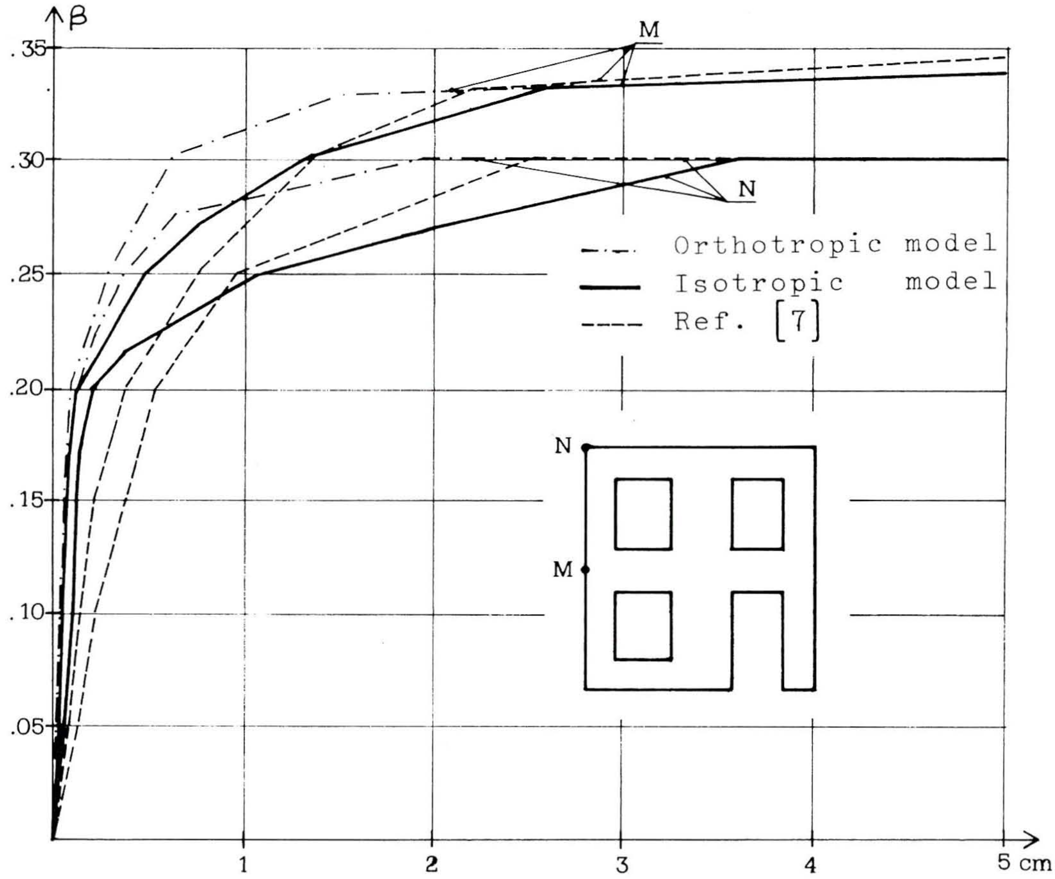



Fig.3.- Displacements and ultimate load behaviour.

It is also numerically analysed the behaviour of the simple column quoted in Fig.5, which experimental results, obtained by prof. Giuffrè at the Istituto di Scienza delle costruzioni of Rome University, are reported in part in [8] .

The mechanical properties of the model are the following.
$f'_c = 8,4$ ; $f'_t = 0,4$ ( MPa ); $\nu = 0,04$ ; $E = 8842$ ( MPa ) ;
$\varepsilon_{cu} = 10^{-3}$ .

The curves p = vertical load/transverse section area versus the strain $\Delta a/a$ are indicated in Fig.5/a, both for numerical and experimental analysis. In Fig.5/b is shown the crack pattern obtained in the numerical model.

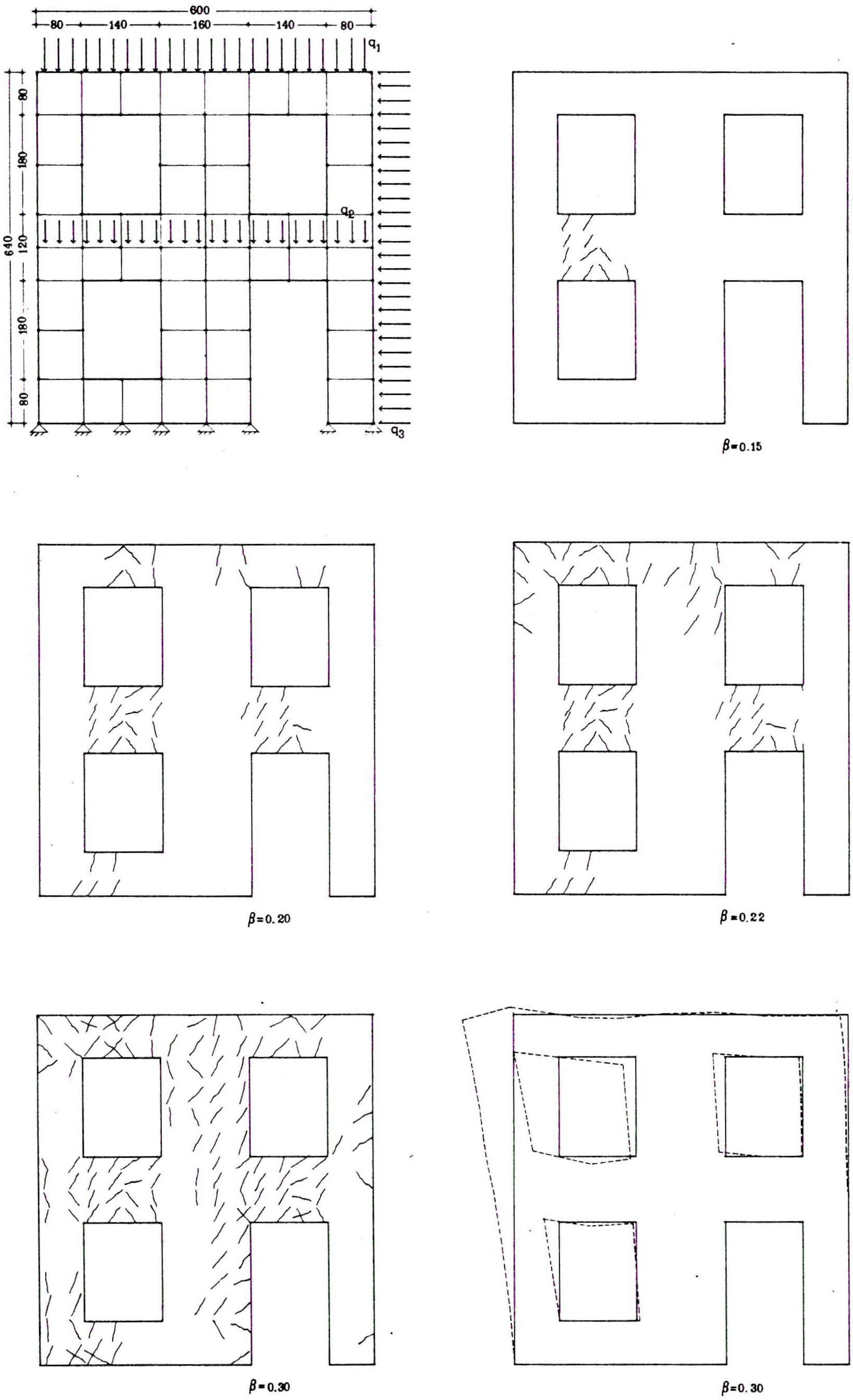



FIG. 4.-Crack pattern at various loading levels

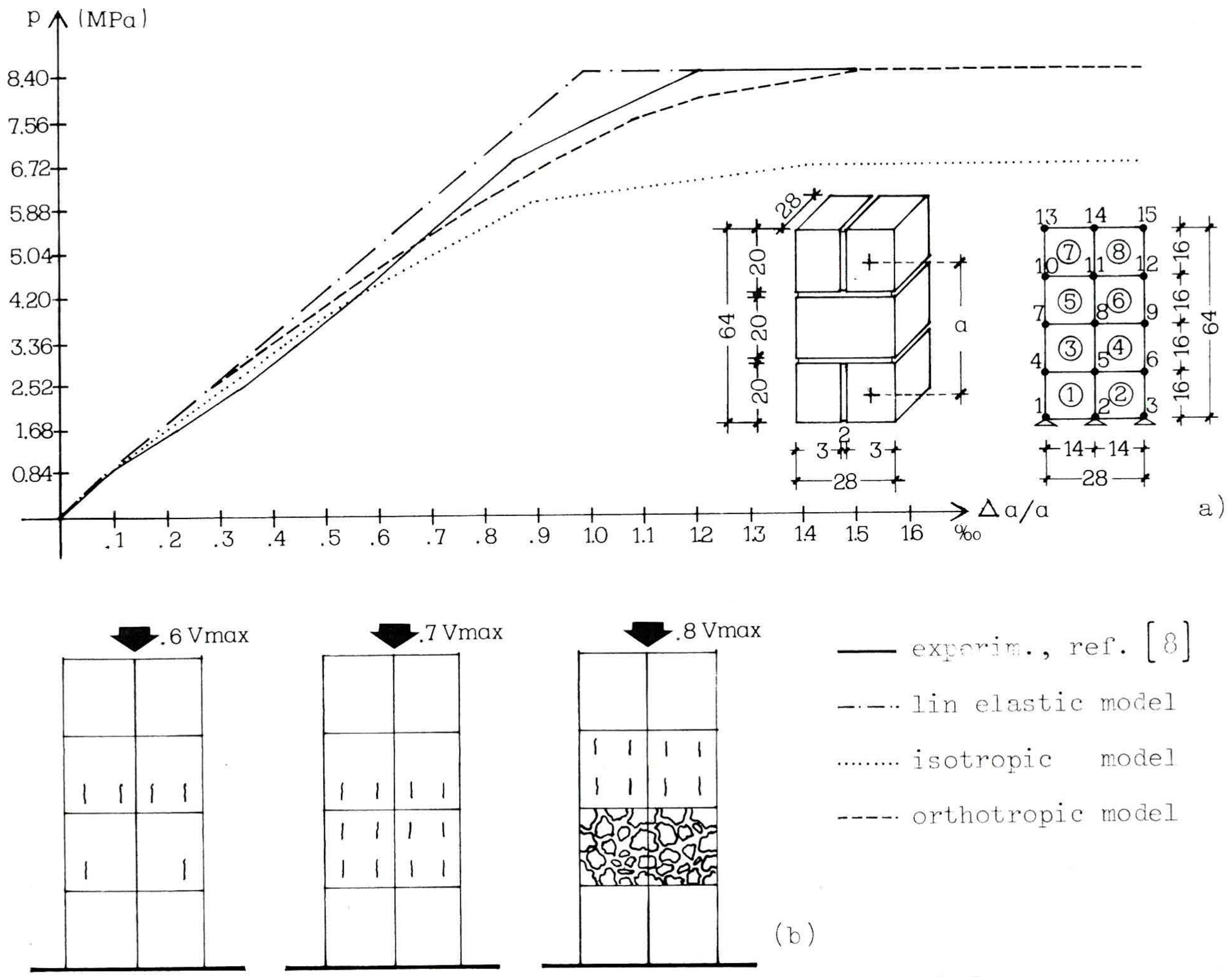



Fig.5.- (a) Medium pressure v/s strain curves,(b). Crack pattern.

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APPENDIX-NOTATION

The following symbols are used in this paper .

$E_o$ = initial tangent modulus in uniaxial loading;

$E_1, E_2, E_i$ = uniaxial tangent moduli in directions 1,2,i respectively;

$E_s$ = secant modulus ;

$f'_c$ , $f'_t$ = uniaxial compressive and tensile strength of masonry;

$G_o$ , $G_s$ , $G_t$ shear modulus (initial-, secant-, tangent-);

$K_o$ , $K_s$ , $K_t$ bulk modulus (initial-, secant-, tangent-);

$d\varepsilon^T = \left[d\varepsilon_1 \; d\varepsilon_2 \; d\gamma_{12}\right]$ and $d\varepsilon_{ij}$ = differential strain vector in material co-ordinates;

$d\sigma^T = \left[d\sigma_1 \; d\sigma_2 \; dt_{12}\right]$ and $d\sigma_{ij}$ differential stress vector in material co-ordinates;

$de_{ij}$ = deviator strain in material co-ordinate;

$ds_{ij}$ = deviator stress in material co-ordinate;

$\delta_{ij}$ = Kronecker delta ;

$\sigma_1$ , $\sigma_2$ = principal stresses;

$\varepsilon_{cu}$ = strain corresponding to $f'_c$ ;

EUS = equivalent uniaxial strain;

$\varepsilon_{ic}$ = EUS corresponding to $\sigma_{ic}$ ;

$\varepsilon_{iu}$ = EUS in i-th direction;

$\sigma_{oct}$ , $\tau_{oct}$ = octahedral normal and shear stresses, respectively ;

$\sigma_{octp}$ , $\tau_{octp}$ = octahedral normal and shear strength, respectively.