RANDOM VARIATIONS IN BRICKWORK PROPERTIES

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ABSTRACT A program of testing the lateral load resistance of non-loadbearing brickwork panels has been continuing at the Experimental Building Station for a number of years. The materials used in this investigation were, as nearly as possible, constant throughout and provide a large body of data on the properties of small specimens of brickwork. Properties which have been considered are flexural stiffness and strength normal to and parallel to the bed joints, the shear strengths of couplets, triplets and torsional specimens, the modulus of rupture of bricks and the compressive strength of mortar cubes. Typical results are presented and discussed. The Normal distribution was found to be the closest fit overall for flexural and shear properties. For brick modulus of rupture a significant negative skew in the distribution, explained as being caused by the presence of fine cracks between perforations in the bricks, causes the Weibull distribution to be the best of those considered. For mortar compressive strength the opposite case, a significant positive skew for which no explanation has been found, indicates a Log-normal distribution as the most appropriate form.

1. INTRODUCTION

In recent years an extensive investigation of the lateral load resistance of non-loadbearing brickwork panels has been carried out at the Experimental Building Station (EBS) in Sydney. This investigation has involved the testing of 32 full-scale wall panels to date, each accompanied by tests on a sample of small specimens to measure various properties of the brickwork. As a consequence a large body of test results on beam and shear specimens has been accumulated. Bricks for the testing program were obtained in separate batches of about 5000 from a single manufacturer, at times approximately six to twelve months apart. Mortar properties were carefully controlled throughout the test program with the many separate batches being mixed to a constant first flow. Because the material properties were, as nearly as possible, constant throughout the testing program, these small specimen results provide the opportunity to examine the extent and form of random variation inherent in each of the various properties.

In 1976, BAKER & FRANKEN(1) discussed the effects of random variation in the flexural strength of brickwork and recent studies(2,3) have stressed the importance of considering this factor and have used Monte Carlo techniques to model its effects in analysis. Because the response of wall panels to lateral loading is related in a stochastic manner to the flexural properties of the material, as measured by tests on beams representing vertical and horizontal strips in the wall, an intimate

knowledge of the nature of random variation in these beam specimens is necessary for a thorough treatment of panel behaviour. In addition, recent moves to introduce limit state design codes, where safety factors are estimated from probabilities of failure, require data on random variation in material properties so that, ultimately, they may attain a uniform level of safety in structures.

The various small specimen tests carried out at the EBS have provided measurements of flexural strength and elastic modulus of vertical and horizontal beams, shear strengths of couplets, triplets and torsional specimens, modulus of rupture of bricks and compressive strength of mortar cubes. The types of vertical and horizontal beam specimens and the characteristic bi-linear load deflection plot of the horizontal beam have been described previously(4) and a fuller description of all test specimens, procedures and detailed results is available(3). For the horizontal beams the initial tangent elastic modulus, the second tangent elastic modulus and the ultimate secant elastic modulus were considered separately. Also, the stress at change of slope (initial cracking) and the ultimate stress were considered separately. Results from approximately 600 beam specimens, 900 shear specimens, 250 modulus of rupture specimens and 530 mortar cubes have been considered.

2. METHODS OF STATISTICAL ANALYSIS

There were seven separate batches of bricks with from three to six test groups per batch and from seven to twenty-four replicates per test group. The technique of analysis of variance was used to investigate differences between batches and between tests within each batch. A basic assumption underlying the analysis of variance is that the parent populations from which the samples have been drawn are distributed Normally and that their variances are the same. The analysis then indicates the likelihood of differences between the population means. This is achieved by comparing the estimate of variance due to each factor with that due to replication, which is considered to be experimental error. A one-way analysis was first carried out for each batch to determine the significance of differences between tests within the batch. Secondly, on the assumption that the differences between tests were not significant, a one-way analysis was carried out on the overall results to ascertain the significance of differences between the seven batches.

It was expected that the distributions dealt with here would be at least close to Normal, and therefore the shape of the distributions would not significantly affect results. It has been demonstrated by Cochran(5) that small departures from Normality do not significantly affect the results of the analysis of variance. The question of homogeneity of variance among the parent distributions was examined by Bartlett's test (Snedecor(6) p.285). This test is considered by Snedecor to be the best method when more than two samples are being considered. In instances where the analysis of variance showed significant differences between sample means, the question of which sample or samples differed from the others was investigated by Tukey's test (Snedecor(6) p.251). In all of these analyses, results are considered to be significant if they have a 5% or less chance of occurring, given that the null hypothesis is true.

The form of random variation in the results for each property was investigated by fitting different types of probability distribution. Three forms were chosen as being suitable candidates: the Normal, Log-normal, and Weibull distributions. The first is an obvious choice, often being relevant for natural phenomena, and the second and third have often been found to be appropriate for material properties. The adopted form of the Weibull distribution is a two-parameter model obtained by setting the third parameter (location or threshold parameter) to zero. This is justified because there can be no negative test results (Benjamin and Cornell(7) pp.166-167).

For fitting the three forms of distribution to the data, the Kolmogorov-Smirnov goodness-of-fit test was employed rather than the more common Chi-squared test. The Kolmogorov-Smirnov test is considered to be preferable, according to Benjamin and Cornell(7) (pp.459-500), although a difficulty arises in the cases considered here because the parameters of the fitted distributions are estimated from the data. Benjamin and Cornell state that the goodness-of-fit test should only be used to reject unsuitable models for the data, not to choose the best fit among contending models. However, because this investigation dealt with the results of some twelve different but related properties, and involved distribution fitting for each of these to each of seven batches of bricks, it was expected that an assessment of the overall results might point to the most suitable of the three forms of distribution. A significance level of 20% was adopted for the Kolmogorov-Smirnov test, using the table of values given by Benjamin and Cornell.

3. RESULTS

In almost all cases Bartlett's test indicates that the variances for test groups within each batch of bricks are homogeneous, showing that the analysis of variance technique can be applied without data transformation. However, the analysis of variance usually shows a significant difference between the means of test groups within each batch. In other words, the results indicate that the specimens in the various tests, even using the same batch of bricks, do not belong to the same population. These differences must be due to factors not controlled in the experiment; their presence indicates the extreme difficulty in controlling the variability inherent in brickwork so that it may be considered to have homogeneous properties in the large scale. Because there is no apparent explanation for these differences, and because they are believed to reflect the differences occurring in practical brickwork, it is considered appropriate to pool the test results within each batch of bricks, and to examine the probability distribution for each batch and the significance of differences between batches.

The analysis of variance by batch indicates that the differences between batches are significant in all cases, even though every effort was made to maintain uniformity between them and to carefully control mortar properties. Despite this, it is considered that differences occurring between batches might be indicative of differences occurring in practical brickwork; the three forms of distribution were therefore also fitted to the data pooled for all batches to give a measure of the overall variation.

The findings from the analysis of variance, Bartlett's test, Tukey's test and the Kolmogorov-Smirnov test were similar for all flexural and shear properties. Because of space limitations the results are only presented here for the elastic modulus of vertical beams but a complete discussion and presentation of results is available elsewhere(3). Results are also presented for brick modulus of rupture and mortar compressive strength because these properties showed a different form of variation.

Table 1 shows statistics for the elastic modulus of vertical beams for each batch, and for all batches taken together. As well as the number, minimum, maximum, mean and coefficient of variation, the skewness is also shown for each group. There is no predominance of positive or negative skew in the data. Fitting of Normal, Log-normal and Weibull distributions to these data result in Kolmogorov-Smirnov statistics which are not significant at the 20% level. In other words all three forms are acceptable to represent the observed data. Considering the results of this analysis overall, the Normal distribution has the lowest Kolmogorov-Smirnov statistic for the majority of the batches and is considered to be the best candidate from the point of view of ease of use. Figure 1 shows the histogram for the elastic modulus of vertical beams for all batches considered together, totalling 311 results. Figure 2 shows the cumulative distribution function with the fitted Normal curve which has the parameters: mean 23.29 GPa, coefficient of variation 0.14.

Batch	Number	Minimum	Maximum	Mean	C.V.	Skewness
1	50	13.86	27.49	20.87	0.14	-0.37
2	54	14.89	23.46	19.83	0.08	-0.53
3	54	15.43	30.88	24.48	0.10	-0.62
4	45	21.50	27.12	23.78	0.05	0.40
5	45	20.40	26.07	22.79	0.06	0.24
6	36	23.66	32.98	27.25	0.09	0.48
7	27	24.18	29.65	27.03	0.05	-0.05
All	311	13.86	32.98	23.29	0.14	-0.06

TABLE 1. ELASTIC MODULUS OF VERTICAL BEAMS (GPa)

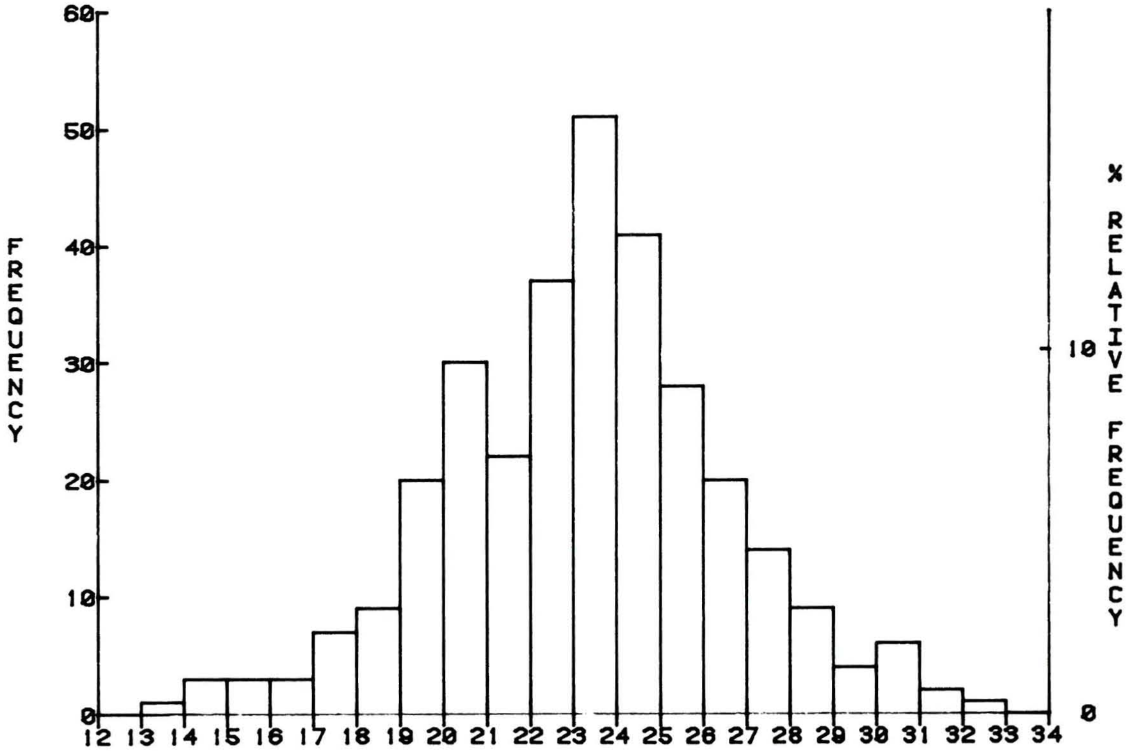

FIGURE 1. ELASTIC MODULUS OF VERTICAL BEAMS - HISTOGRAM

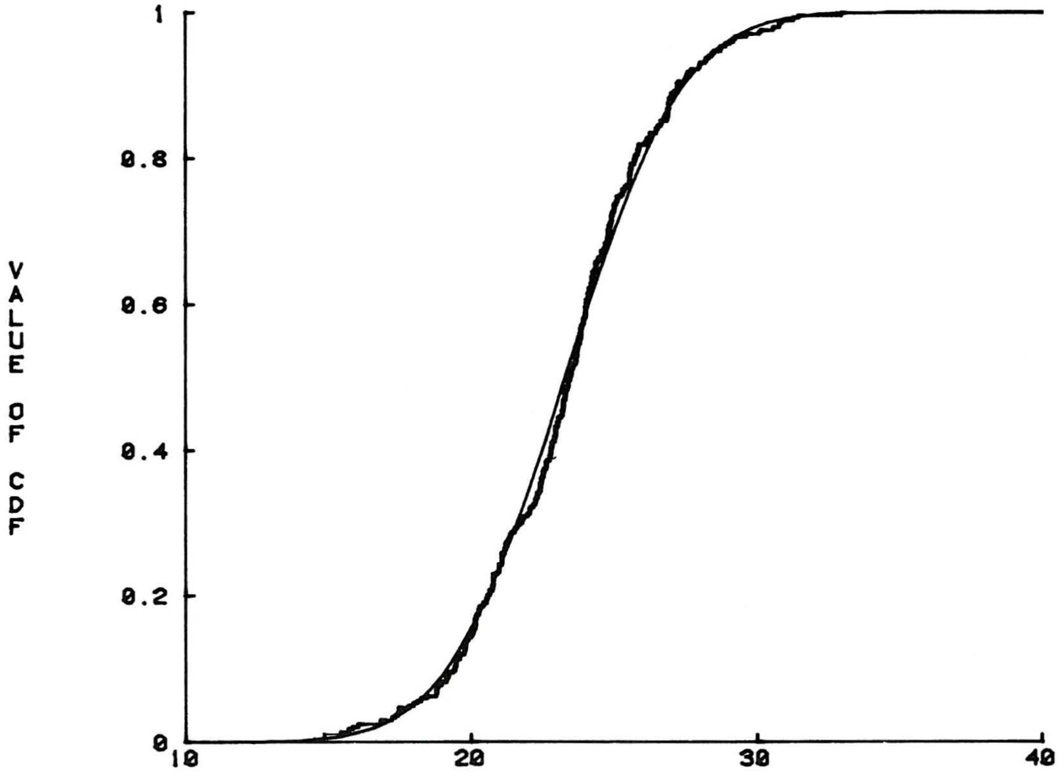

FIGURE 2. ELASTIC MODULUS OF VERTICAL BEAMS - NORMAL CDF

The measurements of brick modulus of rupture considered here were obtained from specimens consisting of three bricks glued end-to-end. These specimens were tested as simple beams to provide a measure of the modulus of rupture, free of end effects, in the mode relevant to out-of-plane flexure. Comparative tests have shown that the results from this test differ from those obtained with the standard transverse strength test(8).

The results for modulus of rupture, summarised by brick batch, are shown in Table 2. In contrast to the findings concerning other properties, these results show a marked tendency towards a negative skew, indicating a long 'tail' of results below the mean. Significance tests of this skewness (Snedecor(6) p.200) show that it is significant in most cases for the three-brick-long modulus of rupture test. It is believed that this skewness is caused by certain low results which arise due to the presence of very fine cracks in the bricks, between perforations, and which have a more marked effect on the modulus of rupture in the 'on-edge' orientation than in the 'standard' orientation. These cracks were observed to be present in some of the bricks, and are a result of the particular perforation pattern and the brittle nature of the bricks.

Batch	Number	Minimum	Maximum	Mean	C.V.	Skewness
1	38	0.89	5.80	3.77	0.30	-0.58
2	32	3.14	7.50	5.73	0.18	-0.57
3	29	2.26	8.36	4.94	0.37	-0.07
4	15	2.05	7.80	6.21	0.25	-1.46
5	23	1.09	6.02	4.31	0.35	-0.94
6	8	1.83	5.22	3.68	0.33	-0.33
7	10	1.57	5.57	3.66	0.33	-0.06
All	155	0.89	8.36	4.70	0.34	-0.18

TABLE 2. 3-LONG MODULUS OF RUPTURE (MPa)

Fitting of Normal, Log-normal and Weibull distributions to these modulus of rupture data give Kolmogorov-Smirnov statistics which are not significant at the 20% level, indicating that all three forms of distribution are adequate representations. However, unlike the other properties, where the Normal distribution generally gives the lowest statistic, in these cases the Weibull distribution almost invariably gives the lowest value. This undoubtedly reflects the significant skewness in the data. Table 3 shows the parameters of the fitted Weibull distribution for each separate batch, and for the aggregate of results.

Batch	Scale Parameter A	Shape Parameter B
1	0.0088	3.27
2	0.0000	6.45
3	0.0070	2.89
4	0.0017	3.27
5	0.0210	2.41
6	0.0114	3.16
7	0.0102	3.26
All	0.0063	3.05

TABLE 3. 3-LONG MODULUS OF RUPTURE WEIBULL DISTRIBUTION PARAMETERS

The modulus of rupture results were examined pooled for all batches as an indication of the variability which might occur in a practical situation, even though the analysis of variance shows that the separate batches should not be considered to be drawn from a single population. There are a total of 155 results with a mean of 4.70 MPa and a coefficient of variation of 0.34. The Kolmogorov-Smirnov statistics for the distribution fitting are not significant at the 20% level, and again the Weibull distribution appears to give the best representation of the data. Figure 3 shows the histogram of these data; the negative skew can be clearly seen.

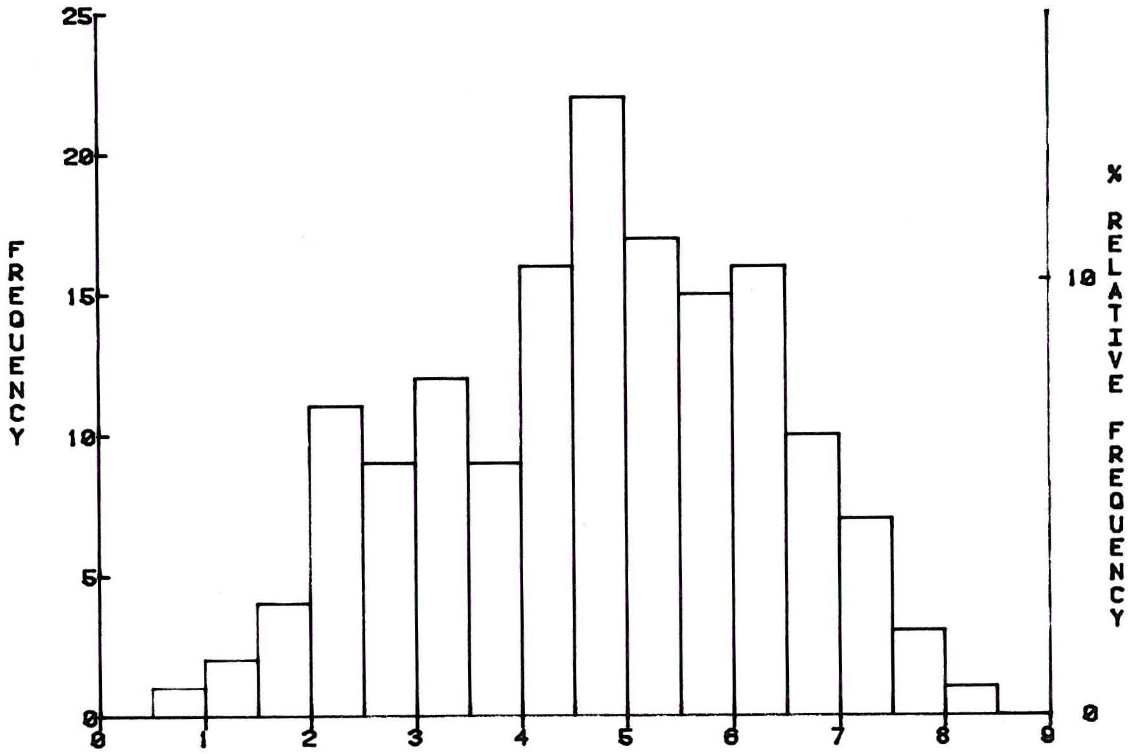

FIGURE 3. 3-LONG MODULUS OF RUPTURE - HISTOGRAM

For the measurements of compressive strength of mortar the logical grouping is by test number and not by brick batch. A one-way analysis of variance showed that there are significant differences between tests. Despite the great care taken in controlling batching of the mortar, using a thoroughly dried sand with a consistent grading and mixing to a constant first flow, the results show statistically significant variation between tests. This must be due to factors beyond the control of the experiment, and which are unlikely to be controlled in a typical practical situation. In fact, the degree of control exercised in these tests would be much greater than that applied on most construction sites. Consequently, it is considered appropriate to examine the pooled results for mortar compressive strength to gain an idea of the variation likely to occur in practice. There are 525 individual observations, with a mean of 2.68 MPa and a coefficient of variation of 0.20. The skewness of these data is 0.39 which is highly significant considering the number of observations in the sample.

Figure 4 shows the histogram of these accumulated compressive strength results and the positive skewness indicated by the extended upper tail in the histogram is obvious. Fitting of the probability distributions gives Kolmogorov-Smirnov statistics indicating that all three forms are valid to represent the data, but examination of the cumulative distribution functions shows that the Log-normal model is clearly the best fit. This form can model the skewness in the data, whereas the Normal distribution cannot. The parameters of the fitted Lognormal distribution are: A = 0.42, B = 0.09.

With $\beta_M = \beta_S$:

$$\beta_W = a \cdot \beta_S^b \cdot \beta_M^c \longrightarrow \beta_W = a \cdot \beta_S^{b+c} \tag{5}$$

In this form, the mean value function for the wall strength is given by

$$\beta_W = 0.787 \cdot \beta_S^{0.953} \tag{2 B}$$

This function as well as the function of the characteristic value are also shown in Figure 1.

The relationship $\beta_{W,5\%} / \beta_W$ lies between 0.78 and 0.82. The function of the characteristic value can thus be given in a simplified form as:

$$\beta_{W,5\%} = (0.61 \div 0.64)\ \beta_S^{0.953} \tag{3 B}$$

The equations (2 B) and (3 B) are valid for the range of the random test $2.5 \leq \beta_S \leq 7.5$.

Group C: masonry wall with thin mortar bed, applied smoothly and flat over the full bed joint face
The evaluation was carried out as for group B in accordance with (5). The mean value function for the wall strength is given by

$$\beta_W = 0.876 \cdot \beta_S^{0.893} \tag{2 C}$$

The relationship $\beta_{W,5\%} / \beta_W$ lies between 0.86 and 0.92, i. e. is more favourable than for group B. The characteristic value function can be written in a simplified form:

$$\beta_{W,5\%} = (0.75 \div 0.81)\ \beta_S^{0.893} \tag{3 C}$$

Normal, Log-normal and Weibull distributions all appear to be adequate to represent the data for the various properties. However, these distributions differ significantly in the tails, where they have a marked effect on the results of analyses and calculations of failure probabilities. It is therefore important to determine the most suitable form if possible. For flexural and shear properties an examination of the Kolmogorov-Smirnov statistics and the cumulative distribution functions indicates that the Normal distribution is the most appropriate form. For brick modulus of rupture, a significant negative skew, explained as being caused by the presence of fine cracks between perforations in the bricks, resulted in the Weibull distribution being indicated as the best form to represent the data. For mortar compressive strength the opposite case, a significant positive skew for which no explanation is apparent, indicated the Lognormal distribution as being the most appropriate form.

These findings concerning variability between samples, which cannot be controlled even in the most carefully executed laboratory tests, indicate what a difficult material brickwork can be to deal with in design. It is essential that any design and analysis, particularly of situations involving the highly variable tensile and shear properties, take due account of the extent of random variation present. The data considered here show that flexural and shear properties can be adequately represented by a Normal distribution with the relevant mean and standard deviation, but that other properties may display significant positive or negative skews and may therefore be more appropriately represented by Weibull or Log-Normal distributions.

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