THE RELATION BETWEEN THE COEFFICIENTS OF VARIATION OF THE COMPRESSIVE STRENGTHS OF UNITS, MORTAR AND BRICKWORK

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ABSTRACT Using the formulae developed from the theory of probability, we can get the relation between the coefficients of variation of the compressive strength of units, mortar and brickwork. If we have these formulae for computing compressive strength of masonry, then the reliability index of a masonry structure can be obtained. A series of tests has been made for comparison.

Legends:

- $\beta$ - index of reliability of strength.
- $\mu$ - mean value of resisting force of structure.
- $\sigma_R$ - standard difference of compressive strength.
- $\mu_S$ - mean value of acting pressure.
- $\sigma_S$ - standard difference of acting pressure.
- $f_m$ - mean value of compressive strength of brickwork.
- $V_m$ - coefficient of variation of compressive strength of brickwork.
- $f_b$ - mean value of compressive strength of units (bricks or block).
- $V_b$ - coefficient of variation of compressive strength of units(bricks or block).
- $f_c$ - mean value of compressive strength of mortar.
- $V_c$ - coefficient of variation of compressive strength of mortar.
- $f_K$ - characteristic compressive strength of brickwork.

## 1. Formulae of Coefficient of Variation for Brickworks

According to the regulations —— " The bases for design of structures " made by the committee of fundamentals of structural design ISO/TC98 , the reliability of structure is established on the basis of the analysis of probability and to be expressed by the index $\beta$ . The equation for calculating $\beta$ is as follows :

$$\beta = \frac{\mu_R - \mu_S}{\sqrt{\sigma_R^2 + \sigma_S^2}} \qquad (1)$$

When $\mu_S, \sigma_S$ are known , the reliability index $\beta$ can be got dependent upon the values of $\mu_R$ and $\sigma_R$ . In order to get $\sigma_R$ , a large amount of experiments should be carried out. The quantity of masonry specimen for $\mu_R, \sigma_R$ is usually over 30 pieces for a single test, and its sizes are very big. In China, the smallest dimension of masonry specimen required for test is 24 x 37 x 70 cms. Futhermore, there are various kinds of bricks and mortars , then their combinations will be numerous. So the works for making experiments are tedious and difficult. This paper suggests that through the formulae we have already had on the principal of probability , we can find out the relations between the coefficients of variation of compressive strength of units , mortars and brickworks , So the variation of brickwork can be found through calculation by using the coefficient of variation of units and mortars theoretically . The compressive strength of brickwork , we get in this way , is much simpler than the same result through numerous experiments.

By mathematical statistics and theory of probability , we know that the independent variables $x_1, x_2, \cdots x_n$ becomes a function:

$$y = f(x_1, x_2, \cdots x_n) \tag{2}$$

The mathematical expectation for $y$ is $E(y)$,

$$\bar{y} = E(y) = f(\bar{x}_1, \bar{x}_2 \cdots \bar{x}_n) \tag{3}$$

In which, $\bar{x}_1, \bar{x}_2 \cdots \bar{x}_n$ are the mean values of $x_1, x_2, \cdots x_n$. The mean square deviation of $y$ is $\sigma_y^2$ which can be obtained approximately as follows :

$$\sigma_y^2 = E(y - \bar{y})^2 \tag{4}$$

To evolve the equation $y$ at $x_1, x_2 \cdots x_n$ into Tayler's series , and take the first item we get following equation :

$$y = f(\bar{x}_1, \bar{x}_2 \cdots \bar{x}_n) + \sum_{i=1}^{n} \left(\frac{\partial y}{\partial x_i}\right)_m (x_i - \bar{x}_i) \tag{5}$$

to substitute $\bar{y}$ and $y$ in equations (3) (5) into equation(4) , after adjustment we get

$$\sigma_y^2 = \sum_{i=1}^{n} \left(\frac{\partial y}{\partial x_i}\right)_m^2 \sigma_{x_i}^2 \tag{6}$$

If equation(6) is used to calculate the compressive strength of the brickwork , we can get the relation between the variation of units, mortar and brickwork. The index $m$ in equation (6) indicates the mean value for $x_i$ to be used.
According to current Chinese Code for masonry , the formulae for compressive strength of brickwork is as follows:

$$f_m = (0.1\sqrt{f_b} + 0.2\sqrt{f_c})\sqrt{f_b + 60} \tag{7}$$

The formula which will be adopted in the revised code for masonry in China is as follows :

For brickwork,

$$f_m = \sqrt{f_b}\,(2.13 + 0.018 f_c) \qquad (8)$$

For blocks 20cm $\leqslant$ height of block $\leqslant$ 40cm

$$f_m = 0.15 f_b (2.13 + 0.018 f_c) \qquad (9)$$

For 40cm $\leqslant$ height of block $\leqslant$ 80cm

$$f_m = 0.08 f_b^{1.2} (2.13 + 0.018 f_c) \qquad (10)$$

The formulae suggested in China and in abroad in the documents ever published are as follows :

$$f_m = a f_b^b f_c^c \qquad (11)$$

$$f_m = a f_b^{0.7} f_c^{0.3} \qquad (12)$$

$$f_m = a f_b^{0.66} f_c^{0.33} \qquad (13)$$

$$f_m = a f_b^{0.4} f_c^{0.2} \qquad (14)$$

formula (8) - (13) can be written in a general form as follows :

$$f_m = a f_b^b (\alpha 1 + \alpha 2 f_c^c)$$

when $a = c = 1$, $b = 0.5$, $\alpha 1 = 2.13$, $\alpha 2 = 0.08$, it turns out the result as the formula(8), when $b = 0.7$, $c = 0.3$, $\alpha 1 = 0$, $\alpha 2 = 1$ it becomes equation(12).

Apply formula (6) we get

$$V_m^2 = (b V_b)^2 + \left( \frac{a c \alpha 2 f_b^b f_c^c V_c}{f_m} \right)^2 \qquad (15)$$

substitute equation(11) into equation(6) , then

$$V_m^2 = (b V_b)^2 + (c V_c)^2 \qquad (16)$$

substitute (8) (12) (13) (14) into the equation (15) and (16) in order, then we get :

$$V_m^2 = (0.5 V_b)^2 + \frac{(0.018 f_c V_c)^2}{(2.13 + 0.018 f_c)^2} \qquad (17)$$

$$V_m^2 = (0.7 V_b)^2 + (0.3 V_c)^2 \qquad (18)$$

$$V_m^2 = (0.66 V_b)^2 + (0.33 V_c)^2 \qquad (19)$$

$$V_m^2 = (0.4 V_b)^2 + (0.2 V_c)^2 \qquad (20)$$

Substitute equation (7) into (6) we get

$$V_m^2 = \frac{(Af_b V_b)^2 + (Bf_c V_c)^2}{f_m^2} \tag{21}$$

In which

$$A = 0.5\sqrt{\frac{f_b+60}{f_b}} + 0.05\sqrt{\frac{f_b}{f_b+60}} + 0.1\sqrt{\frac{f_c}{f_b+60}} \; ;$$

$$B = 0.1\sqrt{\frac{f_b+60}{f_c}}$$

We analysis the equation (17) - (21) , we find that in equation (17) , $V_m$ has no effect to the compressive strength $f_b$ of the brick , but there is close relation to the strength of mortar $f_c$ and the coefficient of variation of the brick and mortar $V_b, V_c$ . In equation (18) to (20) , $V_m$ varies with the coefficient of variation of brick and mortar , but it has no effect to the strength of either brick or mortar , but in equation (21) , $V_m$ varies with all the factor $f_b, V_b, f_c$ and $V_c$. For the sake of investigating the reliability of these equations mentioned above, experiments are carried out for comparison. At the same time , the experiments on the coefficient of variation of brickwork and mortar are also made. The dimension of specimen and testing results are shown in the table . The ratio of height of specimen to its thickness will not exceed 3 . The units consist of clay bricks and hollow blocks . The dimension of brick is 24 x 12 x 5.3cm. and that of hollow block kp1 is 24 x 11.5 x 9cm. , its cavity ratio is 15% ; for hollow block kp2 is 24 x 18 x 11.5cm. , its cavity ratio is 25%.

Table 1 shows the results of experiment , Table 2 shows the ratio of experimental value to the values calculated through the equation (10) - (15). We can find that the results calculated by equation (10) which evolves from equation (8) is more favorable , its mean value is 1.047 . the coefficient of variation is 0.1406. The coefficient of variation , calculated from other equations , are closer to each other , but the mean value deviates greatly . Equation (7) (8) (12) (13)(14) are empirical formulae in which the coefficients are selected so as the values of compressive strength through calculation and that through experiments will be closed to each other , without considering the effect of the coefficients of variation of the strength of units , mortars and brickworks. The way of calculating $V_m$ as montioned above , through equations

THE TEST DATA FOR THE COMPRESSIVE STRENGTH OF BRICKS MORTARS AND BRICKWORKS

Table 1

| Group | Brick type | Mortar type | Dimention of masonry (cm) L x W x H | Test compressive strength of brick | | | Test compressive strength of mortar | | | Test compressive strength of masonry | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | $f_b$ | $\sigma_b$ | $V_b$ | $f_c$ | $\sigma_c$ | $V_c$ | $f_m$ | $\sigma_m$ | $V_m$ |
| 1 | KP1 hollow brick | 75 | 49 x 37 x 102 | 130 | 37.7 | 0.290 | 74.0 | 17.02 | 0.238 | 31.6 | 5.75 | 0.180 |
| 2 | KP1 hollow brick | 75 | 37 x 24 x 70 | 156 | 38.2 | 0.245 | 65.7 | 16.56 | 0.252 | 37.2 | 6.29 | 0.169 |
| 3 | KP1 hollow brick | 25 | 49 x 37 x 102 | 120 | 27.6 | 0.230 | 23.6 | 3.78 | 0.160 | 28.1 | 2.56 | 0.096 |
| 4 | KP1 hollow brick | 25 | 37 x 24 x 70 | 189 | 41.4 | 0.219 | 19.8 | 5.33 | 0.269 | 29.5 | 3.22 | 0.109 |
| 5 | KP2 hollow brick | 25 | 43 x 43 x 114 | 124 | 29.8 | 0.240 | 15.6 | 5.30 | 0.340 | 26.6 | 2.90 | 0.110 |
| 6 | KP2 hollow brick | 25 | 37 x 37 x 100 | 113 | 29.5 | 0.261 | 29.3 | 5.27 | 0.180 | 29.1 | 3.43 | 0.118 |
| 7 | KP1 hollow brick | 10 | 37 x 24 x 70 | 169 | 39.4 | 0.233 | 8.6 | 2.19 | 0.252 | 29.3 | 2.93 | 0.100 |
| 8 | KP2 hollow brick | 10 | 37 x 37 x 100 | 136 | 33.9 | 0.249 | 14.8 | 2.92 | 0.197 | 24.6 | 2.68 | 0.109 |
| 9 | common brick | 25 | 24 x 37 x 72 | 196 | 45.3 | 0.231 | 31.4 | 12.56 | 0.400 | 48.0 | 8.50 | 0.178 |
| 10 | common brick | 75 | 24 x 37 x 72 | 101 | 20.4 | 0.202 | 63.8 | 26.70 | 0.418 | 30.8 | 4.06 | 0.132 |
| 11 | common brick | 25 | 24 x 37 x 72 | 70 | 28.9 | 0.411 | 32.7 | 15.87 | 0.485 | 18.5 | 3.42 | 0.183 |
| 12 | common brick | 10 | 24 x 37 x 72 | 70 | 28.9 | 0.411 | 8.0 | 3.22 | 0.402 | 18.3 | 3.87 | 0.210 |

offers a new rational method from another point of view for determining the coefficients of empirical formulae and the equation for calculating the compressive strength of brick-work.

COMPARISON OF THE CALCULATED VALUES AND THE TEST VALUES FOR THE COEFFICIENT OF VARIATION OF COMPRESSIVE STRENGTH OF BRICKWORK

Table 2

| Group | Test value $V_m$ | Calculated values $V_m'$ | | | | | $V_m'/V_m$ | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | (1) | (2) formula (8) | (3) formula (7) | (4) formula (12) | (5) formula (13) | (6) formula (14) | (2)/(1) | (3)/(1) | (4)/(1) | (5)/(1) | (6)/(1) |
| 1 | 0.180 | 0.1698 | 0.1715 | 0.2144 | 0.2058 | 0.1247 | 0.9437 | 0.9531 | 1.1911 | 1.1438 | 0.6932 |
| 2 | 0.169 | 0.1519 | 0.1586 | 0.1874 | 0.1818 | 0.1102 | 0.8993 | 0.9387 | 1.1090 | 1.0759 | 0.6520 |
| 3 | 0.096 | 0.1180 | 0.1426 | 0.1680 | 0.1607 | 0.0974 | 1.2295 | 1.4860 | 1.7500 | 1.6741 | 1.0146 |
| 4 | 0.109 | 0.1160 | 0.1586 | 0.1732 | 0.1696 | 0.1028 | 1.0650 | 1.4554 | 1.5893 | 1.5561 | 0.9431 |
| 5 | 0.110 | 0.1263 | 0.1667 | 0.1965 | 0.1941 | 0.1176 | 1.1487 | 1.5157 | 1.7867 | 1.7646 | 1.0694 |
| 6 | 0.118 | 0.1353 | 0.1566 | 0.1905 | 0.1822 | 0.1104 | 1.1466 | 1.3273 | 1.6145 | 1.5441 | 0.9358 |
| 7 | 0.100 | 0.1177 | 0.1708 | 0.1797 | 0.1748 | 0.1059 | 1.1774 | 1.7080 | 1.7976 | 1.7482 | 1.0595 |
| 8 | 0.109 | 0.1264 | 0.1660 | 0.1840 | 0.1767 | 0.1071 | 1.1597 | 1.5236 | 1.6885 | 1.6213 | 0.9826 |
| 9 | 0.178 | 0.1427 | 0.1765 | 0.2013 | 0.2016 | 0.1222 | 0.8019 | 0.9921 | 1.1312 | 1.1329 | 0.6866 |
| 10 | 0.132 | 0.1778 | 0.1641 | 0.1889 | 0.1918 | 0.1162 | 1.3475 | 1.2433 | 1.4317 | 1.4533 | 0.8807 |
| 11 | 0.183 | 0.2307 | 0.2420 | 0.3223 | 0.3149 | 0.1908 | 1.2610 | 1.3229 | 1.7617 | 1.7210 | 1.0430 |
| 12 | 0.210 | 0.2070 | 0.2469 | 0.3119 | 0.3019 | 0.1830 | 0.9860 | 1.1759 | 1.4854 | 1.4379 | 0.8714 |
| | | | | mean value | | | 1.0972 | 1.3035 | 1.5281 | 1.4894 | 0.9027 |
| | | | | coefficient of variation | | | 0.1406 | 0.1835 | 0.1622 | 0.1594 | 0.1594 |

COMPARISON OF THE CALCULATED VALUES AND THE TEST VALUES FOR THE COEFFICIENT OF VARIATION OF COMPRESSIVE STRENGTH OF BLOCKMASONRY

Table 3

| Group | Block type | Test values | | Calculated values $V_m'$ | | | $V_m'/V_m$ | | |
|---|---|---|---|---|---|---|---|---|---|
| | | $V_b$ | $V_m$ (1) | (2) formula (22)(23) | (3) formula (24)(25) | (4) formula (18) | (2)/(1) | (3)/(1) | (4)/(1) |
| 1 | height =20cm | 0.113 | 0.132 | 0.1202 | 0.1186 | 0.1198 | 0.9112 | 0.8988 | 0.9077 |
| 2 | " | 0.143 | 0.132 | 0.1488 | 0.1521 | 0.1346 | 1.1274 | 1.1375 | 1.0197 |
| 3 | " | 0.163 | 0.173 | 0.1681 | 0.1711 | 0.1453 | 0.9718 | 0.9893 | 0.8400 |
| 4 | " | 0.101 | 0.103 | 0.1090 | 0.1060 | 0.1144 | 1.0591 | 1.0296 | 1.1111 |
| 5 | " | 0.112 | 0.158 | 0.1193 | 0.1176 | 0.1193 | 0.7553 | 0.7443 | 0.7554 |
| 6 | height =80cm | 0.130 | 0.169 | 0.1613 | 0.1625 | 0.1279 | 0.9547 | 0.9615 | 0.7573 |
| 7 | " | 0.167 | 0.225 | 0.2045 | 0.2087 | 0.1475 | 0.9093 | 0.9277 | 0.6556 |
| | | | | mean value | | | 0.9555 | 0.9555 | 0.8687 |
| | | | | coefficient of variation | | | 0.1151 | 0.1175 | 0.1719 |

2. Formulae of Coefficients of Variation for Blockworks

For block masonry, substitute equation (9) (10) into (6), or apply equation (15) directly with coefficient adopted we can get :

For block masonry $20cm \leqslant$ height of block $\leqslant 40cm$

$$V_m^2 = V_b^2 + \frac{(0.018 f_c V_c)^2}{(2.13 + 0.018 f_c)^2} \tag{22}$$

For block masonry $40cm \leqslant$ height of block $\leqslant 80cm$

$$V_m^2 = (1.2 V_b)^2 + \frac{(0.018 f_c V_c)^2}{(2.13 + 0.018 f_c)^2} \tag{23}$$

Compare equation (17) to equation (22) and (23), we find that the second term for right portion are the same, the only difference is the coefficient of $V_b^2$, which are 0.5,1,1.2 respectively. It reveals that the strength of masonry is directly influenced by the coefficient of variation of compressive strength of units ( bricks or blocks ). Practical calculations show that the value of second term of (22) (23) equation is 5% only of the value of the first term. So it is simpler to express equation (22) (23) approximately as follows :

$$V_m = 1.05 V_b \tag{24}$$

$$V_m = 1.26 V_b \tag{25}$$

Table 3 is the result of experiments of coefficient of variation of block masonry. The coefficient of variation of compressive strength of mortars and units is not checked before expriment, suppose they are tested before delivery from the factory. The dimension of the units consist two types, the small size is 39 x 19 x 19cm with cavity of 50%. The medimum size is 120 x 80 x 20cm with cavity of 60%. The result of calculation of equation (11) is also involved in the table . We noticed that both the mean values and coefficients of variation and the equation (22) (23) or (24) (25) are satisfactory. The mean value is 0.9555 , coefficient of variation is 0.1175 which is better than that from Table 2.

3. Quality Equation

Nowaday , in various design code, the index $\beta$ can not be found out directly , it is expressed by the partial safety factor for loads and materials. When the partial safety factor for loads and materials reaches the value predetermined , it can be recog-

nized the reliability index is satisfactory. The strength of brickwork under inspection should fulfil the formula given below, in which $V_m = 0.17$ according to current Chinese code:

$$\mu_R(1-1.645V_m) \geqslant f_K=(1-1.645V_m)f_m=0.72f_m \quad (27)$$

If we substitute equation (8) and (17) into equation (27)then we get equation (28) which is in terms of $f_b, V_b, f_c, V_c$ :

$$\sqrt{f_b}(2.13+0.018f_c)\left\{1-1.645\sqrt{(0.5V_b)^2+\left(\frac{0.018f_c V_c}{2.13+0.018f_c}\right)^2}\right\} \geqslant f_k=0.72f_m \quad (28)$$

or we can write in other form

$$F(f_b, V_b, f_c, V_c) \geqslant f_K=0.72f_m \quad (29)$$

The right portion of the sign of unequality is called characteristic value of strength of masonry. When the spicemens of bricks and mortars are inspected the value of $f_c, V_b, f_c, V_c$ are already known and we will find out whether the formula (29) is satisfactory or not . In general , the variation of strength of unit or mortar is dependent upon the workmanship, which can be qualified beforehand . So , only the numerious investigations on $f_b$ and $f_c$ are required. Equation (28) and(29) can be called as quality equation.