A SUGGESTED CONCEPT FOR PREDICTING STRENGTH OF UNGROUTED MASONRY PRISMS

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ABSTRACT This paper is prepared to establish a concept which is intended to take a role in predicting the mechanical properties of masonry prisms, focussing particularly on the uniaxial strength of prism piers.

As far as we know, there are many factors affecting prism strength. Some generalized concepts are necessary and the concepts should be a function of the various factors concerning basic materials.

In this paper, 'Biaxial Stress Coefficient' as the primary concept, is proposed in order to predict uniaxial strength. A method of prediction and the possible range of efficiency are discussed against various experimental results.

## 1. INTRODUCTION

The structural performance of load bearing masonry buildings is greatly influenced by the quality and properties of component materials. Prism strength has been esteemed as the most important property in the design of masonry structures, and many experimental and theoretical studies have been done. [1)-6), 8)-10)] Some fundamental progress in the theoretical aspects has been made in the past studies. However, their actual application to the design of structures and the development of necessary basic materials are left in an insufficient state.

## 2. THEORETICAL BASIS

### 2.1 Assumptions

Prediction of the uniaxial compressive strength of ungrouted masonry is conducted on the basis of the following assumptions in this report.

(1) All the parts of masonry are completly elastic until fracture occurs in some of these parts. (linear fracture mechanism).

(2) The biaxial strains of the components of masonry have the same value and their distributions in the two parts are uniform.

(3) The bond between units and joints is complete and no fractures occur at the interfaces.

### 2.2 Basic Equations

The relationships between stress and strain of the two components, masonry unit and joint mortar, are shown as follows:
In masonry unit,

$$\varepsilon_{u1}=\frac{1}{E_U}\left(\sigma_{u1}-2\mu_u\ \sigma_{u2}\right) \qquad \text{------} \quad (1)$$

$$\varepsilon_{u2}=\varepsilon_{u3}=\frac{1}{E_U}\left(-\mu_u\,\sigma_{u1}+\overline{1-\mu_u}\,\sigma_{u2}\right) \qquad \text{------} \quad (2)$$

In joint mortar, in the same manner,

$$\varepsilon_{j1}=\frac{1}{E_j}\left(\sigma_{j1}-2\mu_j\,\sigma_{j2}\right) \qquad \text{------} \quad (3)$$

$$\varepsilon_{j2}=\varepsilon_{j3}=\frac{1}{E_j}\left(-\mu_j\,\sigma_{j1}+\overline{1-\mu_j}\,\sigma_{j2}\right) \qquad \text{------} \quad (4)$$

The uniaxial stress and biaxial strains of the two parts are obviously equal, and the following equations can be obtained on the basis of the assumption.

$$\sigma_1=\sigma_{u1}=\sigma_{j1} \qquad \text{------} \quad (5)$$

$$\varepsilon_1 h=\varepsilon_{u1}\cdot h_u+\varepsilon_{j1}\cdot h_j \qquad \text{------} \quad (6)$$

$$\sigma_{u2}\cdot h_u+\sigma_{j2}\cdot h_j=0 \qquad \text{------} \quad (7)$$

$$\varepsilon_2=\varepsilon_{u2}=\varepsilon_{j2} \qquad \text{------} \quad (8)$$

As the whole masonry, the following equations can be shown.

$$\varepsilon_1=\frac{1}{E}\sigma_1 \qquad \text{------} \quad (9)$$

$$\varepsilon_2=\varepsilon_3=-\frac{\mu}{E}\,\sigma_1 \qquad \text{------} \quad (10)$$

From the above ten equations, Young's Modulus and Poisson's Ratio can be calculated as follows:

$$\frac{E}{E_u}=\frac{(1-\mu_U)\,\alpha\beta+(1-\mu_j)(1-\beta)}{(1-\mu_U)\,\beta+(1-\mu_j)\,(1-\beta)^2+\left\{4\mu_j\,\mu_u+(1+\mu_U)\,(1-2\mu_U)\,\alpha+(1+\mu_j)\,(1-2\mu_j)\times\frac{1}{\alpha}\right\}\beta\,(1-\beta)} \qquad \text{------} \quad (11)$$

$$\mu=-\frac{\mu_j\,(1-\mu_U)\,\beta+(1-\mu_j)\,\mu_U\,(1-\beta)}{(1-\mu_U)\,\beta^2+(1-\mu_j)(1-\beta)^2+\left\{4\mu_j\,\mu_u+(1+\mu_U)\,(1-2\mu_U)\,\alpha+(1+\mu_j)\,(1-2\mu_j)\times\frac{1}{\alpha}\right\}\beta\,(1-\beta)} \qquad \text{------} \quad (12)$$

In the same manner the biaxial tensile stress and the uniaxial strain of the masonry unit are deduced as follows:

$$\frac{\sigma_{u2}}{\sigma_1}=\frac{\beta\,(\mu_U\,\alpha-\mu_j)}{\beta\,(1-\mu_U)\,\alpha+(1-\beta)\,(1-\mu_j)} \qquad \text{------} \quad (13)$$

$$\frac{\varepsilon_{u1}}{\sigma_1/E_U}=\frac{\left\{(1+\mu_U)\,(1-2\mu_U)\,\alpha+2\mu_U\,\mu_j\right\}\beta+(1-\mu_j)(1-\beta)}{(1-\mu_U)\,\alpha\beta+(1-\mu_j)\,(1-\beta)} \qquad \text{----} \quad (14)$$

The biaxial tensile stress and the uniaxial strain of joint

mortar confined in masonry are as follows:

$$\frac{\sigma_{j2}}{\sigma_1} = -\frac{(1-\beta)(\mu_u\alpha-\mu_j)}{(1-\mu_u)\alpha\beta+(1-\mu_j)(1-\beta)} \quad \text{------ (15)}$$

$$\frac{\varepsilon_{j1}}{\sigma_1/E_j} = \frac{\{(1+\mu_j)(1-2\mu_j)+2\mu_u\mu_j\alpha\}(1-\beta)+(1-\mu_u)\beta}{(1-\mu_u)\alpha\beta+(1-\mu_j)(1-\beta)} \quad \text{-- (16)}$$

2.3 Stress in Two Components

In the Equations (13) and (15), when $\alpha$ is greater than $\mu j/\mu u$, and in the general case, where the Young's Modulus of the joint mortar is smaller than that of the masonry unit, the biaxial stress is tensile in the masonry units and compressive in the joint mortar layers.

The stress direction, therefore, can be summarized as shown in Table 1.

In case of $\alpha \leqq \mu j/\mu u$, stronger masonry units are supposed to be used and tensile biaxial stress would occur in clay bricks and concrete blocks. The amount of this stress can be estimated on the basis of Equation(13). On the other hand, joint mortars might be tri-compressive stress conditions. The uniaxial compressive strength of masonry could be supposed to be the minimum value of both the reduced compressive strength of units by the uniform biaxial tensile stress and the increased value of joint mortars by the uniform biaxial compressive stress.

2.4 Criteria of Fracture

The fracture criteria of masonry component materials have been discussed for a long time and several equations have been proposed. The detailed data of experiments, however, are not so sufficient that an appropriate simple fracture criteria might be proposed to examine the general tendency.In this paper, the following equations could be proposed.

In uniaxial compressive and biaxial tensile,

$$\frac{\sigma_1}{f_c}+\frac{\sigma_2}{f_c/k_t}=1 \quad \text{------ (17)}$$

In uniaxial compressive and biaxial compressive,

$$\sigma_1 = k_c\,\sigma_2 + f_c \quad \text{------ (18)}$$

in which, 1/kt is a dutility factor of materials as shown in ft/fc. Kc is a constant which shows the increasing trend of compressive strength by biaxial compression.

## 3. PREDICTION OF UNIAXIAL COMPRESSIVE STRENGTH

3.1 Equations for Prediction

The compressive strength of the component material under both uniaxial compression and biaxial tension can be shown as follows,

$$F_t = f_c/(1+\lambda K_t) \quad \text{------ (19)}$$
$$\lambda = \sigma_2/\sigma_1,\ K_t = f_c/f_t$$

where $\lambda$ can be estimated by Equations (13) and (14).
At the same time, the compressive strength of the other component material under tri-compresion can be predicted by the following equation.

$$F_c = f_c / (1+\lambda K_c) \qquad \text{------ (20)}$$

As a conclusion, the prism strength of masonry can be predicted as the minimum value of the two values obtained by Equations (19) and (20).

3.2 Evaluation of Biaxial Stress

As shown in Art 2, the Biaxial stresses occurred in the two components can be evaluated on the basis of Equations (13) and (15). These two equations contain the same part and they can be rewritten in the followings.

$$A=\frac{\mu_U\,\alpha-\mu_J}{\beta(1-\mu_U)\,\alpha+(1-\beta)(1-\mu_J)} \qquad \text{------ (21)}$$

$$\sigma_{U2}=\beta A\,\sigma_1 \qquad \text{------ (22)}$$

$$\sigma_{J2}=(1-\beta)\,A\,\sigma_1 \qquad \text{------ (23)}$$

Therefore, the biaxial stresses can be represented by A. The coefficient A is determined by the mechanical properties of the two component materials. Figure 3 shows the relationship between the coefficient A and the Young's modulus ratio schematically. The physical meaning of this coefficient, as shown in Figure 3 , should be the difference between the biaxial stresses in the two materials. At the interface a certain amount of shear stress could exist. When this coefficient A is greater than a certain value which could be determined by the bond strength between the two, a different fracture mechanism might happen from the assumption shown previously in Art 2. This A, therefore, could be called "Biaxial Stress Coefficient".

In Figure 3, this coefficient has both the maximum value and minimum one. These values are shown as follows.

$$A\max=\frac{\mu_U}{\beta(1-\mu_U)}, \quad A\min=-\frac{\mu_J}{(1-\beta)(1-\mu_J)} \qquad \text{------ (24)}$$

where, Amax or Amin are dependent upon the Poisson's ratio of masonry units or joint mortars respectively.

3.3 Procedure of Prediction

On the basis of the above discussions, the uniaxial strength can be calculated. The fundamental procedure of calculation can be shown as follows,

(1) Calculation of a biaxial coefficient from elastic constants of the two components.

(2) Calculation of uniaxial strength from both the biaxial coefficient and the fracture constants of the two components.

A schematic relationship between uniaxial strength and biaxial

stress coefficient is shown in Figure 4. As shown in this figure, the maximum value of prism strength can be obtained, if the biaxial coefficient A can be chosen most appropriately. At that time A and uniaxial strength shall be shown as follows,

$$A_{app} = \frac{-f_{cu} + f_{cj}}{(1-\beta) f_{cj} K_{tu} + \beta f_{cu} k_{cj}} \qquad \text{------ (25)}$$

$$F_{max} = \frac{\gamma - (1-\beta) K_{cj} / \beta K_{tu}}{1 - (1-\beta) K_{cj} / \beta K_{tu}} \cdot f_{cu} \qquad \text{------ (26)}$$

## 4. COMPARISONS WITH EXPERIMENTAL RESULTS

### 4.1 Experimental Examination Based on Past Data

The experimental examination of this theory shall require the precise experimental results which don't only include the strength properties of components but also contain the various elastic constants. Such a complete experimental result could not be found in the past experiments except R.H. Brown's.

Figure 5 shows the relationships between the uniaxial compressive strength of brick masonry prisms which include 3 kinds of bricks and 3 kinds of joint mortars. The prism strength increases as decreasing in biaxial coefficient A within a certain range of A, but the strength begins to decrease from a certain point according to the further decrease of A. This experimental examination shows that the relationships between prism strength and the biaxial coefficient shall have a peak which might be corresponded to the most appropriate choice of component materials. That is to say, this point means Aapp and Fmax physically.

The theoretical calculation can be approximated to the experimental results fairly well. However some disagreements between these two values may also be discovered. This might be explained on the basis of the followings.

(1) In the region of high absolute values of A, shear stress could work on the vicinity of the interface of the two components and the amount of the stress might increase beyond the strength. (future problem in theories)

(2) The strength of joint mortars could increase due to suction of water into adjacent bricks. (future problem in experiments)

From the fundamental trends, this experiment offered the good correlation for this theoretical consideration particularly in qualitative aspects. Further necessary studies should be carried out theoretically and experimentally.

### 4.2 Possible Range of Prism Strength

As mentioned above, the possible range of uniaxial prism strength can be determined. Figure 6 shows both the upper limitation and the lower one of the ratio of prism strength to unit strength against the ratio of joint strength to unit strength. In this figure, the upper limitation could be calculated from the equation (26), and the lower limitation could be estimated as the prism strength correspondent to Amin or as the uniaxial strength of joints. That is to say, actual cases can be limited to

when units are stronger than joints.

In the above discussion, the case of bedding fully has been mentioned. The actual cases, however, contain partially bedded prisms. In these cases, it should be noticed that the prism strength might be reduced proportionally to the ratio of the net area against the gross area of prism specimens.

Figure 7 shows the actual distributions of this relationship in the past experimental results. These results could be confined within the range predicted on the basis of the suggested theory in this paper.

## 5. CONCLUSIONS AND OUTSTANDING PROBLEMS

A simple theoretical examination has been done to clear the complexed feature of uniaxial compresive strength of various types of ungrouted masonry. Based on the results of the examination mentioned above, the following conclusions can be drawn.

(1) The end of a thread to predict prism strength could be discovered, and a further precise experimental and theoretical examination might give a complete solution.

(2) Biaxial stress condition could be represented by using the concept "Biaxial Stress Coefficient" as a common scale for biaxial stress.

(3) The fluctuated value of prism strength could be explained briefly and generally by using the concept of biaxial stress coefficient from the experimental data of the elastic constants of component materials.

(4) The possible range of prism strength could be mentioned, and the possible maximum value and a method to obtain the maximum value also could be looked out.

As further extensive examinations, the following two items shall be examined.

(1) A precise analysis of stress distribution in the two parts of masonry shall be clarified including the shear stress distribution at the interface of the two which may be very effective in the case of a low biaxial stress coefficient and also may be supposed to have a great influence on the shear strength of bearing masonry walls.

(2) A precise experimental examination on prism strength shall be carried out including various elastic constants of the two materials. Then, the mechanical properties of joint materials should be tested under the truly actual states including the condition of water sucked by masonry units.

## 6. REFERENCES

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( NOTATIONS )

$\varepsilon_1$: Uniaxial Strain
$\varepsilon_2, \varepsilon_3$: Biaxial Strain
$\sigma_1$: Uniaxial Stress
$\sigma_2, \sigma_3$: Biaxial Stress
$E$ : Young's Modulus
$\mu$ : Poisson's Ratio
$F_c$ : Uniaxial Compressive Strength of Prism
$f_t$ : Tensile Strength of Components
$f_c$ : Compressive Strength of Components
$h$ : Height of Units or Thickness of Joints
Suffix u and j mean units and joint respectively
$\alpha = E_j/E_u$, $\beta = h_j/(h_u+h_j)$

Table 1 Stress Conditions in Units and Joint Mortar under Compressive Loading

| | | $\alpha = Ej / Eu$ | |
|---|---|---|---|
| | | $> \mu j / \mu u$ | $< \mu j / \mu u$ |
| Masonry Unit | Uniaxial Stress | Compressive (-) | Compressive (-) |
| | Biaxial Stress | Compressive (-) | Tensile (+) |
| Joint Mortar | Uniaxial Stress | Compressive (-) | Compressive (-) |
| | Biaxial Stress | Tensile (+) | Compressive (-) |

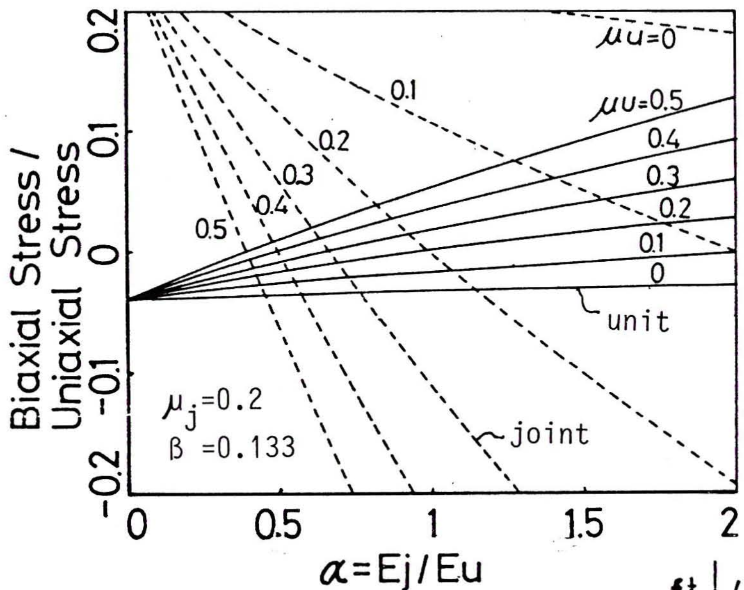



Figure 1 Calculated Relationship between βA or (1-β)A and $\alpha$

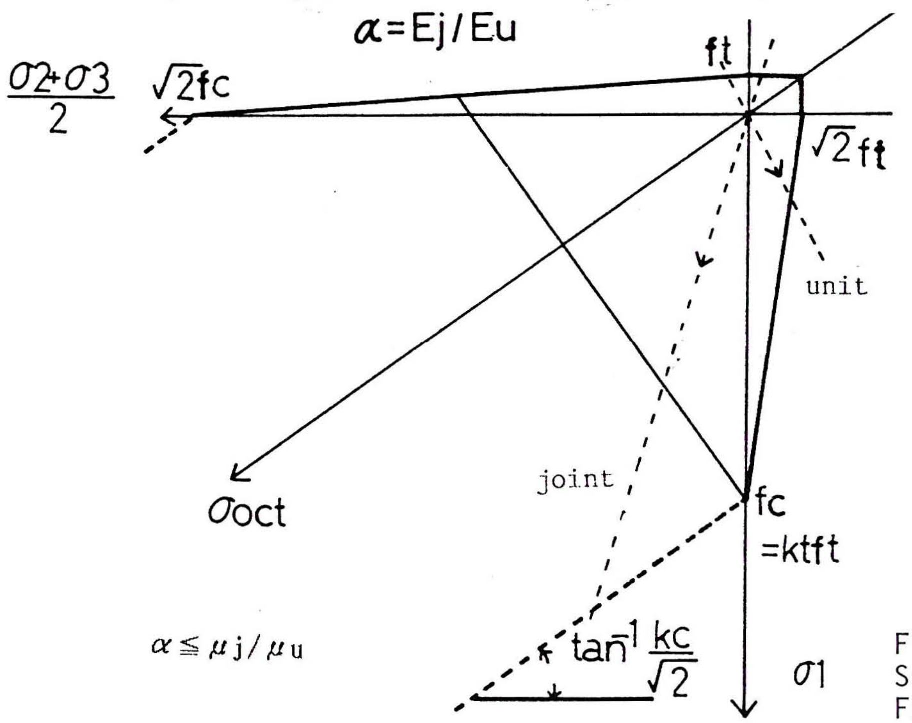
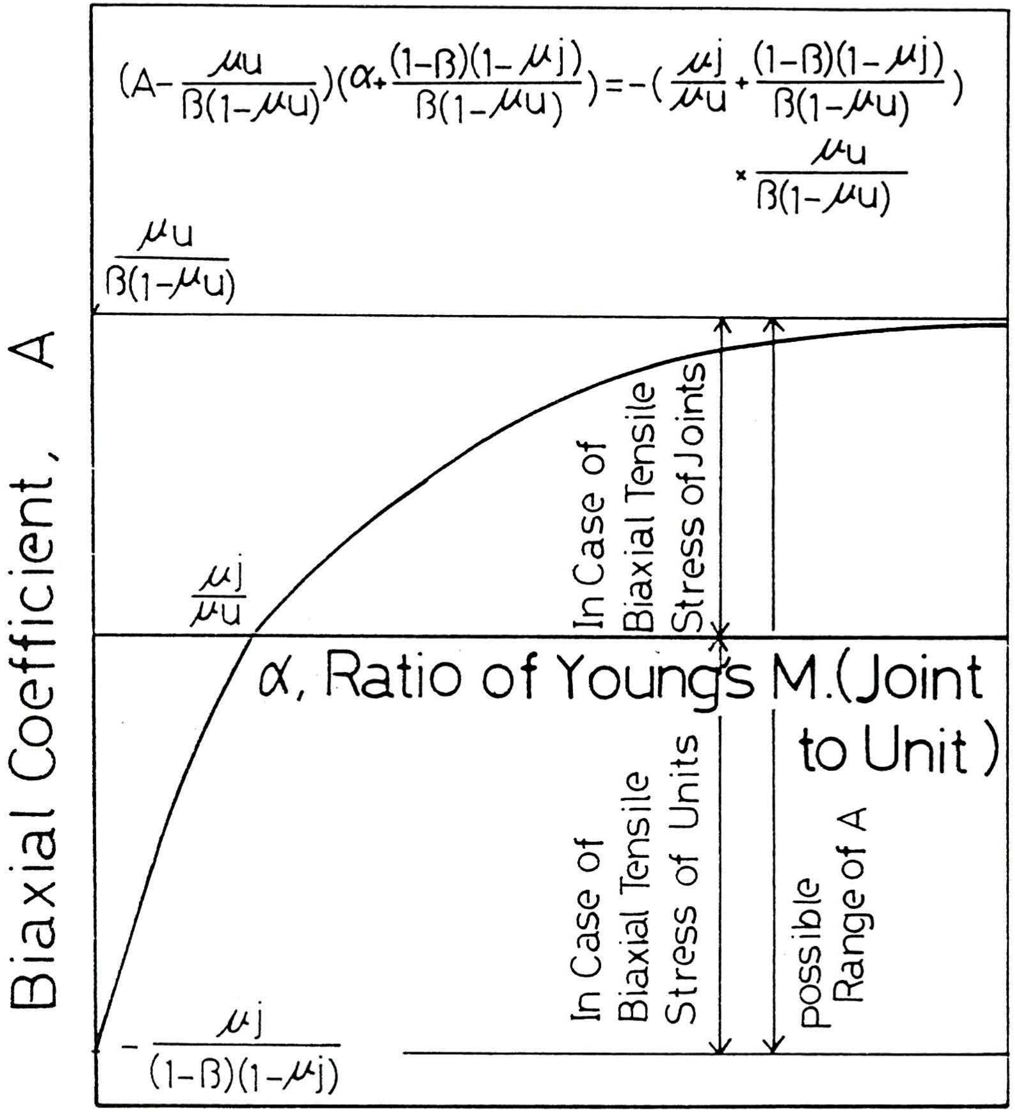



Figure 2 Supposed Criteria of Fracture



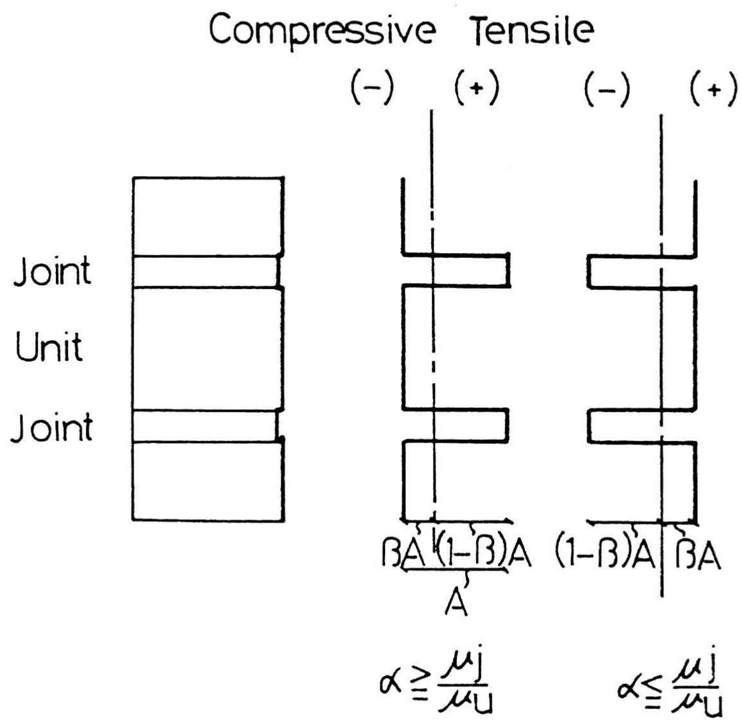




Figure 3 Schematic Relationship between Biaxial Stress Coefficient and Youngś Modulus Ratio

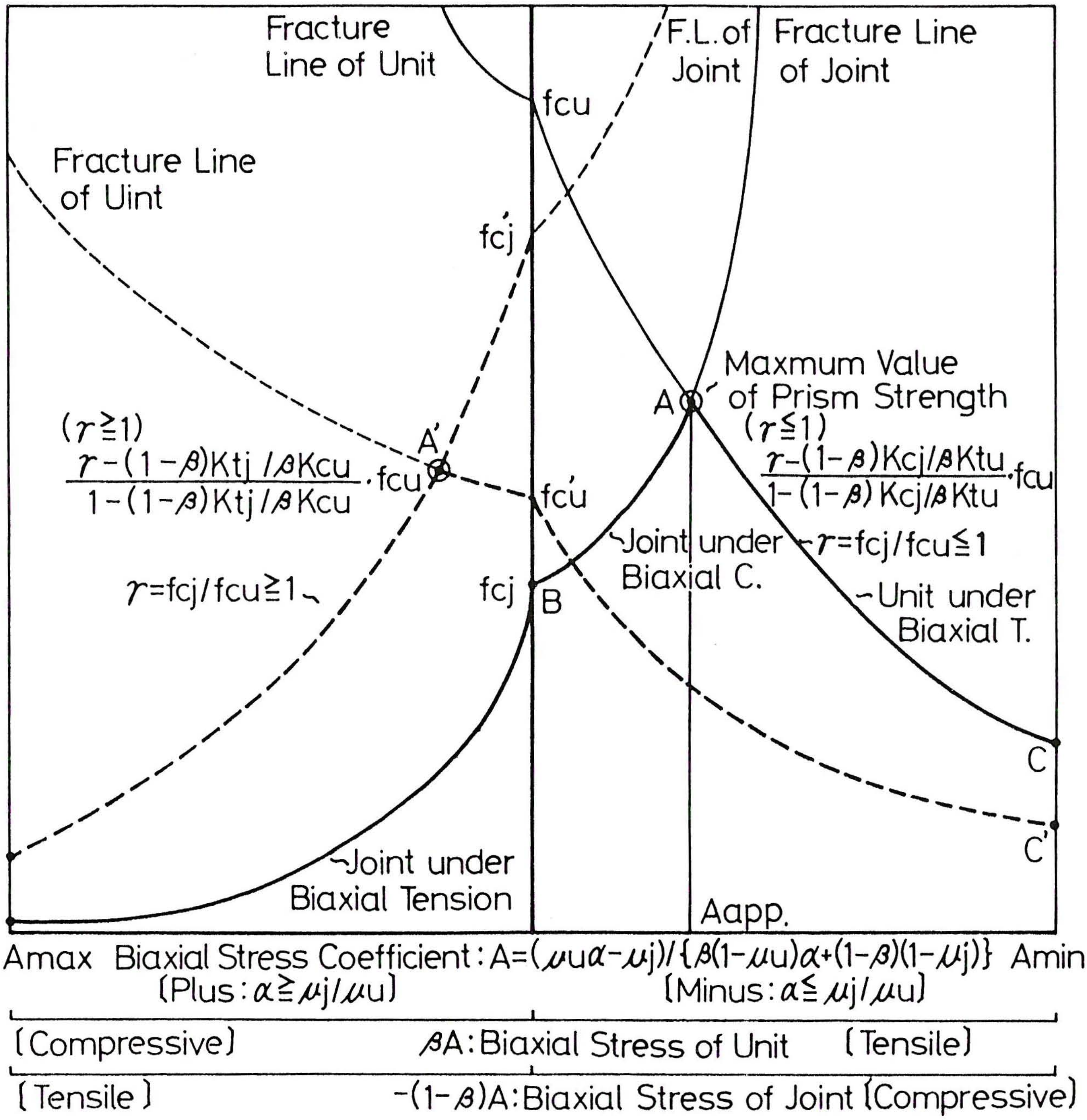



Figure 4 Schematic Relationship between Prism Strength and Biaxial Stress Coefficient

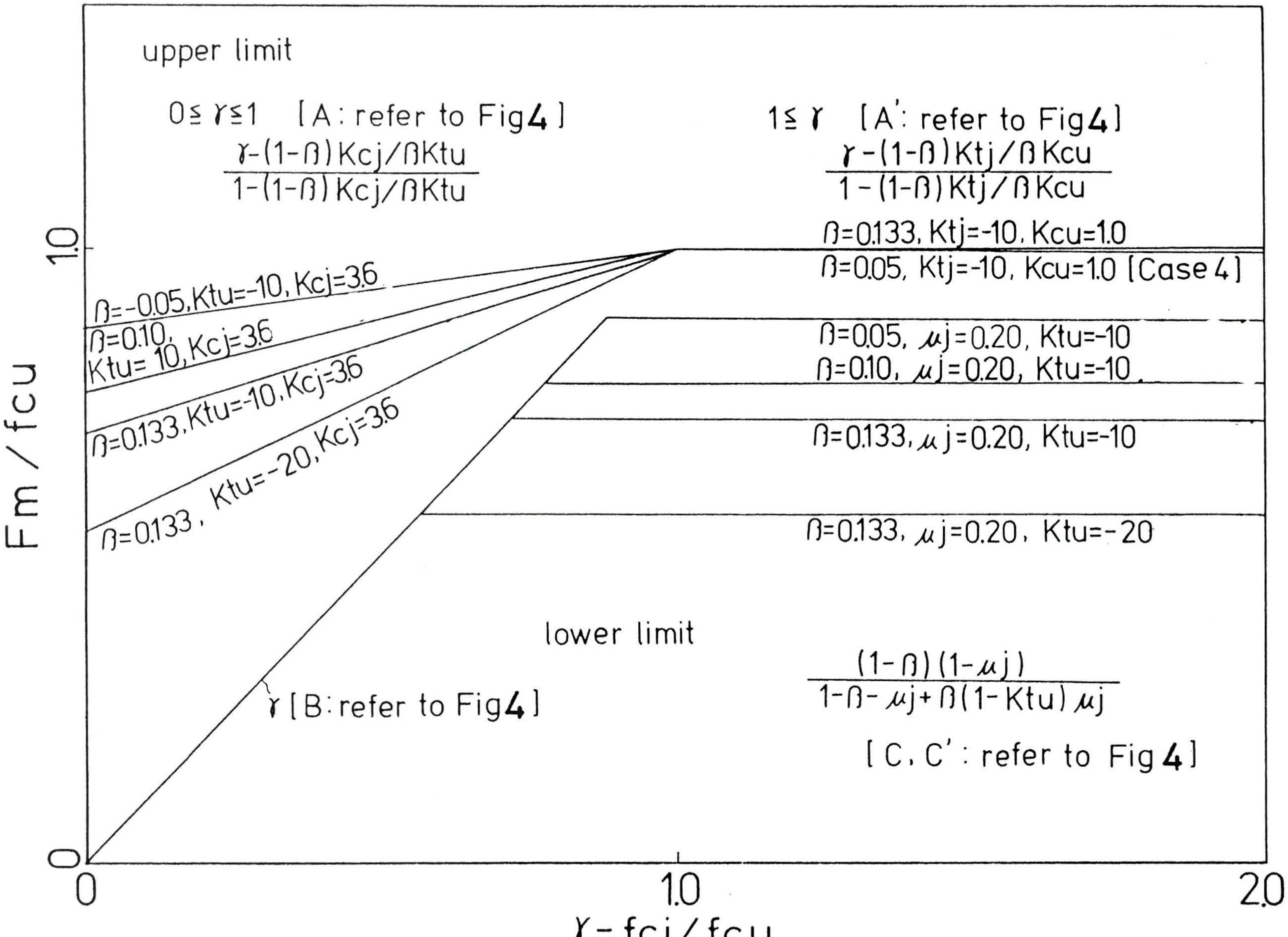



Figure 6 predicted Range of Efficiency Based on Theory

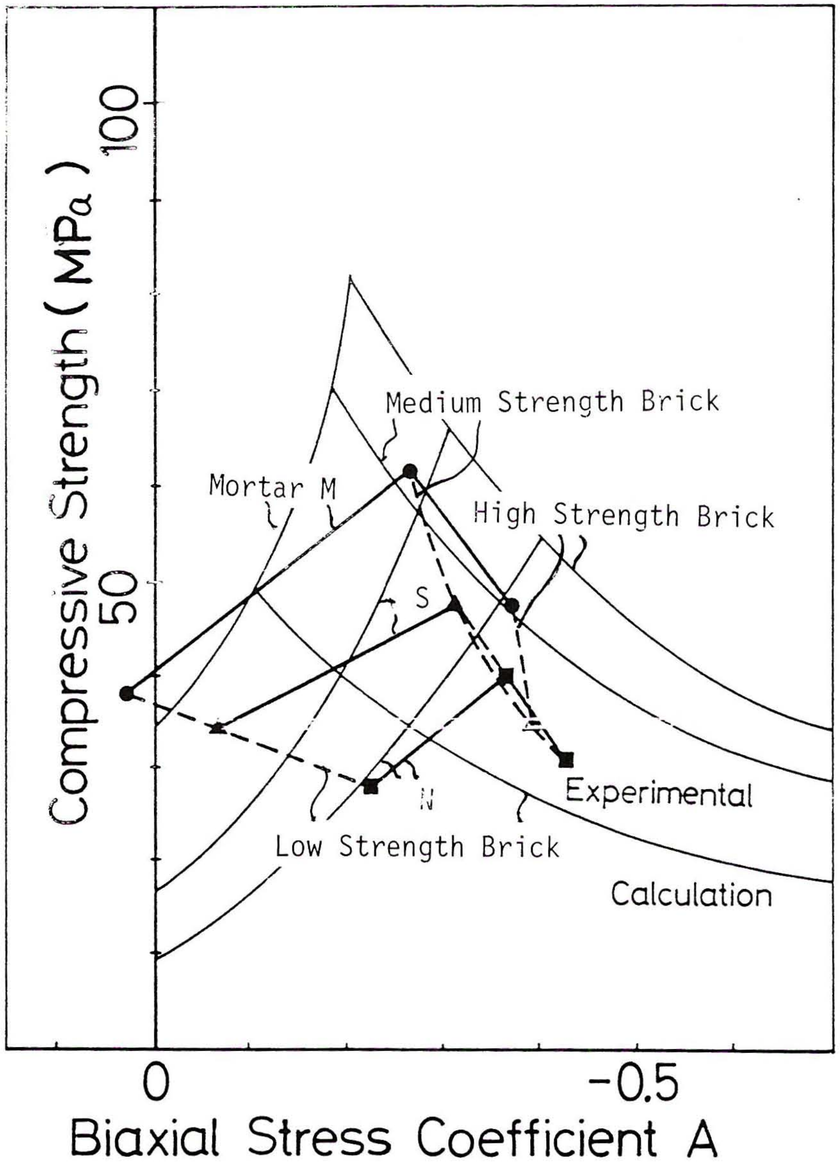



Figure 5 Comparison with Past Experimental Data

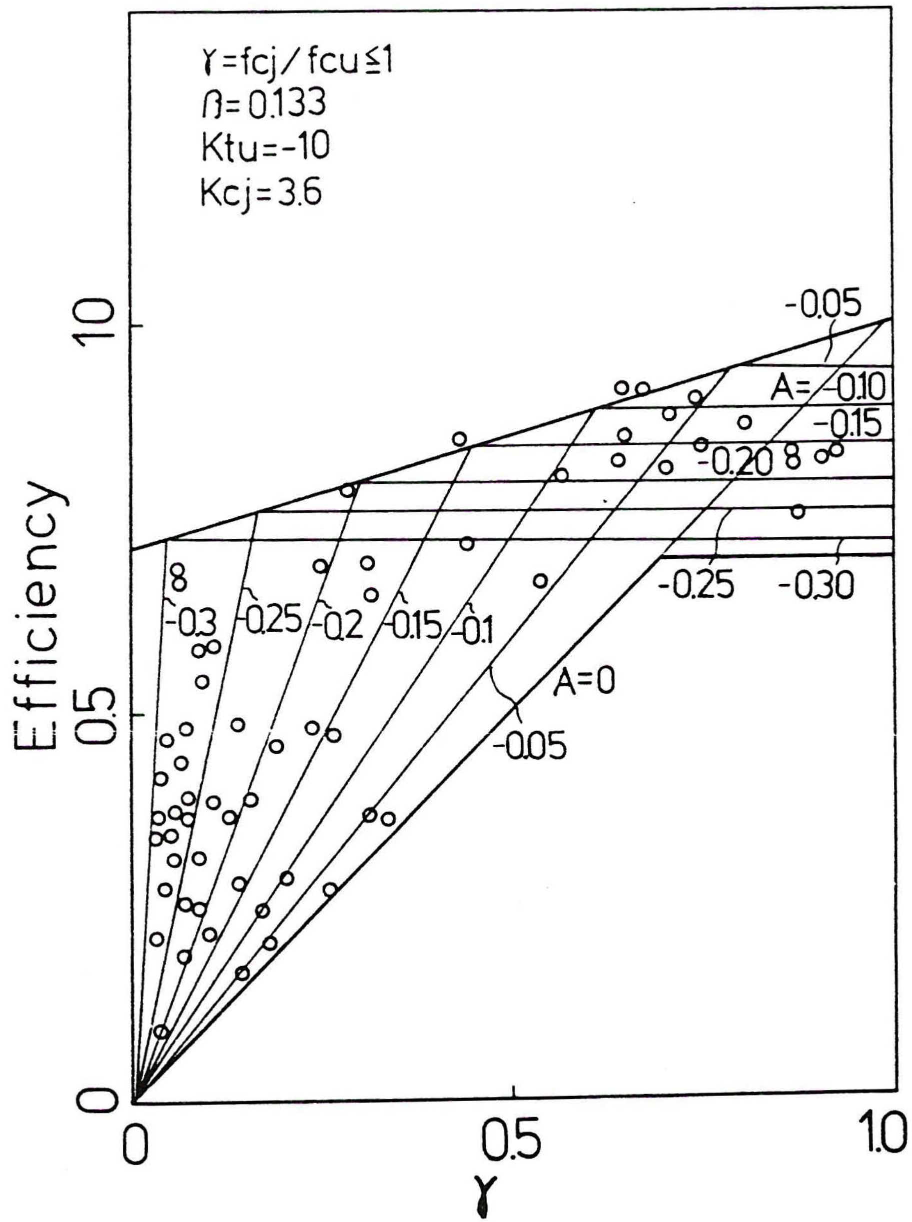



Figure 7 Experimental Examinations of Possible Range of Efficiency