INVESTIGATIONS ON THE BUCKLING EFFECT IN MASONRY WALLS RESTRAINED IN THE FLOOR SLABS

RECHNERISCHE UNTERSUCHUNGEN ZUM KNICKVERHALTEN VON GEMAUERTEN WÄNDEN UNTER BERÜCKSICHTIGUNG DER EINSPANNWIRKUNG DER GESCHOSSDECKEN

by Prof. Dr.-Ing. W. Mann and Dipl.-Ing. E. Leicher,
Technische Hochschule Darmstadt, Germany

ABSTRACT. The degree of restraint of walls in floor slabs increases the load capacity, as the effective height of the wall becomes smaller. The paper shows the effect of the stiffness of wall and floors and also shows the advantage for the calculation of the load capacity of such walls. The theoretical studies are made for homogeneous materials as well as for materials without tensile strength. As a result, it is shown that the effective length of slender walls can be reduced to 3/4 of the storey height if the reinforced concrete floor slabs bear over approximately the full support area. For thick walls simple approximation equations are given for the effective length.

ZUSAMMENFASSUNG. Die Einspannung von Wänden in Geschoßdecken wirkt sich günstig auf das Tragverhalten der Wände aus, da sich die Knicklänge dadurch verringert. Die Arbeit zeigt, wie sich das Verhältnis der Steifigkeiten von Wand und Decken auswirkt und welche Vorteile sich für die Bemessung von Wänden hieraus ergeben. Die theoretischen Untersuchungen werden sowohl für homogenes Material als auch für Baustoffe ohne Zugfestigkeit durchgeführt. Als Ergebnis wird nachgewiesen, daß die Knicklänge von schlanken Wänden auf 3/4 der Geschoßhöhe abgemindert werden darf, wenn Stahlbetondecken annähernd vollflächig aufliegen. Für gedrungene Wände werden einfache Näherungsformeln für die Knicklänge angegeben.

NOMENCLATURE

d_W, J_W, E_W	Wall: Thickness, Moment of Inertia, E-Module
d_B, J_B, E_B	Slab: Thickness, Moment of Inertia, E-Module for Concrete
h	Storey Height
$s_k = \beta \cdot h$	Effective Length of Wall
ℓ_1, ℓ_2	Span of Floor Slab
$P_E = \pi^2 \cdot EJ_W/h^2$	Euler Load
$\alpha = \sqrt{N/EJ_W}$	Parameter
e_o	Accidental Eccentricity of Wall
M_o, M_m, M_u	Wall Moment at Head, Centre, Base

1. INTRODUCTION

The effective length of walls is an important factor in the determination of the bearing load of masonry walls. In the following, walls restrained at only 2 sides are considered, i.e. walls that are restrained in a horizontal direction only by floor slabs and not by cross walls. The slabs themselves are restrained horizontally by walls in both directions.

Often a pin-jointed bearing of the wall is assumed. In this case $s_{k1} = h$ is valid, in accordance with Fig. 1a. This assumption is on the safe side. If the floor slabs consist of reinforced concrete slabs bearing on the full support surface, then a frame system as shown in Fig. 1b or 1c is produced. A restraining moment thus results between wall and slab, which reduces the effective length of the wall to s_{k2}, and increases the loading capacity P_2. As the slabs are also subject to deflection, only an elastic restraint results. The assumption of a rigid restraint in accordance with 1d would, in normal cases, be too favourable.

In the following, external walls as shown in Fig. 1b are examined, whereby the preparatory work [1] and [2] is used as a basis. The behaviour of internal walls according to Fig. 1c can be derived from it.

In order to work with mathematic formulae of a closed order, linear-elastic homogeneous material is at first assumed. For comparison, the same system for linear-elastic material without tensile strength is examined in a subsequent section.

2. FRAME EFFECT FOR HOMOGENEOUS MATERIAL

2.1. GENERAL DERIVATION FOR EXTERNAL WALLS

One storey of the frame system shown in Fig. 2 is examined. The slabs here are assumed to be pin-jointed at the right. This case is most unfavourable for the walls and thus decisive, as a restraint of the slabs would increase their bending strength and thus also increase the buckling load. Only half the value of the bending strength EJ_B of the slabs is used for the one storey, as the other half is required for stiffening the walls in the storeys either above or below.

The following formula is valid for the walls

$$EJ_W \cdot w'' + N \cdot w + N \cdot e_0 \cdot \sin\frac{\pi x}{h} + M_o\left(1-\frac{x}{h}\right) + M_u \cdot \frac{x}{h} = 0 \tag{1}$$

with the solution

$$M(x) = \frac{N \cdot e_0}{1-\frac{N}{P_E}} \cdot \sin\frac{\pi x}{h} + M_o \cdot \frac{\sin\alpha(h-x)}{\sin\alpha h} + M_u \cdot \frac{\sin\alpha x}{\sin\alpha h} \tag{2}$$

and the limiting conditions

$$\begin{aligned} M(x=0) &= M_o \\ M\left(x=\tfrac{h}{2}\right) &= M_m = \frac{N \cdot e_0}{1-\frac{N}{P_E}} + \frac{M_o+M_u}{2} \cdot \frac{1}{\cos\alpha h/2} \\ M(x=h) &= M_u \end{aligned} \tag{3}$$

From the continuity conditions between wall and slabs, the restraining moments M_o and M_u result:

$$M_o = -\frac{N \cdot e_0}{1-\frac{N}{P_E}} \cdot \frac{1}{\frac{1}{\sqrt{N/P_E}} \cdot tg\left(\frac{\pi}{2} \cdot \sqrt{N/P_E}\right) + \frac{2\pi}{3} \cdot \frac{EJ_W}{EJ_B} \cdot \frac{\ell}{h}} - \frac{q\ell^3}{24 \cdot \frac{1}{2} EJ_B} \cdot \frac{h}{\pi} \cdot \frac{1}{\frac{\sqrt{N/P_E}}{tg\left(\frac{\pi}{2}\sqrt{N/P_E}\right)} - \frac{2\pi}{3} \cdot \frac{EJ_W \cdot h \cdot N}{EJ_B \cdot \ell \cdot P_E} - \frac{2}{\pi}} \tag{4a}$$

$$M_u = -\frac{N \cdot e_0}{1-\frac{N}{P_E}} \cdot \frac{1}{\frac{1}{\sqrt{N/P_E}} \cdot tg\left(\frac{\pi}{2} \cdot \sqrt{N/P_E}\right) + \frac{2\pi}{3} \cdot \frac{EJ_W}{EJ_B} \cdot \frac{\ell}{h}} + \frac{q\ell^3}{24 \cdot \frac{1}{2} EJ_B} \cdot \frac{h}{\pi} \cdot \frac{1}{\frac{\sqrt{N/P_E}}{tg\left(\frac{\pi}{2}\sqrt{N/P_E}\right)} - \frac{2\pi}{3} \cdot \frac{EJ_W \cdot h \cdot N}{EJ_B \cdot \ell \cdot P_E} - \frac{2}{\pi}} \tag{4b}$$

and the moment M_m at half wall height

$$M\left(\frac{h}{2}\right) = M_m = \frac{N \cdot e_0}{1-\frac{N}{P_E}} \cdot \left[1 - \frac{1}{\frac{tg\left(\frac{\pi}{2}\sqrt{N/P_E}\right)}{\sqrt{N/P_E}} + \frac{2\pi}{3} \frac{EJ_W \cdot \ell}{EJ_B \cdot h}} \cdot \frac{1}{\cos\left(\frac{\pi}{2}\sqrt{N/P_E}\right)}\right] \tag{5}$$

If a restraint has to be considered at the right bearing, the extension of the rigidity of slabs is realized by reducing the span to an ideal length $\ell_i = 0{,}75\,\ell$. Thereby the degree of restraint is slightly increasing, the effective length slightly becoming minor. For the rest all the formulaes remain valid.

2.2. SPECIAL CASE PIN-JOINTED BEARING

The special case of pin-jointed bearing as shown in Fig. 1a results from the general case, if the bending strength of the slab $EJ_B = 0$. Thus, resulting from (4) and (5)

$$M_o = M_u = 0 \tag{6a}$$

$$M(h/2) = M_m = \frac{N \cdot e_o}{1 - \frac{N}{P_E}} \tag{6b}$$

This case is in accordance with the Euler Case 2 with $s_k = h$. The following general case derives from this. The buckling load is reached when

$$N = P_E = \frac{\pi^2 \cdot EJ_W}{h^2}$$

so that $M_m = \infty$. From this $s_k = h$ is defined.

2.3. EFFECTIVE LENGTH WITH ELASTIC RESTRAINT AT WALL-SLAB JUNCTION

With elastic restraint, the restraint moments $M_{o,u}$ are smaller than M_m, so that M_m becomes decisive for the buckling load. From (5) the buckling case $M_m = \infty$ follows for

$$tg\left(\frac{\pi}{2}\sqrt{N/P_E}\right) = -\sqrt{\frac{N}{P_E}} \cdot \frac{2\pi}{3} \cdot \frac{EJ_W}{EJ_B} \cdot \frac{l}{h} \tag{7}$$

Defining the effective length $s_k = \beta \cdot h$ and thus the buckling load

$$N_K = \frac{\pi^2 \cdot EJ}{s_k^2} = \frac{P_E}{\beta^2},$$

then from (7)

$$\beta \cdot tg\left(\frac{\pi}{2\beta}\right) = -\frac{2\pi}{3} \cdot \frac{1}{\frac{EJ_B}{EJ_W} \cdot \frac{h}{l}} \tag{8}$$

From equation (8) one obtains the reduction of the effective length with factor β. The result is shown in Fig. 3. The limiting values are also contained herein: Euler Case 2, i.e. pin-jointed bearing, is given by $EJ_B = 0$. This is satisfied in (8) with $\beta = 1$. Euler Case 4, i.e. full restraint, is satisfied with $EJ_B = \infty$. Here (8) is satisfied with $\beta = 0.5$. Cases met with in practice lie between these two limits.

The effective length $s_k = \beta \cdot h$ was derived according to Fig. 2 at the system, in which the slabs are without loads, i. e. $q_i = 0$. As a matter of fact, however, a load q_i acts, which causes bending moments also in the wall. They are linearly divided over the height by maximal values on head and foot of the wall

and moments' zero point in the central area of the wall height. These moments have no essential influence on the effective length of the wall. However, they have to be considered for the calculation of the wall. For the analysis of the buckling they have to be taken into account as excentricities. Hereby they are not very much effective, because in the important central area of the wall height with usual regular systems of superstructure they are small. The maximal moments at head and foot of the wall have to be proved, anyhow.

2.4. PRACTICAL APPROXIMATION FOR THE REDUCTION OF THE EFFECTIVE LENGTH $s_k = \beta \cdot h$

The determination of ß from equation (8) appears to be too complicated for practical application. Thus the following linearisation is proposed as an approximation:

$$\beta = 1 - 0.15 \cdot \frac{E_B}{E_W} \cdot \left(\frac{d_B}{d_W}\right)^3 \cdot \frac{h}{\ell} \geq 0.75 \tag{9}$$

This approximation is shown in Fig. 3 as a chain-dotted line. As was given by comparitive calculations, ß = 0.75 is decisive for the interesting practical cases.

2.5. EXPANSION FOR INTERNAL WALLS

The results for external walls can be applied for internal walls in that the bending resistance of both adjacent floor slabs are added:

$$\frac{EJ_B}{\ell} \rightarrow \left(\frac{EJ_B}{\ell}\right)_{left} + \left(\frac{EJ_B}{\ell}\right)_{right} = EJ_B\left(\frac{1}{\ell_1} + \frac{1}{\ell_2}\right)$$

The approximation (9) can thus be expanded:

$$\beta = 1 - 0.15 \frac{E_B}{E_W} \left(\frac{d_B}{d_W}\right)^3 \cdot h \cdot \left(\frac{1}{\ell_1} + \frac{1}{\ell_2}\right) \geq 0.75 \tag{10}$$

For external walls $\frac{1}{\ell_2} = 0$ is to be inserted.

The equation (10) was included in the German "Mauerwerksnorm" (Masonry Standard) DIN 1053, Part 2. This was possible as tests [3] were available which confirmed the reduction of the effective length. The application of the formulae (9) or (10) to the test results produced satisfactory concurrence.

2.6. NUMERICAL EXAMPLES

In the following, several conventional constructions are examined.

Slab E_B =	Concrete B 25 30 000 MN/m^2		Wall E_W = 7 500 MN/m^2			β	
d_B [m]	ℓ_1 [m]	ℓ_2 [m]		d_W [m]	h [m]	(10)	β
0.15	5.0	-	external	0.115	2.60	0.75	0.63
0.15	5.0	5.0	internal	0.115	2.60	0.75	0.57
0.15	5.0	-	external	0.175	2.60	0.80	0.78
0.15	5.0	5.0	internal	0.175	2.60	0.75	0.69
0.15	5.0	-	external	0.24	2.60	0.92	0.88
0.15	5.0	5.0	internal	0.24	2.60	0.85	0.81

One can see that for normal slender walls, where the reduction of the effective length is useful, generally $\beta = 0.75$. For thicker walls, where the effective length is not of great importance, occasionally $\beta > 0.75$ may be valid.

3. FRAME EFFECT FOR MATERIAL WITHOUT TENSILE STRENGTH

3.1. ASSUMPTIONS AND CALCULATIONS

Linear-elastic material is assumed which cannot accept tensile forces, i.e. cracks when subjected to tensile forces. Using this assumption, the system shown in Fig. 2 and described in Section 2 was examined in order to investigate the effect of cracked zones in the wall. The investigation was carried out using an electronic calculation program, which took into account the reduced bending resistance on cracking of the cross-section over the wall height. The load N subjected to the frame system was increased until wall fracture took place.

3.2. RESULT OF CALCULATIONS

The result of the calculations is shown in Fig. 4 as broken lines. One can see that the factor ß is less than with homogeneous material, and that the effective length s_k is reduced and thus more favourable. This is easily explained: By cracking open of the tensile zones in the wall, the bending strength of the wall is reduced, the restraint in the slab is thus larger than for a homogeneous wall. This has a favourable effect on the effective length of the wall. The exact equation (8) for ß obtained from homogeneous material and the approximation equations (9) and (10) for external or internal walls are thus on the safe side.

4. SUMMARY

Reinforced concrete floor slabs, which bear over approximately the full wall support area, can appreciably reduce the effective length of the walls due to the frame effect. For normal dimensions, the effective length of slender walls can be taken as $s_k = 0.75\ h$. For thick walls, the reduction may be somewhat less. Equation (10) represents a simple approximation. For slabs which do not fully bear on the support surface, e.g. floor slabs that turn around the inner face of the wall due to deflection, and thus lift up at the external face, or for slabs consisting of timber beams, $ß = 1$ should be applied.

LITERATURE:

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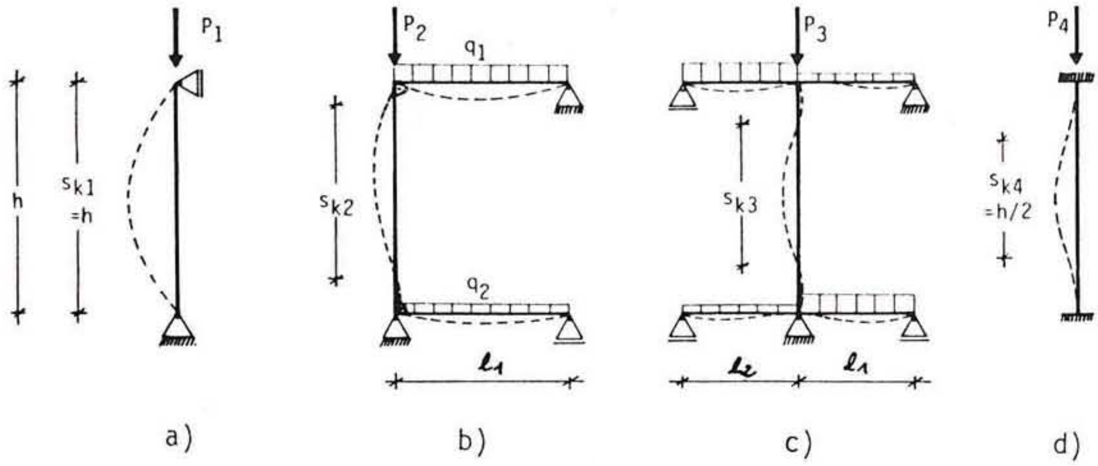

$$h = s_{k1} > s_{k2} > s_{k3} > s_{k4}$$

$$P_1 < P_2 < P_3 < P_4$$

Fig. 1: Systems and Effective Lengths s_k
a) pin-jointed bearing, b) and c) frame effect for external and internal walls
d) full restraint

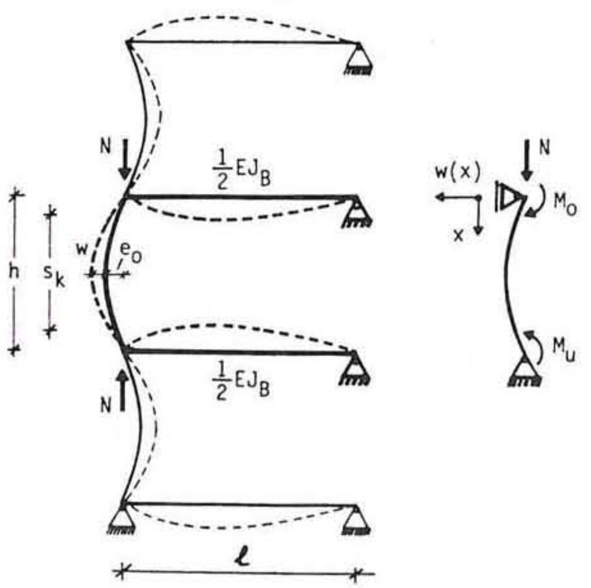

Fig. 2: Examined System of an External Wall

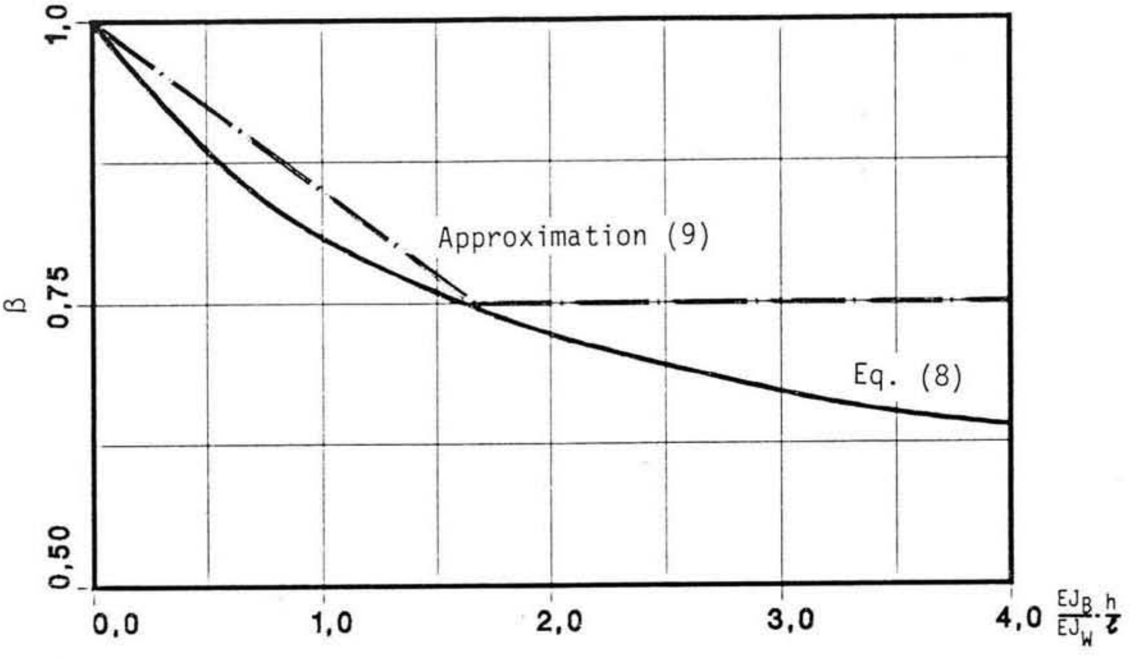

Fig. 3: Reduction of Effective Length $s_k = \beta \cdot h$ According to Equation (8) and Approximation

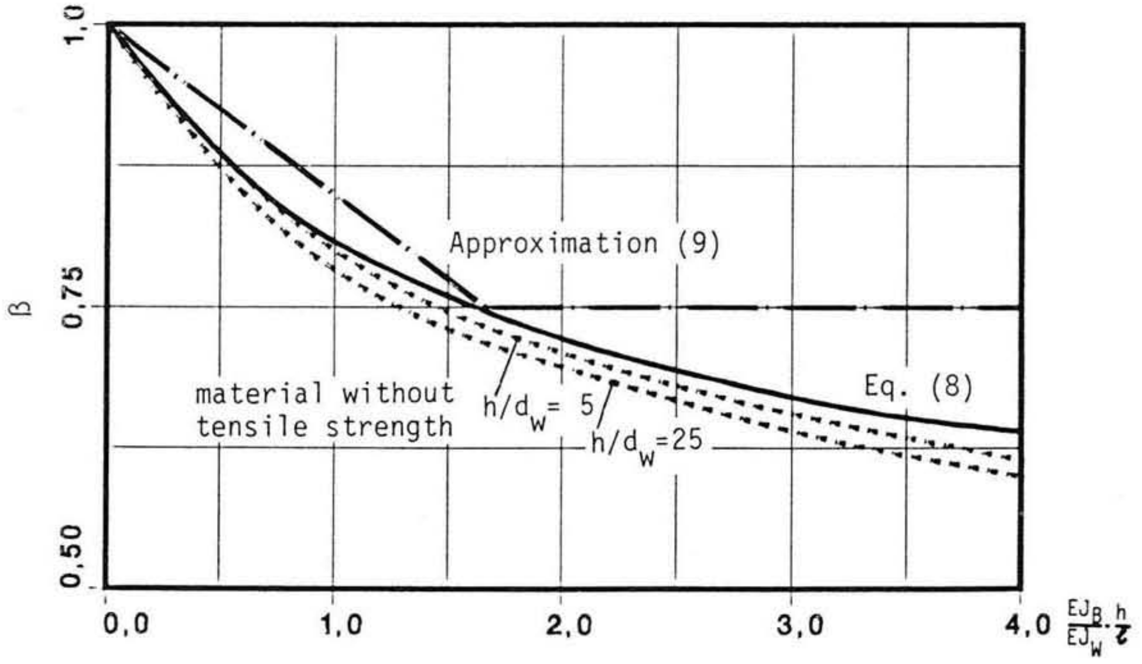

Fig. 4: Comparison of Reduction of Effective Length $s_k = \beta \cdot h$ for Homogeneous Material and for Material Without Tensile Strength