THE STRENGTH OF RECTANGULAR WALL PANELS CARRYING VERTICAL LOAD

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ABSTRACT The vertical load carrying capacity and the stiffness of a wall panel supported or restrained along its vertical edges will be enhanced by the two way bending action of the panel. The paper presents a theoretical model of this bending action under progressive cracking which predicts the behaviour up to failure. The results were confirmed on a full scale test.

1. INTRODUCTION

An isolated brick wall or column carrying axial load with some eccentricity and undergoing one way bending may crack on the bed planes formed by the brick and mortar joints (1), (2), (3), (4). This cracking leads to a reduction in the flexural rigidity of the column section resulting in an increase in lateral deflections and compression stresses normal to the bedjoints (5) and can have a significant influence on the ultimate failure of a column either by overall buckling or by material failure in the brickwork (6), (7).

Finite elements have been used to determine the distribution of stresses in cracked brickwork, and moment curvature relationships have been obtained for brickwork walls acting as columns. (5). The tension field stiffening in the bricks and possible non-linear material behaviour in the mortar have been included in parametric studies involving stocky and slender brick walls (7), (8).

In many load bearing buildings, the brickwork is constructed so that it is effectively restrained laterally on three or four sides. As a result, a wall panel subjected to vertical eccentric loads may deflect laterally under two-way bending and torsion, or twisting, in the brickwork. In this plate-type action, cracking may occur both in the horizontal bedjoints and in the vertical perpend joints if the tension stresses across the brick-mortar interfaces exceed the bond strength or the tension strength of the mortar. The cracking may occur progressively with increasing lateral deflections in the panel and may lead to reductions in the flexural and torsional stiffnesses of the brickwork. It follows that the load-deformation behaviour and the ultimate strength of a cracked brickwork panel may be significantly different from those of an uncracked panel.

In this paper, results of investigations into the reduction of the flexural and torsional stiffnesses of cracked brickwork two way panels are summarized. It is shown how the stiffness values obtained by finite element analysis may be substituted into the differential equations describing the behaviour of a plate of varying thickness carrying eccentric axial loads. As in the case of brick columns, it is convenient to write these plate equations in a finite difference form and to use an iterative computation to solve the resulting equation system (8), (10). Computer-controlled experiments on a full-scale brickwork panel are described in which both the lateral deflections and the load capacity of the panel are compared with values calculated using the

finite element-finite difference method. Results of a parametric study are presented; the study was carried out to determine the factors which significantly influence the vertical load-carrying capacity of eccentrically-loaded brickwork panels supported on four sides.

3. THEORETICAL ANALYSIS

Brickwork constructed in conventional stretcher bond may be subdivided into small brick-mortar "modules" as shown in figures 1 and 2. The properties of a single module may be used to analyse the behaviour of a whole panel by investigating the load-deformation characteristics of such a module with the brickwork in either an uncracked or cracked state. This analysis may be carried out by using the finite element method.

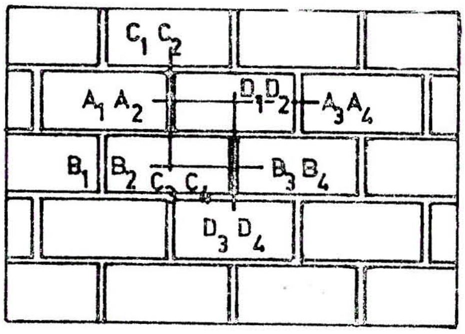


Fig 1 Brickwork in Stretcher Bond

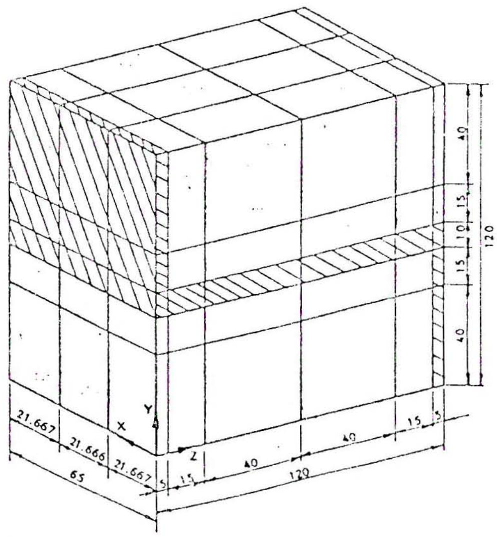


Fig 2 Finite Element Details

The module shown in figure 2 was analysed by using ninety twenty-mode isoparametric elements, coupled at the brick-mortar interfaces for the uncracked condition and uncoupled wherever tension stresses could occur at the brick-mortar interfaces. It was found that the extent of the cracking on the bedjoints depends on the effective eccentricity of the vertical force resultant. It was also shown that brickwork subjected to bending parallel to the bedjoints may crack at the brick-mortar interfaces in the perpends, resulting in a reduced flexural stiffness parallel to the bedjoints (Table 1).

The effect of cracking on the torsional stiffness of brickwork was also investigated using the single brickwork "module" with the same finite element subdivision employed to check bending parallel to the bedjoints. By simulating cracking on the bedjoints, caused by eccentric vertical load, and on the perpends caused by bending parallel to the bedjoints, the reduction in torsional stiffness due to cracking was calculated as shown in Table 2.

Ratio of Elastic moduli (Brick/Mortar)	Relative Stiffness (Finite Element Method)(a)			
	Depth of Bedjoint Crack (b)			
	Zero	$^d/6$	$^d/3$	$^d/2$
1.0	0.768	0.751	0.724	0.699
2.5	0.766	0.745	0.718	0.691
10.0	0.765	0.733	0.705	0.676

(a) Bending stiffness of cracked brickwork relative to uncracked brickwork with the same ratio of elastic moduli.
(b) Perpend and bedjoint cracks on the same panel face.

Table 1 Flexural Stiffness of Brickwork in Bending parallel to the Bedjoints

Ratio of Elastic moduli (Brick/Mortar)	Relative Stiffness (Finite Element Method)(a)			
	Depth of Bedjoint Crack (b)			
	Zero	$^d/6$	$^d/3$	$^d/2$
1.0	0.906	0.855	0.800	0.766
2.0	0.909	0.858	0.805	0.774
5.0	0.928	0.869	0.826	0.806

(a) Torsional Stiffness of cracked brickwork relative to uncracked brickwork with the same ratio of elastic moduli.
(b) Perpend and bedjoint cracks on the same panel face.

Table 2 Torsional Stiffness of Brickwork

The stiffness values for cracked and uncracked brickwork were used to determine the load-deformation behaviour of a full brickwork panel by modelling the panel as a plate of varying thickness. It was found convenient to work in a coordinate system in which the x and y axes are located in the undeflected compression (crack free) face of the panel. The thickness of the plate at any point depends on the effective eccentricity of the vertical load resultant. (This is, in fact, similar to the situation used in the analysis of walls behaving as columns).

If the resultant vertical in-plane load, acts within the kern of the section, the panel is uncracked and the thickness of the plate, t, is equal to the full thickness of the panel. If the vertical resultant acts outside the kern, the section is cracked and the effective plate thickness is

$$t = \frac{6M_y}{N_y} \qquad (1)$$

in which N_y is the vertical load resultant,

M_y is the bending moment normal to the bedjoints, calculated about an axis at $t/2$ from the compression face of the panel.

In areas where the panel is cracked the plate thickness, t, must be calculated by iteration.

2.1 Equilibrium Equation

By taking moments about axes situated at $t/2$ from the compression face of a panel, the equilibrium equation for a panel carrying in-plane forces becomes -

$$\frac{\partial^2 M_x}{\partial x^2} + \frac{\partial^2}{\partial x \partial y}\left(M_{xy} + M_{yx}\right) + \frac{\partial^2 M_y}{\partial y^2} = q + N_x\left(\frac{\partial^2 w}{\partial x^2} + \frac{1}{2} \cdot \frac{\partial^2 t}{\partial x^2}\right)$$

$$+ 2N_{xy}\left(\frac{\partial^2 w}{\partial x \partial y} + \frac{1}{2} \cdot \frac{\partial^2 t}{\partial x \partial y}\right) + N_y\left(\frac{\partial^2 w}{\partial y^2} + \frac{1}{2} \cdot \frac{\partial^2 t}{\partial y^2}\right) \qquad (2)$$

where q is the lateral load per unit area.

2.2 Constitutive Equations

The constitutive equations for a plate of varying thickness are -

$$M_x = - \frac{E_x(t).t^3}{12(1 - \nu_{xy}.\nu_{yx})} \cdot \left(\frac{\partial^2 w}{\partial x^2} + \nu_{xy} \cdot \frac{\partial^2 w}{\partial y^2}\right) \qquad (3a)$$

$$M_y = - \frac{E_y(t).t}{12(1 - \nu_{xy}.\nu_{yx})} \cdot \left(\frac{\partial^2 w}{\partial y^2} + \nu_{yx} \cdot \frac{\partial^2 w}{\partial x^2}\right) \qquad (3b)$$

$$(M_{xy} + M_{yx}) = -2. \frac{G(t).t^3}{6} \cdot \frac{\partial^2 w}{\partial x \partial y} \qquad (3c)$$

where $E_x(t)$, $E_y(t)$ and $G(t)$ are effective elastic constants, depending on the plate thickness, t, and which incorporate the results of the finite element calculations.

ν_{xy} and ν_{yx} are Poisson's ratios for an orthotropic plate.

The elastic constants $E_x(t)$, $E_y(t)$, $G(t)$ may be taken as

$$E_x(t) = C_x.E_b. \frac{\left[\frac{H(L+p)}{L+(E_b/E_m).p} + \frac{b}{(E_b/E_m)}\right]}{(b + H)} \qquad (4a)$$

$$E_y(t) = E_b \cdot \frac{(b + H)}{(H/\alpha) + \frac{E_b}{E_m} . b} \qquad (4b)$$

$$G(t) = C_{xy} \cdot \frac{\sqrt{E_x(t)_{t=d} \cdot E_y(t)_{t=d}}}{2(1 + \nu_b)} \qquad (4c)$$

In equations (4a), (4b) and (4c),

E_b is the brick elastic modulus
E_m is the mortar (initial tangent) elastic modulus
H is the brick height
b is bedjoint thickness
d is the thickness of the uncracked panel
p is perpend thickness
L is brick length

C_x is $\begin{cases} 1,0 \text{ for uncracked bricks} \\ 0.75\ (d/t)^3 \text{ for cracked perpends} \end{cases}$

α is a factor which accounts for tension stiffening in the bricks

C_{xy} is $\begin{cases} 1.0 \text{ for uncracked brickwork} \\ 0.85\ (d/t)^3 \text{ for cracked brickwork} \end{cases}$

Bricks of dimensions 230 mm x 110 mm x 65 mm, laid on edge were used to determine values of C_x and C_{xy} in order to check the analysis experimentally.

Deflections of the panel, using the notion of a plate of varying thickness, must be calculated by iteration because the depth of cracking and hence the plate thickness, t, is not known initially.

Each of the equations (2) to (4) can be written in finite difference form, but in contrast to the column analysis, the increments Δx and Δy are larger than the dimensions of a single brick. The solution procedure and the treatment of boundary conditions are described in detail in reference (8).

3. EXPERIMENT

A full scale experiment was carried out on a brickwork panel approximately 3600 mm long by 2400 mm high in order to compare the theoretical behaviour predicted from the computer program with test observations. For this purpose, idealised edge conditions and loading patterns were chosen rather than the less well identified conditions and patterns found in practice. The experimental apparatus was designed to meet the following criteria.

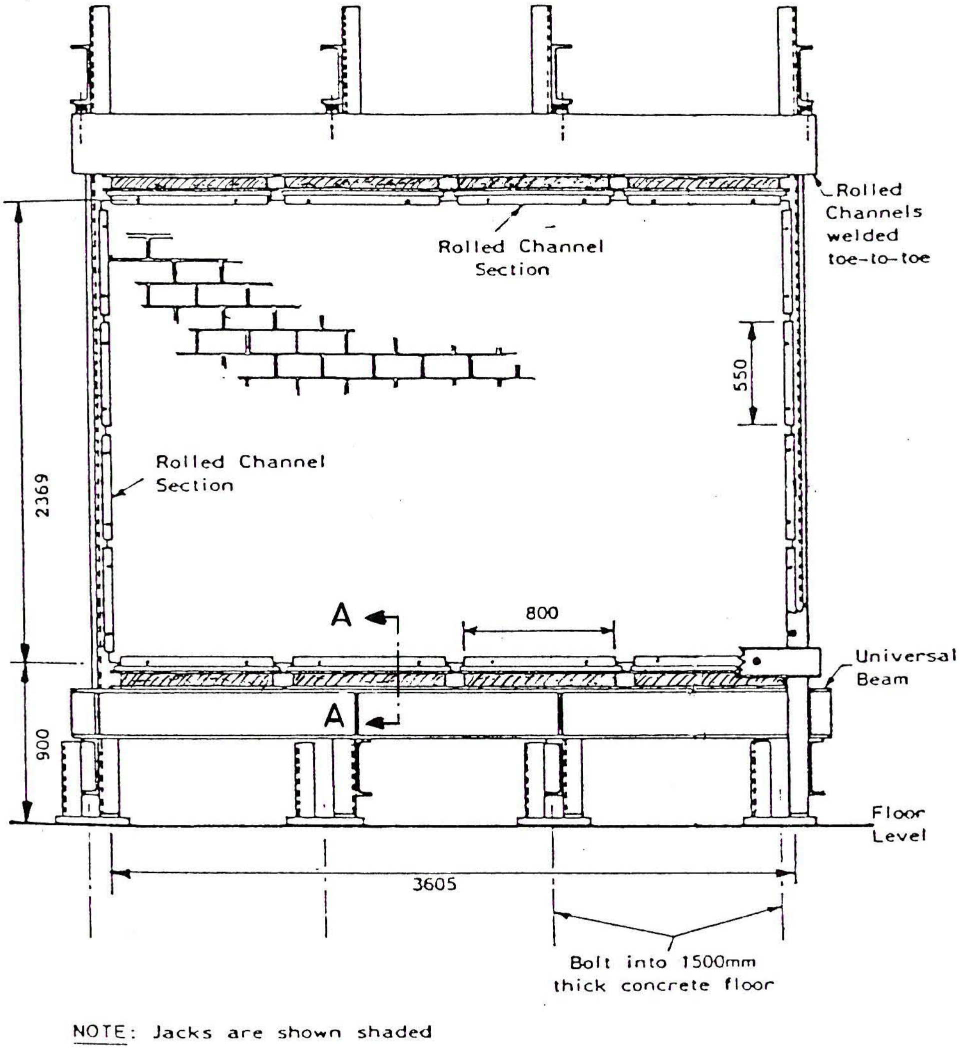


Fig 3(a) Panel and Loading Frame

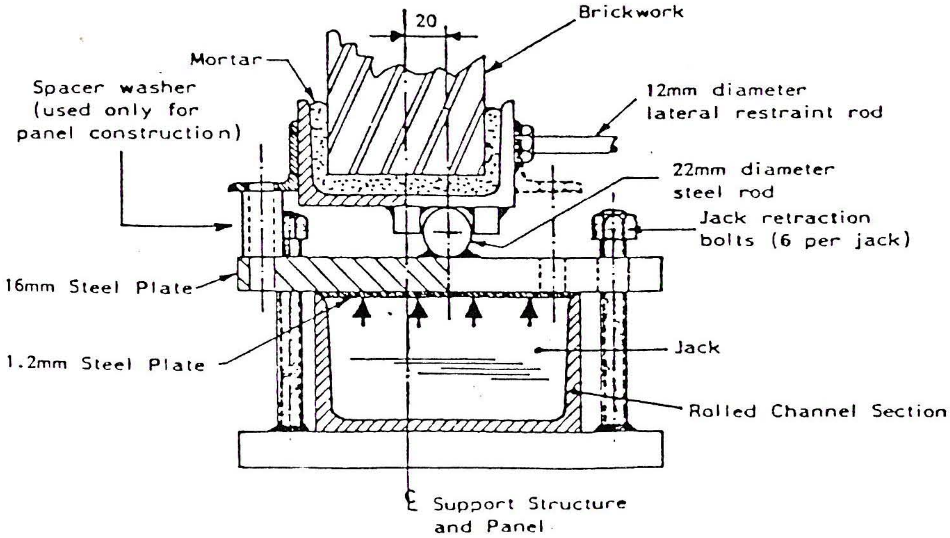


Fig 3(b) Section AA Loading Jack

(a) The panel should be free to rotate at its four supported edges with no out-of-plane movements at the edges (that is, simply-supported).
(b) The load should be applied uniformly along the top and bottom edges with no load-shedding towards the vertical edges as the out-of-plane deflections increased.

In order to make the test results as extensive as possible, the test panel itself was designed -
(a) to avoid excessive vertical compression stresses (causing vertical splitting) so that failure should occur by lateral buckling,
(b) to ensure that the mortar would not be stressed into the non-linear range in a state of triaxial compression.

Preliminary calculations showed that a panel 65 mm thick loaded at a nominal eccentricity of 20 mm along both top and bottom edges would behave in accordance with these conditions.

3.1 Apparatus

The main elements of the steel support structure and loading frame are shown in figures 3(a) and 3(b). In order to provide for the design loading condition, hydraulic jacks were fabricated from 800 mm lengths of rolled steel channel over the toes of which 1.2 mm mild steel plate was welded to form low-profile jacking units. The ends of the jacks were sealed by welding in 6 mm steel plates. Eight jacks were used to load the panel, four at the base and four at the top. The load was applied to the panel eccentrically through lengths of mild steel rod offset nominally 20 mm from the structure centreline.

The panel edges were restrained from out-of-plane displacement by building the brickwork into lengths of rolled steel channel section which were, in turn, connected to the main support frame by lengths of 12 mm diameter steel rod.

3.2 Panel Construction

All bricks were selected solid extruded wire-cut clay bricks approximately 230 mm x 115 mm x 65 mm. The panel was constructed by laying nineteen courses with the bricks on-edge in 10 mm thick mortar joints. The mortar was 1 cement:1 lime:6 sand by volume and the bricks were laid in a saturated surface-dry condition against a double string line to obtain a plane panel.

Six brickwork prisms, each of four bricks laid on edge, were constructed to test the brickwork compressive strength and six prisms each of six bricks laid on edge were built to test bond strength of the bedjoints. Three mortar prisms 25 mm x 25 mm x 50 mm were cast with each of the ten mortar batches used in the panel construction in order to determine the mortar elastic modulus. All the brickwork and mortar prisms were cured in polythene sheeting for 14 days, after which the top edge channels were grouted to the brickwork.

The measured height of the completed panel was 2369 mm between the top and bottom edge restraints and the length was 3605 mm. The mean measured eccentricity of the load at the base was 20.1 mm and at the top was 20.8 mm with an overall mean of 20.5 mm (for use in the computer analysis).

3.3 Instrumentation

Twenty-six electrical resistance strain gauges were used to measure surface

strains on the brick; nine dial gauges and eight LVDTs monitored the out-of-plane displacements of the panel.

The pressure in the low-profile hydraulic jacks was monitored by connecting all jacks to a manifold which incorporated a high output pressure transducer accurate to within 10 KPa. The pressure was applied through a hand-operated high pressure hydraulic pump and was read directly on a calibrated digital voltmeter. All the strain gauges and L.V.D.T.'s were connected to a computer-based data logger and the dial gauge readings were entered manually into the data-collection program to obtain computer plots of the deflection contours. All data were reduced by computer as the test progressed to give a continuous assessment of the experimental behaviour of the panel.

3.4 Material Properties

The mean elastic modulus of the bricks obtained by testing six bricks in compression was 9400 MPa and the mean mortar prism elastic modulus was 11200 MPa. The mean elastic modulus of the six brickwork prisms was 9100 MPa, and the mean compressive strength was 26.1 MPa, varying between 23.9 MPa and 28.7 MPa. Tests on the six brickwork prisms used to determine bond strength in flexure gave very low results (albeit with considerable scatter) indicating that the assumption of zero flexural bond strength made in the theoretical analysis was not unreasonable.

3.5 Results

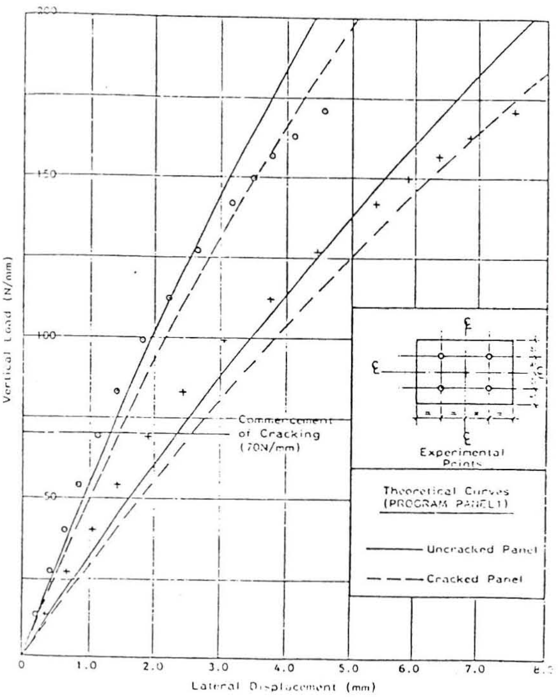
A preliminary load test conducted to check the apparatus and instrumentation showed that the lateral displacements presented the best method of assessing the panel behaviour. It was found that at low loads the load-deformation characteristics were linear throughout the load range but as the loading progressed, the load-deformation relationship became non-linear as the brickwork cracking took place (figure 4). Comparisons between experimental and theoretical displacements showed that the measured values (panel uncracked) were initially less than the calculated values. However, once cracking was initiated, the differences between the measured and calculated displacements decreased.

In a second series of tests, the panel was reloaded and unloaded. The results (figure 5) indicate that the slopes of the uncracked brickwork and cracked brickwork load-displacement curves differed by approximately 20 percent. The calculated ratio was somewhat lower.

The panel was finally loaded to failure which was initiated by a torsion failure at one of the bottom corners followed by lateral buckling of the central region of the panel as the effectiveness of the vertical edge restraints decreased. The lateral displacements measured to the point of failure agreed closely with values calculated by the computer program. The calculated failure load was approximately 173 N/mm length of wall compared with 185 N/mm obtained in the test.

4. PARAMETRIC STUDY

A parametric study was carried out to determine to what extent the load capacity of brickwork walls of different geometry may be increased by restraining the vertical edges against lateral translation.



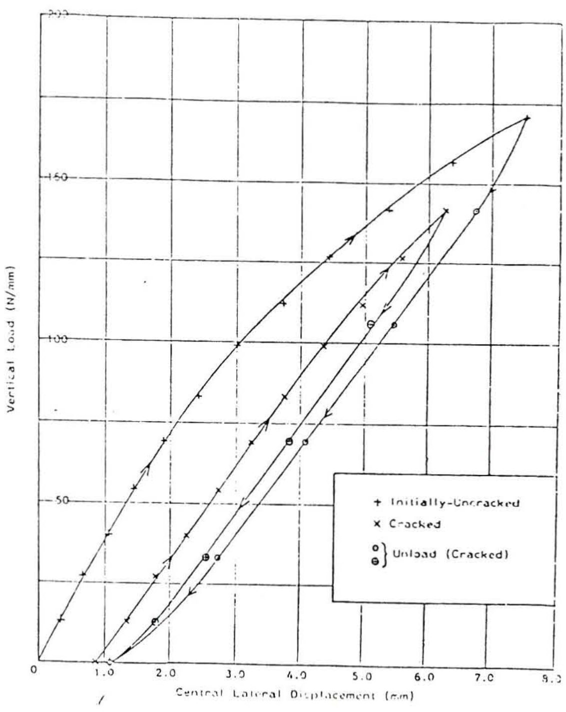


Fig 4

Fig 5

In the study, the material properties were assumed to be as shown in Table 3. The analysis allows for three possible modes of material failure (vertical splitting, flexural failure in bending parallel to the bedjoints and torsional cracking at 45 degrees to the bedjoints) in addition to possible buckling failure. The panels were loaded with equal eccentricities top and bottom and all edges were assumed to be free to rotate. Figure 6 shows calculated failure loads, plotted non-dimensionally for panels 2400 mm high constructed from standard bricks 230 mm x 110 mm x 76 mm laid on edge. It was assumed that for practical purposes the relative bending and torsion stiffnesses for cracked brickwork are those values presented earlier in this paper.

Material Property	
Brick Compressive Strength,	60 MPa
Mortar: Cement:Lime:Sand by Volume	1:1:6
Brick Elastic Modulus,	18000 MPa
Brick Tensile Strength,	3.5 MPa
Brickwork Compressive Strength,	26.0 MPa

Table 3 Material Properties used in Parametric Study on Panels

The calculated panel load capacities are compared in figure 6 with load capacities specified by Australian Standard AS1640-1974. The Code results are similar to the computed panel load capacities for a load eccentricity of d/6 but at eccentricities of d/1000 and d/3, the two sets of values differ considerably. The results show clearly that for panels with aspect ratios length-to-height greater than 0.5, the Code load capacities are conservative and take no account of the reserve of strength in a panel beyond the strength of a wall with the same height-to-thickness ratio.

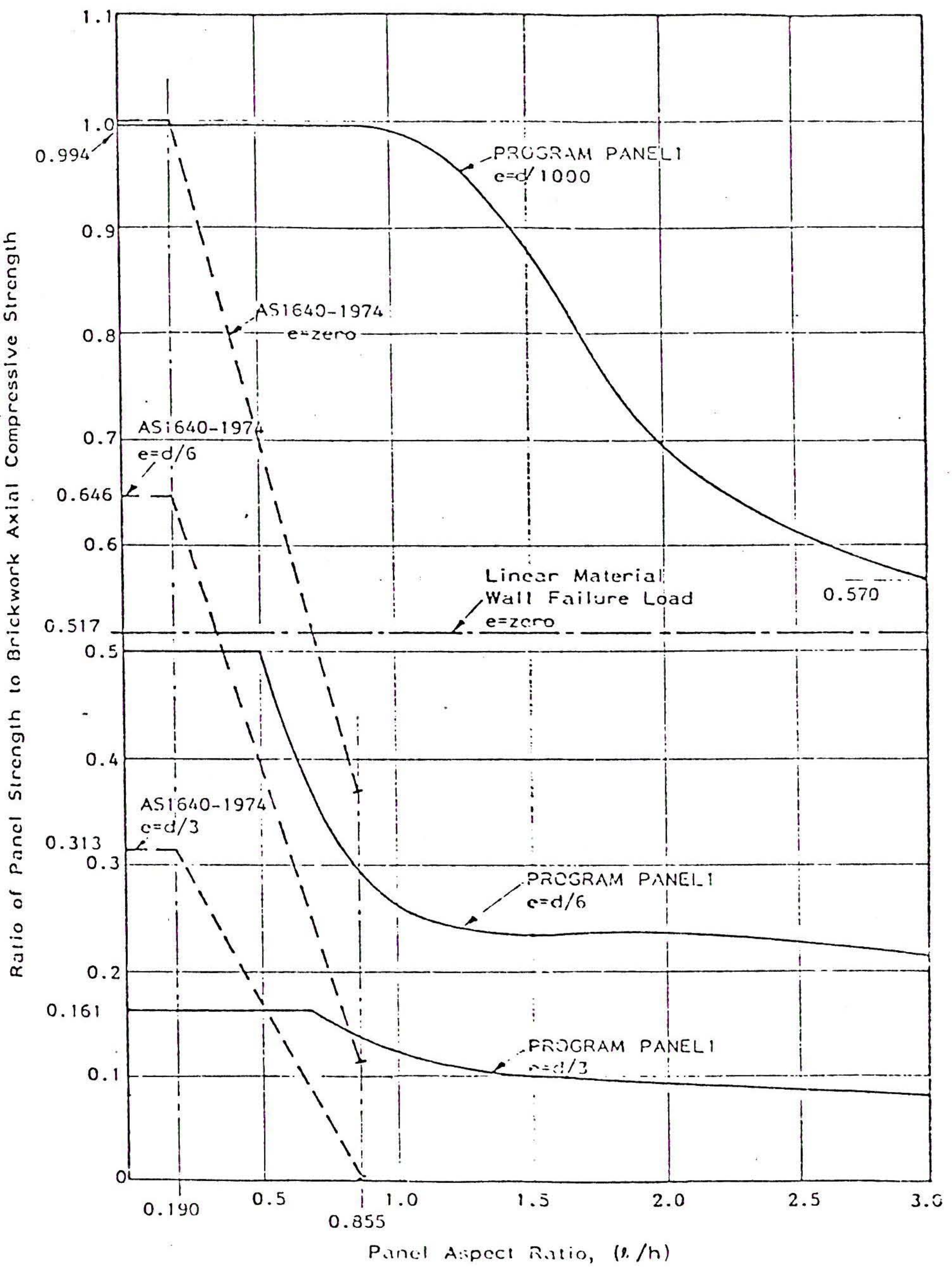


Fig 6 Results of Parametric Study

5. CONCLUSIONS

The behaviour of a rectangular wall panel, supported or restrained along four edges and carrying an eccentric vertical load is modelled. The modelling technique used combines finite element and finite difference calculations.

The accuracy of the model was verified on a full scale (3600 mm long x 2400 mm high) panel. A special frame, with its own loading device was built to apply a maximum 670 kN load, with 20 mm eccentricity, uniformly distributed along the horizontal edges.

The agreement between calculated and measured behaviour was good.

A parametric study was carried out to investigate the strengthening and stiffening afforded by the two way action associated with the support given to the vertical edges of the panel.

The study shows that for panels with aspect ratios greater than 0.5 the Australian Brickwork Code (11) load capacities are conservative and take no account of the reserve strength in a panel beyond the strength of a wall with the same height to thickness ratio.

6. ACKNOWLEGEMENTS

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