N. G. Shrive, Professor
The University of Calgary, Civil Engineering Department, 2500 University Drive, N.W., Calgary, Alberta, Canada, T2N 1N4

ABSTRACT A generalized theory of crack initiation is described, based on the concept of cohesive strength. The theory is shown to explain many experimental data in uni- and multi-axial stress states. In masonry prisms, splitting occurs across faces prone to conditions of plane strain. Capping can have positive or negative effects with respect to stresses in the prism. The presence of mortar joints causes little disturbance to the vertical stresses, lateral confinement of the mortar and small tensile stresses in the units. It is suggested that capping be a thin layer to reduce the effect of prism surface irregularities, and that researchers consider a toughness approach to compressive failure.

1. INTRODUCTION

In current masonry design codes and in limit states design codes under development (eg: 1), compressive strength is an important parameter for design. There are two approaches taken for specifying the masonry design compressive strength. One is to use a table or graph, where the main input variable is the result of a specific test for unit compressive strength, and there is some adjustment for mortar type. The tables and graphs have been developed empirically from experience and extensive experimental testing: they are conservative for most combinations of unit and mortar. The second approach is to test representative samples of the masonry and obtain a value of compressive strength from the results.

When loaded in compression, bricks in a stack-bonded masonry prism usually crack first, parallel to the direction of compression. The observation is widely thought to be the result of the mortar being more flexible than the units. Under compression, the Poisson effect in the mortar is greater than that in the units, which therefore become subject to lateral tension. However, if a column of bricks stacked one on top of another is compressed, bricks will still crack parallel to the direction of compression. The widely accepted mechanism of failure does not provide an answer to this observation. A fundamental explanation of the cause of cracking in compression is necessary if we are to improve masonry design. The work described here is part of a study aimed at understanding the mechanism of failure of masonry and what is measured in a compression test. A large range of strength values can be obtained from such tests, depending on the shape and size of the masonry sample used. For example, tables are provided in codes to allow for variations in height to width ratio of prisms (2). The results of tests also depend on any capping placed between specimen and test machine (3).

Thus there is some difficulty in defining what the real 'compressive strength' is. Which number obtained from which test, table or graph actually indicates the strength of the masonry constructed? The problem has not been solved. We observe phenomena such as 'strain gradient effects' (4, 5) and differences in strength between walls and piers constructed of nominally the same masonry (6); yet we have no satisfactory explanation of such phenomena, nor any consistent method of dealing with the relative compressive strengths.

There are many theories of failure for masonry, most dealing with particular stress states (7), yet a common feature of all failures is cracking. No matter what the overall stress field, cracking is observed prior to and/or during failure. 'Strength' must therefore be related to the cause(s) of crack ini-
tiation and propagation. Cylinders of height to diameter ratio 2:1 of glass, sulphur mortar, concrete, mortar and rock (8, 9) all have similar cracking patterns. Failure of these so-called 'brittle' materials in biaxial compression results from cracking between planes of compression (10, 11, 12). In triaxial compression, states of stress close to hydrostatic produce 'plastic' behaviour in glass (13), concrete (14) and rocks (15). The similarity of behaviour of these materials leads to the conclusion that there is a common cause of crack initiation: each material then has its individual 'toughness' to resist crack propagation.

A generalized theory of crack initiation is that of 'cohesive strength' - once a stress large enough to break interatomic bonds occurs, a crack can initiate. For 'brittle' materials subject to uniaxial tension, both the Griffith criteria (cohesive strength and energy) independently result in strength being inversely proportional to the square root of the length of an infinitely sharp crack: failure usually occurs by the rapid, unstable propagation of a single crack. In contrast, the stresses induced in a specimen in a standard compression test are neither uniaxial nor uniform (3, 16); cracks tend to grow stably. Some form of energy or stress intensity criterion with the different materials having different toughnesses would therefore seem more appropriate; but is not yet available. Nevertheless, the cohesive strength approach has been shown to predict theoretical behaviour remarkably similar to experimental observation (8, 17, 18). Thus, following a brief description of the theory, factors which affect masonry 'strength' as determined from a prism test will be discussed.

The Theorem and Its Application in Simple States of Stress

The cohesive strength approach to the fracture of brittle materials is simply that a stress must be created in the material which is large enough to break interatomic/intermolecular bonds. Examination of the bond force vs bond spacing curve for any directional bond reveals that only interatomic tension can break the bond: shear and compression cannot. Thus, in a material subjected to any stress, a tensile stress equal to the 'cohesive strength' must be created to break bonds and to initiate a crack. It is well recognized that in tension, voids and flaws act as stress raisers. The same is true in compression. Many analyses deal with the stresses around two dimensional voids (eg: 17) but in general such flaws do not occur naturally. Three dimensional voids do, and analysis of the stresses around spherical (16) and ellipsoidal voids (8, 18) is revealing.

An ellipsoidal void is depicted in Figure 1. It has three semi-axes a, b and c in three directions x, y and z. The points A, B and C are on the surface of the void where the semi-axes intersect the solid. Principal stresses in the X, Y and Z directions are applied and stresses determined in the x, y and z directions at each of A, B and C. Cases involving rotation of the ellipsoid axes away from the principal axes have not been examined.

For uniaxial compression in the Z direction, tensile stresses occur in the x and y directions at C. The magnitude of these stresses is between 0.6 and 1.0 of the applied compressive stress. The magnitude depends on the shape of the void and the Poisson's ratio of the solid. Interestingly, when semi-axis a is infinite (the problem thus reverting to one of 2-D) the tensile stress in the y direction at C is independent of Poisson's ratio and of equal magnitude to the applied stress. 2-D analyses are thus seen to be deficient.

If biaxial principal stresses are applied to the solid in directions Y and Z, the location and direction of the dominant tensile stress can change. For compression in the Y direction and tension in the Z, the dominant tensile stress occurs in the z direction at B. Cracking will therefore occur perpendicular to
A triaxial ellipsoidal void in a solid, showing co-ordinate axes and locations of points A, B and C where stresses were determined (from ref. 8).

the z direction, as observed (10, 12). In biaxial compression, the dominant tensile stress is in the x direction at either B or C, depending in which of the Y and Z stresses is the greater in magnitude. Thus cracking will be perpendicular to the x direction, i.e. the exposed plane will be a y-z plane, which is the plane of compression, again as observed experimentally (10, 11, 12). Between the two cases (small Z stress), we would expect a transition between the two failure modes as reported from experiment (10). The theoretical results can be presented in the form of a 'crack initiation' diagram like Figure 2. Here, stress combinations around two differently shaped voids have been increased until a 'cohesive strength' tensile stress occurs at some point around the void. Naturally, crack initiation should be reflected in measured strength: the diagrams are comparable in shape to those reported from experimental work (eg: 10, 19). The ratio of compressive to tensile strength can be increased from a minimum value of 3 by merely changing the shape of the void considered.

In triaxial compression, two interesting points arise. The first concerns cases involving stress combinations with high compression in one direction (Z) and equal (lower) compression in the two orthogonal direction (X and Y). Predictions for combinations giving tensile stresses fit reasonably with observed strength increases (7, 8, 18). Indeed the linear relationship given by the theory for a spherical void in a solid with Poisson's ratio 0.2, is

\[ \sigma_Z = \sigma_c + 4 \sigma_{LAT} \]

where \( \sigma_c \) is the uniaxial compressive stress required to generate a tensile stress at the void equal to the cohesive strength of the solid. This equation compares with empirical equations for concrete:

\[ \sigma_Z = \sigma_{cs} + 4.1 \sigma_{LAT} \] (\( \sigma_{cs} \) = uniaxial compressive strength) (ref. 20),

\[ \sigma_Z = \sigma_{cs} + 4 \sigma_{LAT} \] (for 0 < \( \sigma_{LAT} < 0.75 \sigma_{cs} \)) (ref. 21).

Secondly, when the equal bilateral stresses are above about 90% of the third (axial) stress, then there is no value of Poisson's ratio and no shape of void
A crack initiation diagram for biaxial stress, $\sigma_1$ and $\sigma_2$ for different void shapes and Poisson's ratio. The shape ratios $\rho_1 = \frac{b}{a}$ and $\rho_2 = \frac{c}{b}$, (a, b and c as in Figure 1) define the shape of the ellipsoid (from ref. 8).

It is reasonable to conclude therefore, that the theorem fits many experimental data and observations. The Poisson's ratio of the material and the distribution (in shape, and presumably size) of surface irregularities, internal voids and pore structure, together with the macroscopic stress distribution in the specimen will play important roles in crack initiation and subsequent strength measurement. In tension, the void which is the worst stress intensifier will be the culprit for failure. In compression, since cracks often grow stably, total porosity will perhaps be of more importance since a number of cracks usually develop during failure. In masonry specimens, the macroscopic stresses can vary with the shape and size of the specimen, with capping and from the existence of
mortar joints. These factors are now examined.

Shape and Size of Specimens

The stresses which develop in specimens subject to a standard, supposed uniaxial compression test have been examined before (eg. 3, 9, 16, 22, 23). One platen of a test machine is displaced towards a second, with the specimen between. The platens thus impose a displacement on the specimen and the load created in the specimen is called the 'applied load'. The lateral Poisson strain of the specimen is restricted by friction at the specimen platen interface. The lateral stresses induced in the specimen are compressive near the platens followed by tension (if the specimen is deep enough) (3, 16, 23). Cubes are subject to more restraint for their volume than cylinders of height to diameter ratio 2:1, and will therefore give greater 'strengths' than cylinders. Larger stresses are required for crack initiation in zones of triaxial compression (7, 8, 18).

Prisms of rectangular cross-section show an interesting feature which must be accentuated in a wall restrained along its length top and bottom. The stresses across the narrow width are higher than those observed across the wider length (3). This occurs because there is a tendency for the longer length to be restrained more towards plane strain conditions. Certainly in a restrained wall, there will be little or no strain along the length of the wall in its central portions. The plane strain conditions lead to the higher lateral stresses (3), Figure 3. Hence prisms will split predominantly up the narrow face, as observed. In walls, failure of one section, usually a quadrant, resulted in the wall no longer being able to carry the load (6). The one section often failed with the longitudinal splitting although other cracks were observed (6).

As the height of a restrained prism is increased beyond about 5-high stack-bond (for clay bricks), there will be little further effect due to the height alone (assuming failure is not due to buckling). The non-uniformity in the stress distribution due to platen restraint has essentially disappeared in the central section of the 5-high prism. However, two factors might affect 'measured strength': the stability of any columns formed from extensive crack propagation, and the number of mortar joints (see below). Prisms or cubes would give reasonable estimates of compressive strength if there were no platen restraint.

Considerable attention has been paid to methods of removing the restraint. A few researchers (eg. 12, 17) use brush platens. If the brushes are small enough (but not so small they buckle), and there are sufficient in number in orthogonal directions on the cross-section, then there will be a substantial reduction in the zone of triaxiality at the specimen platen interface. Discontinuities will still exist at the edge of each brush, but the zone of stress disturbance should be small when compared with that caused by solid platens. Brush platens are currently too expensive for the majority of testing laboratories. Hence, consideration must be given to the standard compression test. In that test, capping is used between specimen and platen, sometimes in an attempt to reduce the effects of platen restraint.

Capping

Capping should ensure full contact between specimen and platen - reducing the effect of small irregularities in the surface of the specimen. The effects of larger irregularities can only be reduced by using a capping material of similar mechanical properties to the specimen. The existence of capping generally complicates the stress distributions in the prism considerably (3).
Variation of lateral stress down the centre-line of a prism of height to width ratio of 2:1, for two values of Poisson's ratio, v. The sketch to the left indicates that a tendency to plane strain can be expected across the narrow width, and a tendency to plane stress across the wider length, in a prism of rectangular cross section.

Many types of capping material exist. However, a prism of brick masonry has rectangular cross-section, so the lateral stresses across the two faces are different. Therefore it does not seem possible to find a capping combination (thickness, modulus, Poisson's ratio) which will remove all lateral effects in both directions (3). The capping can reduce or reverse the platen effect in each of the two orthogonal directions. For example, strains measured on cylinders near the capping/cylinder interface, were greater than the 'free' Poisson value when thin sheets of neoprene or teflon were used as capping (22). For a specimen of a particular stiffness, there appears to be a particular thickness of capping of given material properties which will minimize the platen effect (3). The effect will only remain a minimum if the capping maintains its behaviour until failure or follows the non-linear behaviour of the masonry (one may question here whether non-linear behaviour is an artefact of stable crack growth in a restrained specimen). Hence, a series of thicknesses of a capping material would be needed for masonry of different combinations of brick and mortar - unwieldy for a code. A single thickness may 'minimize' the lateral effect for one type of masonry but not others.

Thus one may conclude that capping would be helpful if it only reduced the effects of surface irregularities. Then only the problem of what is 'strength' or 'toughness' from a platen restrained test would require solution. At the moment, capping merely appears to complicate the problem.
Mortar Joints

Mortar joints are known to affect the distribution of stress in masonry. Different theories have been proposed using mortar joints to account for the 'lateral tensile splitting' of masonry in a compression test (eg: 24, 25). However, prisms of masonry crack in similar overall patterns to similarly shaped prisms of concrete (7) and rock (23). Clearly the mortar joints can not be the sole cause of cracking. Nevertheless, in masonry prisms one or more bricks usually crack first with subsequent crack propagation through mortar. Since the presence of mortar joints does induce small lateral stresses in both the mortar and the brick, these stresses must influence those which develop around voids in each of the two materials. Crack initiation will depend on which material reaches its cohesive strength first. The measured masonry strength will depend on subsequent crack propagation through both materials. Atkinson and Noland (25) agree with these latter two statements, finding that in uniaxial compression, stresses in the units reach unit cohesive strength, but stresses in the mortar do not reach mortar cohesive strength. With other applied stress states, cracking along head and/or bed joints can occur (eg: 12). These planes of weakness could be accommodated in a cohesive strength theory by considering stresses at locations around the surface of the void appropriate to the plane of weakness and the applied principal stresses.

The lateral tensile stresses which develop in the units from unit/mortar interaction vary in both distribution and magnitude with location in the prism. 2-D plane stress analysis will not reveal these variations. Further, the lateral tensile stresses are small (Figure 4) compared with those which develop around voids (0.6 to 1.0 times the vertical stress). 'Average' stress values, calculated using a simple 3-D analysis (6) and assuming isotropy in one joint and one unit are not unreasonable estimates of the stresses at point A in the prism (Figure 4). The stress distribution at the edges (B and C) are more variable. Only in the brick nearest the platen does the lateral tensile stress drop away from the simple analysis value. Recalculating the simple analysis values on the basis of there being four units but only three joints in the prism does not appear to lead to better estimates of the lateral stresses.

The lateral compressive stresses in the mortar are about one tenth the vertical stress for the material properties used. Thus the mortar can be expected to both stiffer and stronger than if subjected to uniaxial compression. It is worth noting that the simple analysis (6) shows that the lateral stresses are not linearly dependent on the differing material properties. Increasing the lateral Poisson effect of the mortar (by decreasing its stiffness) to six times that of the unit only slightly more than doubles the average values shown in Figure 4. The confining stress in the mortar can thus be a reasonable proportion of the vertical stress, but the lateral tensile stresses in the units remain small - much smaller than the tensile stresses which develop at voids.

The vertical stress distributions also change through the units and mortar, (Figure 5). The distortion of the finite elements used in the model around the edges of the mortar joint (curves a, b, and c, Figure 5) indicate that the estimates of stresses (and the curves drawn therefrom) will not be accurate in that region. The reduced stress at the very edge, with the increased stress just inwards, indicates that the mortar/unit interface does not remain plane. Shear deformation occurs. The unit does not impose the same restraint on the mortar as steel platens are assumed to do to a specimen. The shear deformation would be reduced as the stiffness of the unit increases relative to the mortar. There would be a concomitant change in the vertical stress distributions towards those imposed by a rigid platen. The maximum change in vertical stress here is only about 6% from the stress in a uniform prism subject to the same load. In a stack bond prism then, the joints do not cause gross disturbance to the axial...
Variation with height of lateral tensile and compressive stresses in bricks and mortar respectively, at different locations in a four-high prism. The stresses were determined from a finite element analysis of one-eighth of a prism (shown to the left, with positions A, B, C and D marked). The boundary conditions were an imposed displacement on top of the prism, with no platen restraint. What is seen is the effect of mortar joints alone. The stresses have been normalized against the unit stress which would occur in a similar uniform block subject to the same load. Comparisons are shown with results from a simple 3-D analysis (6). Material properties were 'average' values of Young's moduli of 7 GPa (units) and 4 GPa (mortar) with Poisson's ratios of 0.2 (ref. 26).
stress field. The lateral confinement of the mortar is the most significant effect.

The thickness of the mortar joint may be expected to affect the magnitude of the lateral stresses. The simple 3-D analysis (6) shows that varying the thickness of the joint has greater effect on the tensile stress in the unit than on the confining stress in the mortar (Figure 6). This is because the mortar, being more flexible than the unit, shows little change in stress for small changes in lateral strain. The small strain changes have greater effect on the stiffer unit. Figure 7 shows data for a series of tests with different joint thicknesses, normalized against the strength with the 3 mm (nominal) thick joints. The variation of average lateral tensile stress is shown for comparison and obviously can not be the lone cause of failure. The triaxial behaviour of the unit subject to compression-tension-tension will dictate crack initiation in the unit. Using differently shaped voids and the 'average' stresses calculated from the simple 3-D analysis (6), gives a range of unit crack initiation curves (based on cohesive strength requirements).

A problem with application of the cohesive strength approach is revealed here. What shape of void should be used for analysis with which stresses in the prism? There might not be a void at a particularly highly stressed point in a prism, and the shape of the void will affect the magnitude of the tensile stress which develops around it.

Finally, the old practice of using thin mortar joints is advantageous from four points of view, two indicated here - it increases the confining stress in the mortar and will thus increase mortar strength, and it decreases the lateral tensile stress in the unit, reducing its influence on crack initiation. The other two reasons are that thinner joints reduce rain penetration, and bond strength increases considerably as joint thickness is reduced (27).
Figure 6
Variation of lateral stresses with the ratio of mortar joint thickness ($h_m$) to brick height ($h_b$). The values were calculated using the simple 3-D analysis of reference (6), with the material properties used in the analysis shown in Figures 4 and 5.

Figure 7
Variation of strength of 5-high prisms (5 units, 4 joints) with mortar joint thickness, normalized against the strength of the prisms with 3 mm (1/8 in.) joints. Values show mean and standard deviation of 6 tests. The lower curve is for variation of strength based on variation of lateral tensile stress in the units alone: the upper curves are for cohesive strength with differently shaped voids subject to stresses calculated from the simple 3-D analysis (6). $\rho_1$ and $\rho_2$ are as defined in Figure 2.
CONCLUSION

The cohesive strength approach to the failure of brittle materials subject to compression shows considerable promise. Consistent explanation of a vast quantity of experimental results can be provided. A problem lies in the choice of void shape considered, since a wide range of theoretical predictions are possible. Rather than studying pore size and shape distribution, it is thought that a stress intensity approach might be more profitable. Can an 'average' void dimension for a material be defined from a compression toughness, determined from experimental results? This 'average' void dimension would then be applied through stress intensity to other structural situations.

With respect to masonry in particular, prism tests should be able to provide reasonable estimates of compression strength or toughness. Capping should be a thin layer to reduce the effect of surface irregularities: other capping appears to complicate the stress distributions unnecessarily. The major effect of mortar joints is the development of confining stresses in the mortar. Variations to the vertical stress distribution are small, as are the lateral tensile stresses in units. These latter influence crack initiation in units but are not the major cause thereof. A return to the practice of using thin joints would be beneficial.

ACKNOWLEDGEMENTS

The research has been supported by the Natural Sciences and Engineering Research Council of Canada and the Canadian Masonry Research Council. Countless discussions with many colleagues are acknowledged. Reg Rodney made the prisms, Jim Stein helped analyse the finite element data, Manal El-Rahman computed the stresses at the voids: many thanks.

REFERENCES

(8) El Rahman, M., Shrive, N.G., "Understanding the Cause of Cracking in Concrete: A Diagnostic Aid", In press, J. of American Concrete Institute.


