INVESTIGATIONS ON THE STRESSES IN MASONRY WALLS SUBJECTED TO CONCENTRATED LOADS

RECHNERISCHE UNTERSUCHUNGEN ÜBER DIE BEANSPRUCHUNG VON GEMAUERTEN WÄNDEN UNTER KONZENTRIERTEN LASTEN

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ABSTRACT. Concentrated loads occur in walls, e.g. at the bearing surfaces for window lintels or under beams and columns. If only small areas of the wall cross-section are subjected to loading, then it is normal to calculate with increased permissible stress forces as partial area load. The thus resulting tensile forces in homogeneous material cannot be just applied to masonry walling with its inhomogeneous structure. Loading with partial area load for homogeneous walling as well as for masonry walling is investigated. A failure theory is developed and compared with test results. The investigation was carried out for numerous loading cases, especially for partial area loads at wall centre and at the edge of the wall. The favourable effect of horizontal support, e.g. by floor slab, ring beam or by the loaded beams themselves, is shown.

ZUSAMMENFASSUNG. Konzentrierte Lasten treten bei Wänden z.B. am Auflager von Fensterstürzen oder unter Balken und Pfeilern auf. Falls nur kleine Teile des Wandquerschnitts belastet werden, ist es üblich, mit erhöhten zulässigen Spannungen als Teilflächenpressung zu rechnen. Die hieraus entstehenden Spannungen in homogenem Material können nicht ohne weiteres auf Mauerwerk mit seiner inhomogenen Struktur übertragen werden. In der Arbeit wird die Beanspruchung unter Teilflächenlast sowohl für homogene als auch für gemauerte Wände rechnerisch untersucht. Es wird eine Bruchtheorie entwickelt und mit Versuchsergebnissen verglichen. Die Untersuchungen wurden für mehrere Lastfälle durchgeführt, insbesondere für Teilflächenlasten in Wandmitte und am Rand der Wand. Die günstige Auswirkung von horizontaler Abstützung, z.B. durch Deckenscheibe, Ringanker oder durch die belastenden Unterzüge selbst, wird dargestellt.

NOMENCLATURE

c/b	Load Width / Wall Width
a	Edge Spacing of Partial Area
$\sigma_{x,o}$	Horizontal Tensile Stress in Brick from Transverse Strain Difference
$\sigma_{x,1}$	Horizontal Tensile Cracking Stresses in Brick
σ_y	Vertical Pressure
p_R/p_U	Cracking Load / Failure Load on Partial Area
$\beta_{W,R}/\beta_{W,U}$	Cracking Load / Failure Load for Load Width = Wall Thickness
$\beta_{z,St}$	Brick Strength
$\varkappa = p_U/\beta_{W,U}$	Increase Factor for the Acceptable Partial Area Load
$k_z = \beta_{z,St}/\beta_{W,U}$	Relationship Factor

1. INTRODUCTION

The loadbearing capacity of masonry walls is determined by tests whereby the walls are subjected to a constant load over their full width. If the loading is not applied over the full wall width, but instead is concentrated over a partial area, then the acceptable stress force is altered. This effect is known from tests and applies analogously to masonry walling as well as to other building materials, e.g. concrete.

In the following, a theory is developed for the behaviour of masonry walling subjected to partial area loading, enabling subsequent calculation of test results and their interpretation to be made as well as producing findings for further studies. A distinction is made between loading at wall centre and at the edge of the wall, as the position of the partial area loading is of great influence on the loadbearing capacity.

The theory takes into account the specific loading behaviour of masonry walling: The tensile cracking forces resulting from the spreading of the partial area load can only be transferred via the bricks and not via the vertical butt joints. Furthermore the vertical pressure produces horizontally directed tensile stresses in the bricks as the transverse strain of bricks and mortar are not the same. The tensile forces which result from these two effects lead to cracking of the bricks; this leads to failure of the wall.

2. PARTIAL AREA AT CENTRE OF WALL

2.1. WALL CONSTRUCTED USING HOMOGENEOUS MATERIAL

Firstly the simpler case of a wall in accordance with Fig. 1a constructed in homogeneous material is investigated. The tension trajectories for partial area loading are shown in Fig. 2a. The horizontally directed tensile cracking forces $\sigma_{x,1}$ are given in Fig. 1b dependent on the loading width c. Evaluating the curves, the maximum tensile cracking stress can be approximately formulated as follows, in accordance with Fig. 1c:

$$\max \sigma_{x,1} \cong p \cdot \frac{c}{b} \cdot 0.41 \left(1 - \frac{c}{b}\right) \qquad (1)$$

The depth y_1, where the maximum value of $\sigma_{x,1}$ occurs, can be given approximately from Fig. 1b by:

$$\frac{y_1}{b} \cong 0.3 + 0.25 \cdot \frac{c}{b} \qquad (2)$$

The wall will crack when max $\sigma_{x,1}$ exceeds the tensile breaking strength of the material.

2.2 MASONRY WALL

2.2.1. TENSION CONDITION IN THE WALL

Two influences must be taken into account for a masonry wall compared to a homogeneous wall: The tensile cracking forces are interrupted by the vertical butt joints; also tensile stresses occur in the bricks caused by varying transverse strain between bricks and mortar. The tensile forces occurring in the homogeneous wall cannot therefore be applied as such to masonry walling.

The tensile cracking stresses $\sigma_{x,1}$ cannot run continuously over the height of the wall as the vertical butt joints cannot take up tensile forces. Only the bricks transfer tensile forces. As bricks only exist in every second course in the area of the butt joints, the tensile cracking forces in the bricks must be doubled compared with equation (1) for the first approximation.

In order to determine the stress forces more accurately, masonry walls of bricks with varying sizes were investigated with the aid of an electronic program [2]. The stress trajectories are shown in Figs. 2 b - 2e. Fig. 3 also shows the tensile cracking stresses $\sigma_{x,1}$ in the wall axis. Evaluating these results, the maximum tensile cracking stress in the bricks, analogous to equation (1), can be given by the following:

$$\max \sigma_{x,1} \cong p \cdot \frac{c}{b} \cdot k_s \cdot \left(1 - \frac{c}{b} \right) \tag{3}$$

The determination of factor k_s is problematic. This is examined more closely in 2.2.3.

Due to varying transverse strain between brick and mortar, tensile stresses $\sigma_{x,o}$ occur in the bricks, proportional to pressure σ_y at the examined point, given by:

$$\sigma_{x,o} = k_o \cdot \sigma_y \tag{4}$$

The factor k_o can be determined from wall tests with c/b = 1: Under cracking load of the wall $\sigma_y = \beta_{W,R}$, the breaking strength of the brick $\sigma_{x,o} = \beta_{z,St}$ is attained. Thus:

$$k_o = \frac{\beta_{z,\, St}}{\beta_{W,\, R}} \tag{5a}$$

The influence of the tensile stresses from transverse strain differences perpendicular to the wall plane (three-dimensional stress condition) is contained by the factor k_o.

The first crack naturally does not lead to failure of the wall; the load can be increased until fracture takes place. The evaluation of tests, described in[1], has led to following relationships of cracking load p_R to failure load p_U for normal small bricks:

$$\frac{p_R}{p_U} = 0{,}8 - 0{,}3 \cdot \frac{c}{b} \tag{6}$$

For the wall loaded over the full area with c/b = 1, this signifies that $\beta_{W,R} = 0.5 \cdot \beta_{W,U}$. Thus (5a) can also be written as

$$k_o = 2 \cdot \frac{\beta_{z,\,St}}{\beta_{W,U}} = 2\,k_z \tag{5b}$$

with

$$k_z = \frac{\beta_{z,\,St}}{\beta_{W,\,U}} \tag{5c}$$

For the evaluation of (4), the vertical compressive stress σ_y is required. The assumption made in practice is that the partial area load within the wall body spreads to both sides with a slope of 1:2. A good degree of concurrence was attained using this assumption compared with more exact calculation results for inhomogeneous walling. Thus follows:

$$\sigma_y\,(y) \cong \frac{p}{1 + y/c} \geq \frac{c}{b} \cdot p$$

Taking the position $y = y_1$ of the maximum tensile cracking stress from (2), then one obtains for the pressure at this point

$$\sigma_y\,(y_1) \cong \frac{p}{1{,}25 + 0{,}3 \cdot b/c} \geq \frac{c}{b} \cdot p \tag{7}$$

2.2.2. CRACKING LOAD AND FAILURE LOAD

The hatched brick shown in Fig. 4 is investigated at Sections I and II. It is subjected to maximum tensile cracking stress at depth y_1. Courses positioned above, especially in the direct load introduction area, are more favourable as large horizontal pressures act here in the wall plane (see Fig. 1b), which favourably influences the 3-dimensional stress condition. For this reason the tensile stresses acting perpendicular to the wall plane caused by transverse strain differences in the load introduction area are of less importance and are not given further consideration in this paper. In contrast, tensile forces act on the hatched brick not only in the wall plane but also perpendicular thereto. This unfavourably influences the three-dimensional stress condition, and is thus given further regard in the following.

Section I runs through the butt joints, so that the brick has to accept the full tensile cracking stress $\sigma_{x,1}$ acting on it. The tensile stress $\sigma_{x,o}$ resulting from the transverse strain is approximately zero in this section, as σ_y is very small in the area of the butt joints, so that no appreciable transverse strain differences occur. This is verified by more exact calculation of a small wall section as shown in Fig. 5.

In Section II however $\sigma_{x,o}$, as shown in Fig. 4, is fully effective. The tensile cracking stress is however distributed over 2 bricks as there is no butt joint, so that in this section only $\sim 1/2 \cdot \sigma_{x,1}$ acts on the examined brick. Thus the cracking criteria at position $y = y_1$ are given by

Section I: $$\sigma_{x,1} = \beta_{z,St} \tag{8a}$$

Section II: $$\frac{1}{2}\sigma_{x,1} + \sigma_{x,o} = \beta_{z,St} \tag{9a}$$

Inserting the cracking tension forces $\sigma_{x,1} = p_R \cdot \frac{\sigma_{x,1}}{p}$ and $\sigma_{x,o} = p_R \cdot \sigma_{xo/p}$ then one obtains the cracking load p_R.

Section I: $$p_R^I = \frac{\beta_{z,St}}{\sigma_{x,1/p}} \tag{8b}$$

Section II: $$p_R^{II} = \frac{\beta_{z,St}}{\frac{1}{2}\sigma_{x,1/p} + \sigma_{x,o/p}} \tag{9b}$$

The object of the investigation is to determine the relationship of partial area load p_U to wall strength $\beta_{W,U}$. The increase factor $\varkappa$ is therefore defined:

$$\varkappa = \frac{p_U}{\beta_{W,U}} = p_R \cdot \frac{p_U}{p_R} \cdot \frac{1}{\beta_{W,U}} \tag{10}$$

Inserting (8b) and (9b) here, and taking into account p_U/p_R according to (6), $\sigma_{x,1}$ according to (3), $\sigma_{x,o}$ according to (4) and (5b) as well as σ_y from (7), one obtains the increase factors $\varkappa$ for Sections I and II:

$$\varkappa^I = \frac{k_z}{k_s} \cdot \frac{1}{(0{,}8 - 0{,}3 \cdot \frac{c}{b}) \cdot \frac{c}{b} \cdot (1 - \frac{c}{b})} \tag{11}$$

$$\varkappa^{II} = \frac{k_z}{k_s} \cdot \frac{1}{(0{,}8 - 0{,}3 \cdot \frac{c}{b}) \cdot \left[\frac{1}{2} \cdot \frac{c}{b} \cdot (1 - \frac{c}{b}) + \frac{k_z}{k_s} \cdot 2 \cdot \underbrace{\frac{1}{1{,}25 + 0{,}3\, b/c}}_{\geq c/b}\right]} \tag{12}$$

The smaller of the two $\varkappa$-values is decisive. For the boundary value $c/b = 1$ $\varkappa^{II} = 1.0$ follows from (12), i.e. $p_U = \beta_{W,U}$. For $c/b = 0$ one obtains $\varkappa = \infty$.

2.2.3. EVALUATION AND COMPARISON WITH TEST RESULTS

The evaluation of the derived equations was carried out for $k_S = 0.6$ (maximum average values of tension forces in the bricks in accordance with Fig. 3), $k_S = 2 \cdot 0.41 = 0.82$ (double value of tension forces in the homogeneous wall, according to equation (1), and for $k_S = 1.14$ (maximum tension force peaks in the bricks in accordance with Fig. 3). Here 0.2 was inserted for $k_Z = \beta_{Z,St}/\beta_{W,U}$. The results are given in Fig. 6. It became apparent that for $k_S = 0.82$ and for $k_S = 1.14$, $\varkappa$-values < 1 partly resulted. These results do not correspond with the test results described in [1] for walls constructed using normal small-sized bricks with $k_z = 0.2$, where $\varkappa > 1$ was always obtained.

The evaluation of the equations for $k_S = 0.6$ with $k_Z = 0.2$, 0.3 and 0.4 are shown in Fig. 7. This figure also gives the test results mentioned above. The path of the increase factor for the partial area pressure in accordance with the German 'Mauerwerksnorm' (Masonry Standard) DIN 1053, Part 2, is also represented. These $\varkappa$-values obtained from tests are given by the following in the here used nomenclature:

$$\varkappa = 1 + 0.1 \cdot \frac{a}{c} \leq 1.5 \tag{13}$$

One can see that the $\varkappa$-values from DIN are on the safe side compared to the test results.

Also a good degree of consistency is shown between the equations and the test results according to [1] , when using the obtained maximum average values of 0.6 for k_S. A theoretical reason for this could not yet be found. Possibly tension peaks, as calculated according to the elasticity theory, do not occur in this size due to tension re-arrangement in the vicinity of the failure condition.

Finally, it is pointed out that horizontal reinforcement in the bed joints in the area of large σ_x-stresses can, in this case, improve the loadbearing capacity, as the horizontal tensile strength of the wall is thus improved. This effect, which is verified by the tests according to [1] , is readily explained by the presented theoretical investigations.

The investigations are valid for walling constructed in normal small-sized bricks. Additional considerations are necessary for large-sized elements, i.e. blocks.

3. PARTIAL AREA AT EDGE OF WALL

3.1. SIGNIFICANCE OF HORIZONTAL SUPPORT

It is of decisive importance for partial area loading at the edge of the wall if a horizontal support can be provided. If no support is possible, then the resulting force also acts vertically within the wall. If a support is possible, e.g. by means of walls facing one another or a retaining ring beam, then the acting load with the support force forms an inclined resulting force, which forms much more favourable loading conditions within the wall. The conditions are shown in fig. 8. The tension trajectories in a homogeneous panel which cannot transfer tensile stresses at its lower edge are also shown here; the consequence of a support is clearly seen.

3.2. WALL WITHOUT HORIZONTAL SUPPORT

If neither a horizontal support nor retaining ring beam are present, then no substantial load spread in the masonry wall is possible as the acting force in its line of action runs vertically. This is demonstrated by the tension trajectories in Fig. 9a. The tensile stresses, especially in the binding bricks, can lead to a breaking off of the bricks so that only a kind of column remains for load transfer with a width of that of the loaded partial area. In this case a stress increase is not permissible, i.e. $\varkappa = 1.0$ is valid. These theoretical considerations were fully verified by the test results according to [1] : The failure stresses p_U are of the same size as the wall strength $\beta_{W,U}$. The $\varkappa$-values from equation (13) of the German 'Mauerwerksnorm' (Masonry Standard) DIN 1053, Part 2, take these into account.

For an edge spacing distance of the loaded area a = 0, one obtains $\varkappa = 1.0$ in (13).

A horizontal reinforcement of the joints should then be of advantage if it can apply a support force A and can alter the system shown in Fig. 8a into a system as shown in Fig. 8c. A short reinforcement inserted in the joints which does not represent an anchorage, should not be of any great effect. Related investigations are however not known.

3.3. WALL WITH HORIZONTAL SUPPORT

Figures 9b and 9c show the trajectories in a masonry wall with horizontal support. The inclination of the results from acting load and support force allows a load spread without large tensile stresses. No tests have been carried out for this case in [1]. The trajectories show however that similar tension increases $\varkappa$ as for partial area loading at wall centre are likely.

LITERATURE:

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[2] Schwing, H. and Rutemöller, U.: Benutzeranleitung für das Programmsystem ZEFE (Zusammenwirken von einzelnen Fertigteilen als großflächige Scheibe)
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[3] Schulenberg, W.: Theoretische Untersuchungen zum Tragverhalten von zentrisch gedrücktem Mauerwerk aus künstlichen Steinen unter besonderer Berücksichtigung der Lagerfuge
Dissertation, Darmstadt 1982

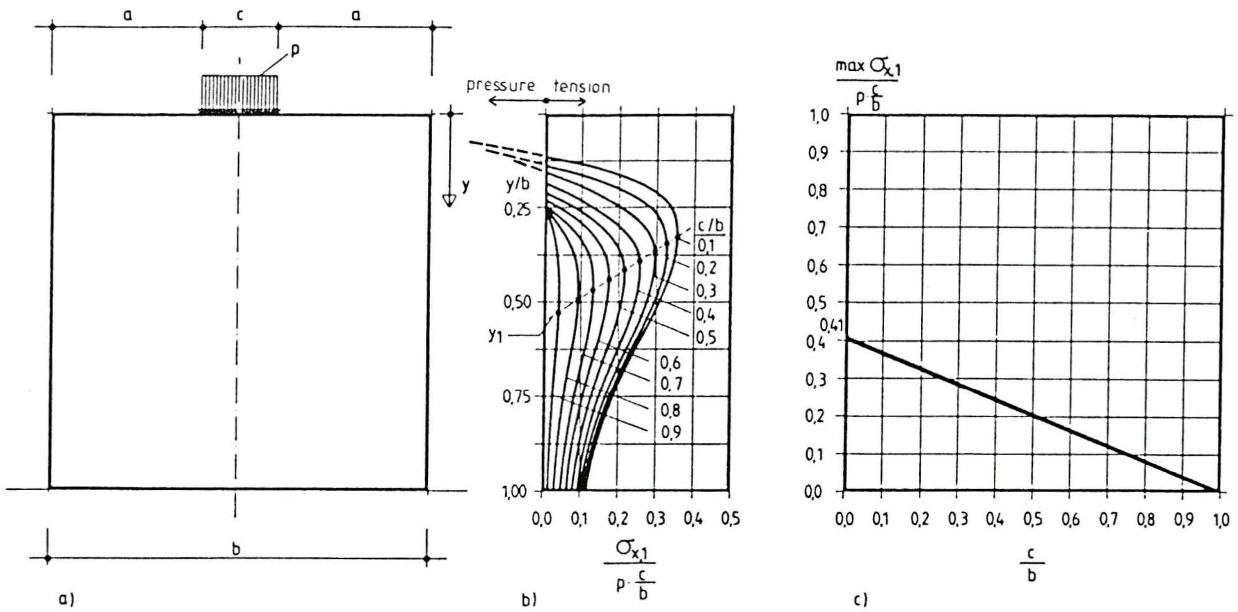

Fig. 1: a) Wall System Subjected to Partial Area Load
b) Horizontal Tensile Cracking Stresses $\sigma_{x,1}$ in Wall Axis
c) Maximum Tensile Cracking Stress $\sigma_{x,1}$ dependent on c/b

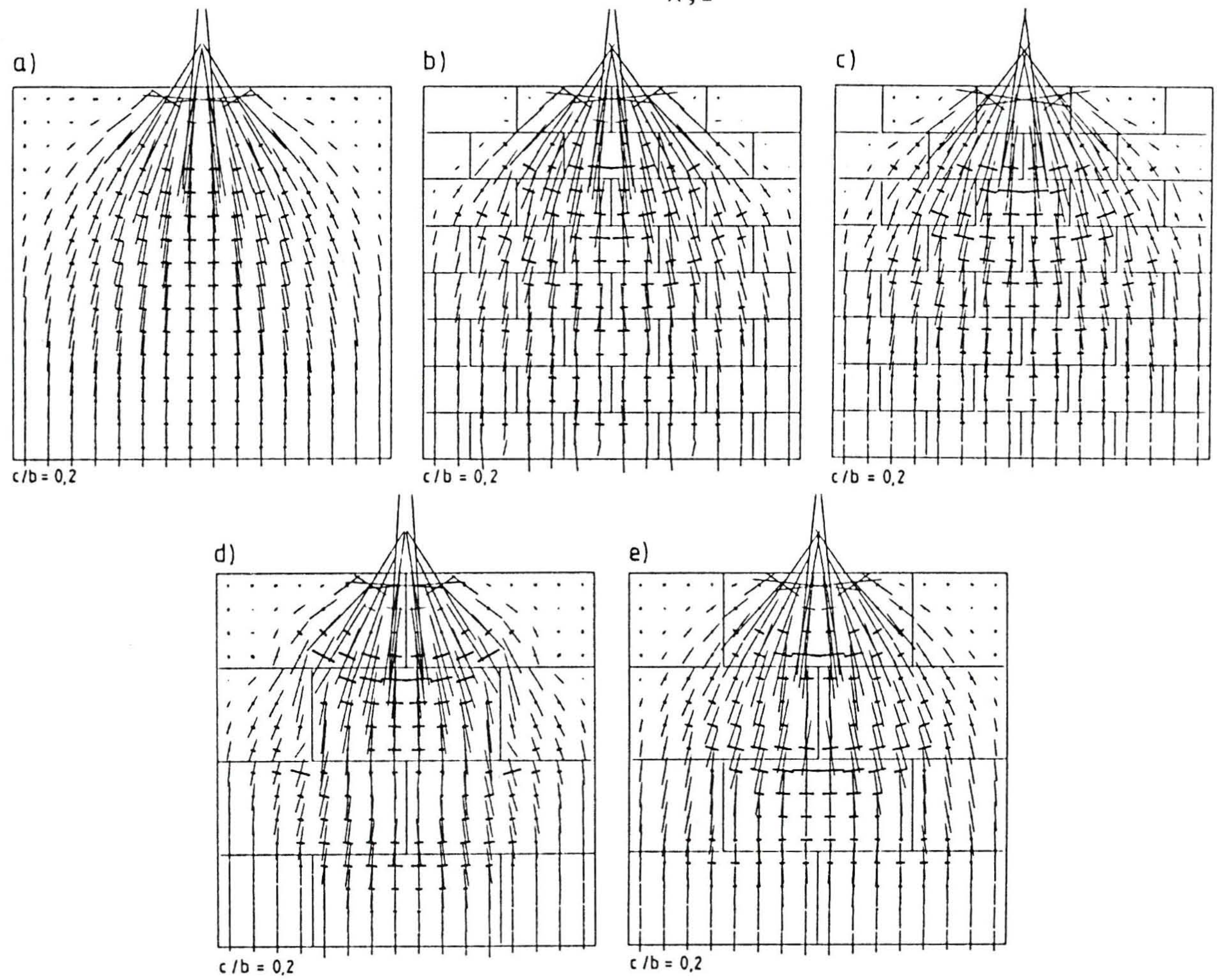

Fig. 2: Stress Trajectories for Partial Area Load
a) homogeneous wall, b) - e) masonry walls

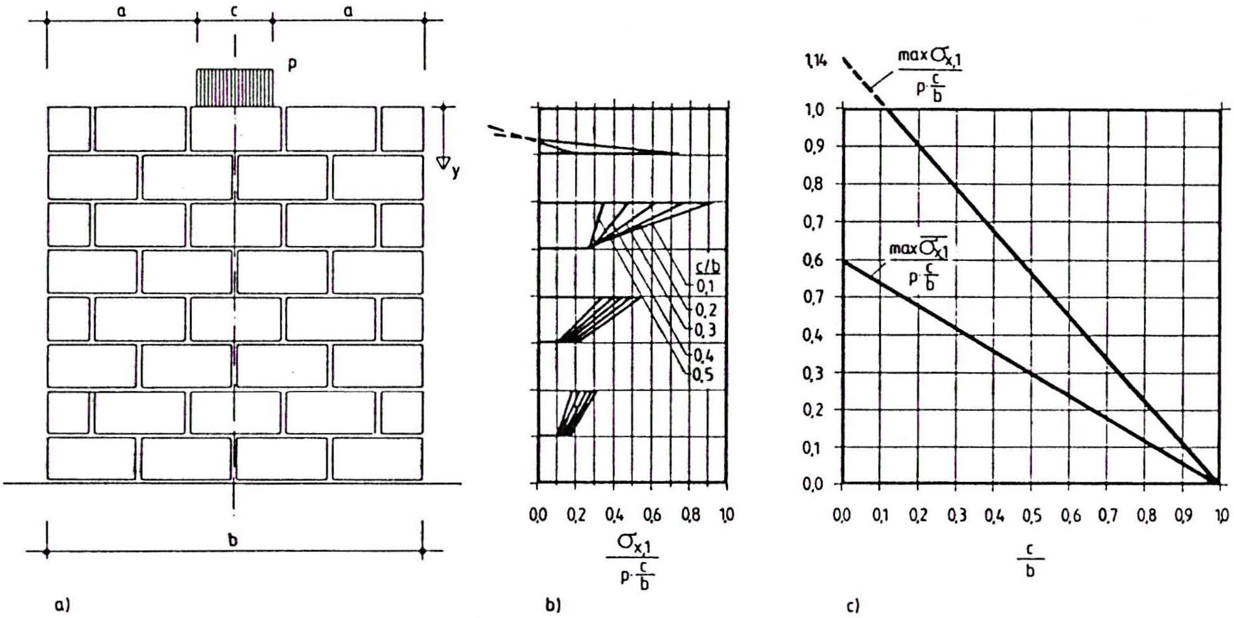

Fig. 3 : a) Wall System Subjected to Partial Area Load

b) Horizontal Tensile Cracking Stresses $\sigma_{x,1}$ in Wall Axis

c) Maximum Tensile Cracking Stress $\sigma_{x,1}$ dependent on c/b

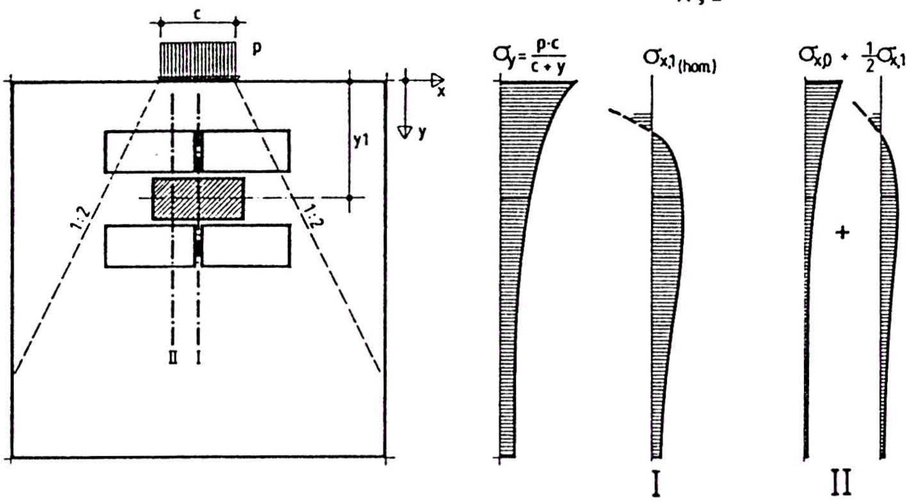

Fig. 4: Stress Forces in the Sections I and II

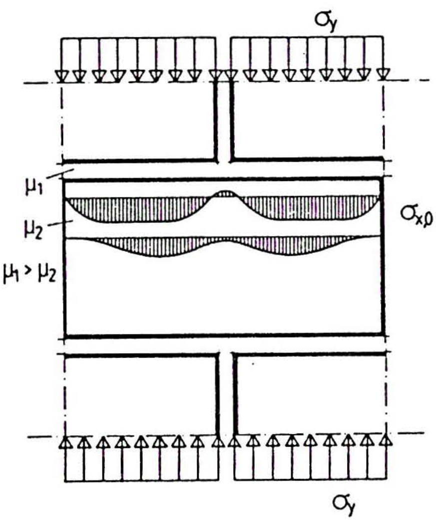

Fig. 5: Path of Horizontal Tensile Stress $\sigma_{x,o}$ in Brick from Transverse Strain Differences

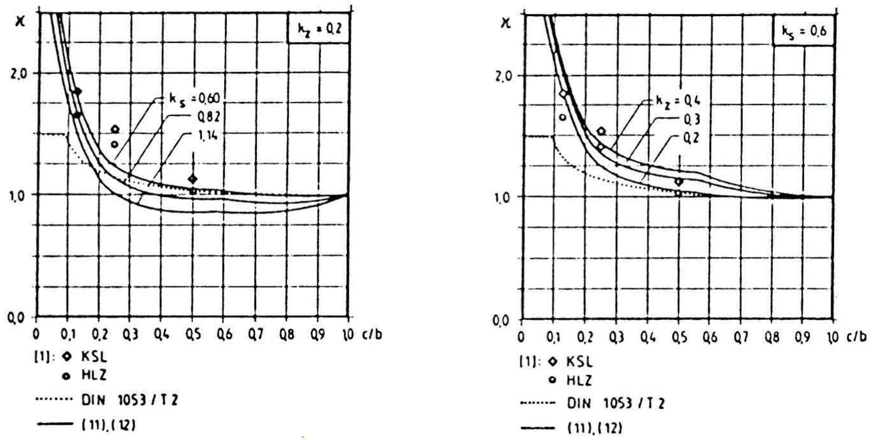

Fig. 6/7: Increase Factors $\varkappa = p_U \,/\, \beta_{W,U}$ from Theory, Tests and DIN 1053, Part 2

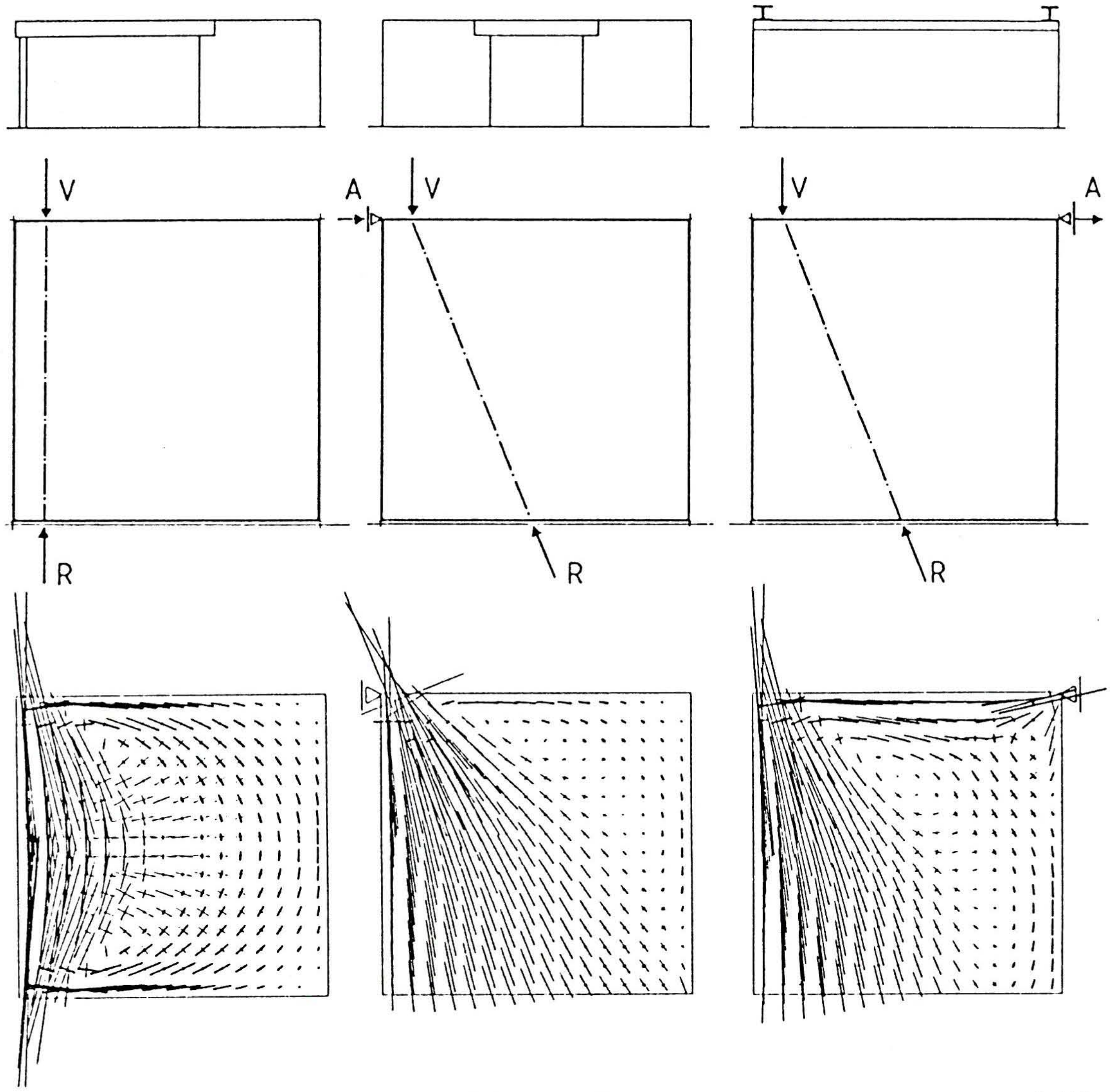

Fig. 8: Partial Area Loads at Edge of Wall for Systems Without and With Support Forces A
Construction, System and Trajectories in Homogeneous Wall

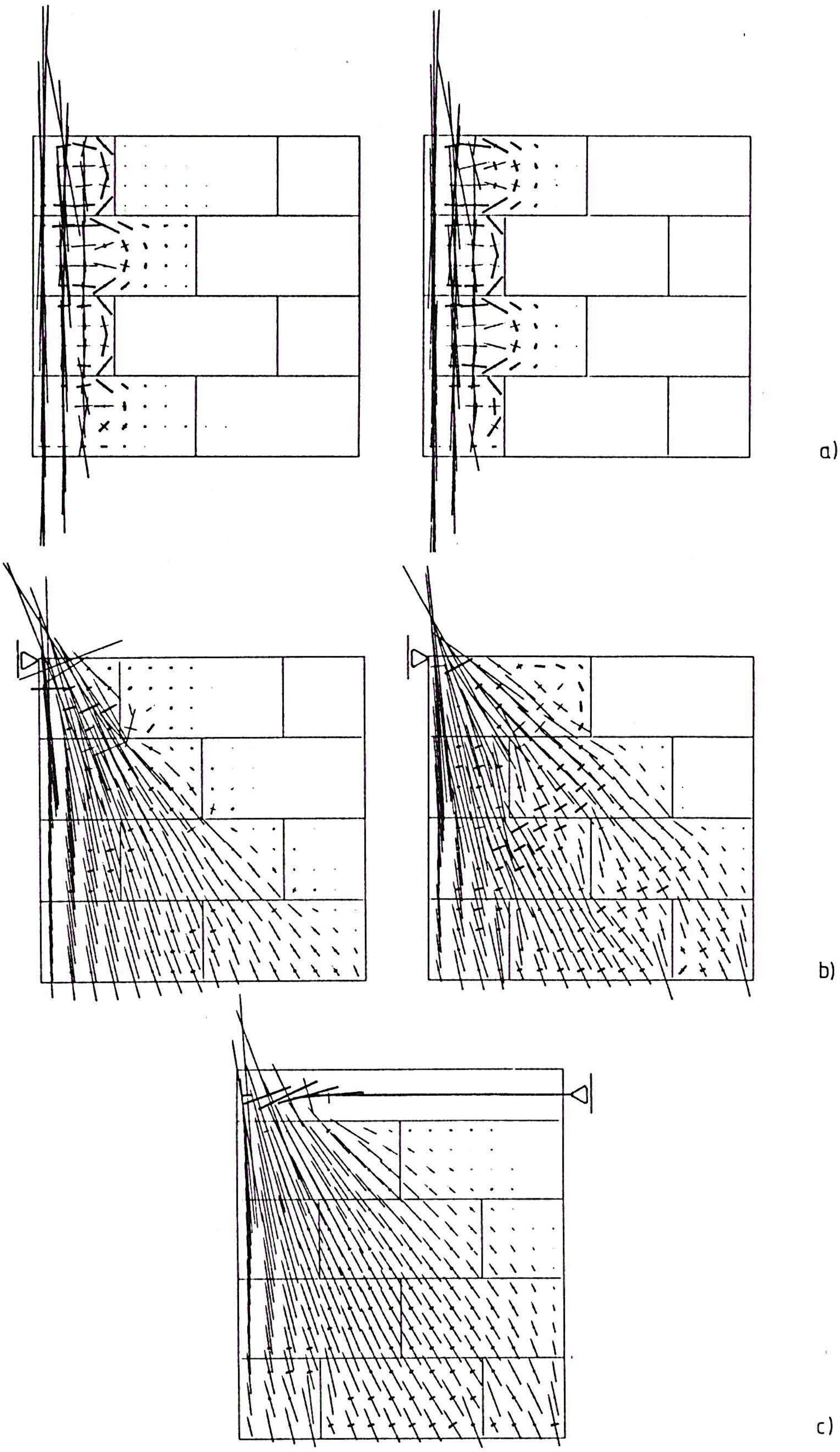

Fig. 9: Trajectories in Masonry Walls for Partial Area Loads at Edge of Wall
a) without support
b) and c) with horizontal support