HEAT CONDUCTANCE DETERMINATION OF HOLLOW LIGHTWEIGHT CLAY BLOCKS BY THE FINITE ELEMENT METHOD

C. BONACINA, A. STEVAN and M. STRADA
Institute of Technical Physics
Faculty of Engineering
PADUA University Italy

ABSTRACT

In this paper the finite element method is applied to the calculation of steady state heat transfer through a wall composed of hollow building lightweight clay tiles. The input data necessary for these calculations are examined with main regard to the determination of the heat transfer in the cavities of the brick. The evaluation of the equivalent conductivity of the cavities was carried out taking into account both the convection and the radiation fluxes, utilizing experimental and numerical correlations. The conductances calculated were compared with some experimental ones obtained in different laboratories. In the paper the isotherms inside the hollow bricks are shown so that it is possible to draw interesting conclusions on the influence of the thickness of the walls and of the cavities. From these considerations an optimized design of hollow lightweight clay blocks can be easily performed.

INTRODUCTION

The finite element method is widely used in the field of structural investigation where by now, most of the planning and design is carried out by ever more reliable, efficient and cheaper computers.

Such method can just as easily be used to resolve thermal problems especially in the case of complex shapes which would call for excessive simplification should analytic models be used.

The present paper utilizes finite element method for the thermal analysis of the cavities in hollow tile blocks,
comparing, at the same time, the results with experimental data already published.

The calculus code used in the study is based on eight node isoparametric elements which automatically gives the value of the coordinates of the median nodes and can deal with different thermal boundary conditions.

GEOMETRIC AND THERMOPHYSICAL CHARACTERISTICS OF THE STRUCTURE

A certain type of block (A) in particular was examined, chosen from among the most common used in Italy. The block measured 25 x 11.5 x 17.5 cm, the shape of the internal cavities being as shown in Fig. 1. The blocks were studied together with the vertical mortar joints, arranging them as shown in Fig. 1, in order to realize the predetermined thickness of the wall.

Figure 1. Wall composed with hollow blocks type A.

A second type of block (B1, B2) represents, on the other hand, the final result of a series of numerical experiments carried out in order to determine how the heat insulation of the structure could be improved.

The blocks measured 34 x 17 x 25 cm, having a particular T shape which allows for the building of a wall using only blocks and without, therefore, any need to use vertical mortar joints as can be seen from Fig. 2.

Figure 2. Wall composed with hollow blocks type B1 and B2.
A horizontal section, bounded laterally by the adiabatic surfaces which corresponded to the symmetrical planes was also examined. The configuration of the structure is such that the heat flow can be regarded as nil over all the horizontal sections in that the problem can, therefore, be considered a two dimensional one.

With reference to the evaluation of the thermophysical properties which should be assigned to the matrix of the blocks, it was decided to take the official values reported in the current Italian standards (UNI CTI FA 101) as a reference point, even though some experimental data were available.

Table 1 indicates the values utilized in the calculation.

<table>
<thead>
<tr>
<th>Type of block</th>
<th>Density ($\text{kg/m}^3$)</th>
<th>$kd$ (W/mK)</th>
<th>$kp$ (W/mK)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A - B1</td>
<td>1800</td>
<td>0.63</td>
<td>0.81</td>
</tr>
<tr>
<td>B2</td>
<td>1250</td>
<td>0.34</td>
<td>0.57</td>
</tr>
</tbody>
</table>

$kd$ is the conductivity of the dry material
$kp$ is the conductivity used in the calculation:

it takes into account the actual building conditions (moisture, horizontal mortar joints).

**THERMAL ANALYSIS OF THE CAVITIES**

The internal configuration of each block can be drawn up by means of an even series of rectangular cavities separated by flat layers of tile material.

The cavities and the tile layers were laid down vertically and are placed in series, perpendicular to the direction of the heat flow in that the orthogonal surfaces to the outside surfaces of the layer of blocks were assumed to be adiabatic.

The layers of mortar which, under actual working conditions, lie along the upper and lower surfaces of the blocks, are, therefore, equal to the height of the same block.

In the model used for numerical analysis, the basic components of the structure, that is the internal and surrounding tile layers which is where thermal conduction takes place, were separated from the vertical cavities where convective and radiative heat exchange occurs. The total heat transmitted is, therefore, the sum of these two mechanisms.

However, in order to make the elaboration of the model simpler and quicker to develop it is more convenient to assimilate each cavity into an equivalent conductive layer of the same thickness whose equivalent conductivity can be determined.
The equivalent conductivity is thus composed of two terms, one for each mechanism:

\[ k_{\text{eq.tot}} = k_{\text{eq.conv.}} + k_{\text{eq.rad.}} \]  

where:

- \( k_{\text{eq.conv.}} \) is the equivalent convection conductivity in the cavity
- \( k_{\text{eq.rad.}} \) is the equivalent radiation conductivity in the cavity.

The convective flow was calculated as indicated in [2] in the case of closed vertical cavities with flat parallel walls:

\[ q_{\text{conv.}} = h_{\text{conv.}} (t_{p1} - t_{p2}) \]  

where:

- \( h_{\text{conv.}} \) is the average convection coefficient in the cavity, and \( t_{p1}, t_{p2} \) are the temperatures at the active surfaces of the cavity. And:

\[ \frac{h_{\text{conv.}}}{k_a} = \frac{l}{\text{Nu}} \]  

where:

- \( \text{Nu} \) Nusselt’s number for the cavity
- \( l \) thickness of the cavity
- \( k_a \) thermal conductivity of the cavity fluid

The equivalent conductivity can therefore be defined by:

\[ q_{\text{conv.}} = \frac{k_{\text{eq.conv.}}}{l} (t_{p1} - t_{p2}) = h_{\text{conv.}} (t_{p1} - t_{p2}) \]  

that is:

\[ k_{\text{eq.conv.}} = k_{\text{conv.}} l \]  

or:

\[ \frac{h_{\text{conv.}}}{k_a} = \frac{l}{\text{Nu}} \]  

To calculate the convection inside closed flat cavities, adimensional correlations are currently in use which state that \( \text{Nu} \) is related to the characteristic parameters of the cavity, that is:

\[ \text{Rayleigh’s number: } Ra = \frac{l^3 g D t}{n \ a} \]
The following correlation has been used in the present study:

\[ \text{Nu} = 0.258 \text{Ra}^{0.248} \text{AR}^{-0.20} \]  

(8)

as advised in [3] where Ra < 3.1 \times 10^4

The radiative heat flow in the cavities has been calculated by considering them as indefinite flat parallel walled interspaces. This model can be applied if the high values of vertical form factor are taken into consideration.

The radiative flow is therefore [2]:

\[ q_{rad} = \frac{\text{sn} (T_{p1}^4 - T_{p2}^4)}{1/e_1 + 1/e_2 - 1} \]  

(9)

where:

- \text{sn} is the constant of radiation of the black body (5.67 \times 10^{-8} \text{ W/m}^2\text{K}^4)
- \text{T}_{p1}, \text{T}_{p2} are the absolute temperatures of the surfaces of the cavity;
- e_1, e_2 are the total hemispheric emission of the surface.

Table 2 shows, together with the geometric values of the various cavities which relate to the blocks, the Ra and Nu numbers and the partial and total equivalent thermal conductivity.

<table>
<thead>
<tr>
<th>Type of cavity</th>
<th>a</th>
<th>b</th>
<th>c</th>
<th>d</th>
<th>e</th>
</tr>
</thead>
<tbody>
<tr>
<td>h (mm)</td>
<td>175</td>
<td>175</td>
<td>250</td>
<td>250</td>
<td>250</td>
</tr>
<tr>
<td>l (mm)</td>
<td>26</td>
<td>94</td>
<td>10</td>
<td>11.5</td>
<td>28.5</td>
</tr>
<tr>
<td>AR</td>
<td>6.7</td>
<td>1.9</td>
<td>25</td>
<td>21.7</td>
<td>8.8</td>
</tr>
<tr>
<td>Ra</td>
<td>7051</td>
<td>17627</td>
<td>251</td>
<td>381</td>
<td>15089</td>
</tr>
<tr>
<td>Nu</td>
<td>1.59</td>
<td>2.56</td>
<td>0.53</td>
<td>0.61</td>
<td>1.82</td>
</tr>
<tr>
<td>(k_{eq.,conv.}) (W/m²K)</td>
<td>0.041</td>
<td>0.066</td>
<td>0.026</td>
<td>0.026</td>
<td>0.040</td>
</tr>
<tr>
<td>(k_{eq.,rad.}) (W/m²K)</td>
<td>0.016</td>
<td>0.420</td>
<td>0.044</td>
<td>0.050</td>
<td>0.127</td>
</tr>
<tr>
<td>(k_{eq.,tot.}) (W/m²K)</td>
<td>0.157</td>
<td>0.486</td>
<td>0.070</td>
<td>0.076</td>
<td>0.174</td>
</tr>
</tbody>
</table>

The temperatures of the inside and outside air were fixed respectively at:
The average temperature in the block was, therefore, \( t_a = 7.5 \, ^\circ C \). All the thermophysical properties of the air in the cavities relate to this temperature. Therefore, the data in [2] are as follows:

\[
\begin{align*}
\bar{b} &= 0.00356 \, \text{K}^{-1} \\
\bar{n} &= 15.713 \times 10^{-6} \, \text{m}^2/\text{s} \\
\bar{a} &= 0.2216 \times 10^{-4} \, \text{m}^2/\text{s} \\
\bar{k} &= 0.026 \, \text{W/mK}
\end{align*}
\]

The temperature difference was distributed over the cavities only, thus obtaining values of 4 °C, 10 °C, 2.5 °C, 2.5 °C and 6.5 °C for cavities a, b, c, d and e respectively. The values shown above, calculated from equations (7) and (8), give the Rayleigh and Nusselt numbers in Table 1 from which the equivalent convective conductivity can be deduced.

It should be noted that for smaller cavities \( \text{Nu} < 1 \). Convection under such circumstances is therefore reduced to pure conduction in the layer of air, such that \( \text{Nu} = 1 \).

In reality, the cavities within the blocks have different average temperatures. Thus, strictly speaking, they should be calculated individually, applying to each of them the thermophysical properties of the fluid at the average temperature pertaining to each cavity.

But the weak relationship between the thermophysical properties of the air and the temperature, together with the values given by the total thermal jump imposed on the block means that the differences between the various cavities are negligible and justify using the average temperature of the entire block.

If then the average thermal conditions (\( t_m = 7.5 \, ^\circ C \)) are applied to the block, the following temperatures are found at the surfaces of the cavities: a (283 K, 279 K), b (286 K, 276 K), c and d (282 K, 279.5 K), e (284 K, 277.5 K).

If the value \( e_1 = e_2 = 0.94 \) is assumed for the emission then from (9), the radiative flow can be determined and consequently the equivalent thermal conductivity for the radiative exchange in the cavities, as defined by the equation:

\[
d_{\text{rad}} = \frac{k_{\text{eq.rad.}}}{1} (t_{p1} - t_{p2})
\]

With reference to the thermal radiation, strictly speaking, the single cavities should also be dealt with individually, because the various internal surface temperatures are different.

An accurate estimation of this influence indicates however only a modest variation in the equivalent conductivities with respect to the values shown in Table 2; it is, therefore, justifiable that the conditions found in
the intermediate cavity of the block are referred to also for the radiative component of the thermal exchange throughout the cavity.

The last line of Table 2 shows the total equivalent thermal conductivity used in the finite element calculation.

The comparative examination of the equivalent thermal conductivity values in Table 2 indicates that smaller cavities are more efficient in terms of insulation.

For low values of the thickness of the cavity convective eddies are not established and the thermal flow is of the pure conductive type through the layers of air.

The radiative flow rather than convective flow always prevailed and is reduced because of the effects of the inner dividing walls in that they act as radiation screens. Their number in fact increases as the thicknesses of the cavities decreases.

**MESHES AND ISOTHERMS**

The subdivision of the whole into elements, conditioned by the arrangement of the cavities and their dividing walls, gives rise to meshes which have the following characteristics:

<table>
<thead>
<tr>
<th>Type of block</th>
<th>Numbers of Elements</th>
<th>Numbers of Nodes</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>195</td>
<td>642</td>
</tr>
<tr>
<td>B1 - B2</td>
<td>216</td>
<td>719</td>
</tr>
</tbody>
</table>

The isotherms were calculated automatically by the ISOPLOT post-processor linked to finite element code.

The difference in the temperature values of two consecutive isotherms was equal to 1/10 of the temperature difference between the faces, that is 2.5 °C.

**CALCULATION OF THE THERMAL CONDUCTANCE**

The finite element program gives the thermal flows on the internal and external surfaces of the structure.

Given the average thermal flow \( q \) for the internal and external surfaces, the thermal conductance of the wall can be found from the equation:

\[
q = \frac{C}{S \times Dt} \quad (11)
\]

Table 3 shows the thermal conductance values \( C_d \) for a dry matrix and the thermal conductance \( C_p \) which relates to
the real completed wall.

### TABLE 3

<table>
<thead>
<tr>
<th>Block Type</th>
<th>Density (kg/m²)</th>
<th>kd (W/mK)</th>
<th>kp (W/mK)</th>
<th>Cd (W/m²K)</th>
<th>Cp (W/m²K)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>1800</td>
<td>0.63</td>
<td>0.81</td>
<td>1.410</td>
<td>1.599</td>
</tr>
<tr>
<td>B1</td>
<td>1800</td>
<td>0.63</td>
<td>0.81</td>
<td>0.885</td>
<td>1.043</td>
</tr>
<tr>
<td>B2</td>
<td>1250</td>
<td>0.34</td>
<td>0.57</td>
<td>0.618</td>
<td>0.831</td>
</tr>
</tbody>
</table>

A comparison of the thermal conductance of A and B1 (the matrices having the same thermophysical characteristics) shows a notable improvement in thermal quality as a result of a more accurate design of the geometric configuration of the cavities and vertical joints.

A comparison of the thermal conductance of B1 and B2 (having the same shape) demonstrates that a a porous matrix is better.

Therefore, to optimize the thermal conductance of block built walls, certain rules should be followed:

- accurate design of the cavities and their dividing walls
- using material which have low density (porous tile).

For B2 type block wall, the global coefficient of heat transfer obtained numerically was compared with the experimental value in accordance with the A.S.T.M. C 236 method and resulted in being equal to 0.71 W/m²K.

This latter value refers to the wall inclusive of plaster (1.5 cm thick, k = 0.9 W/m K) and takes into account the adduction coefficients at the internal and external surfaces (a1 = 7.8 W/m² K and a2 = 15.4 W/m²K respectively). The conductance realized numerically (C = 0.815 W/m² K) was therefore modified to include the effects of adduction resistance and the plaster layers, obtaining a value of 0.70 W/m²K which entirely agrees with the experimental data.

The trend of the isotherms relating to the B2 type block wall is shown in Fig. 3. It should be particularly noted that there were no areas of high values for the thermal flow density (thermal bridges) due to the effects of the inner dividing walls.
CONCLUSIONS

The finite element method allows for the numerical calculation of thermal conductance of tile blocks once their geometry and type of matrix material has been specified.

The reliability of the code used and its suitability for complex shaped structures, such as those examined in the current paper, means that it may be considered a very useful tool to use when studying the thermal insulation of buildings.

The flexibility of the method makes it easy to carry out comparative studies of cavities of different sizes together with dividing walls having different thicknesses and thermophysical properties.

The heat fluxes in the cavities have been drawn up considering separately the contribution of thermal convection and thermal radiation and evaluating the respective equivalent conductivity. In determining the convective contribution use was made of a recent and reliable experimental correlation.

It is possible therefore, at the design stage, to optimize the thermal insulation characteristics of hollow blocks before beginning mass production.
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REFERENCES

