ESTIMATING LATERAL DRIFT OF REINFORCED MASONRY STRUCTURES

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ABSTRACT

This paper discusses a variety of methods for estimating lateral deflections of reinforced masonry building structures subjected to earthquake motions, and provides correlations with measured dynamic response of two shaking-table test structures. Lateral drift is estimated using several different computational methods ranging from the conventional to the state of the art. For small-amplitude motions, recorded drifts are compared with those computed on the bases of linear models. These include simple and approximate pier models, frame models and finite element models. For nonlinear response, an approximate method is used for estimating drift maxima which is based on an equivalent linear stiffness for the complete building system. Also, a method is presented for computing nonlinear dynamic response histories of multi-degree-of-freedom of systems in terms of a single generalized coordinate.

INTRODUCTION

Since the evolution of engineered masonry structures, the primary design constraint has been one of allowable stresses. Even with newly developed strength design approaches, constraints are still expressed in terms of permissible forces. Recent experiments, including the one presented in this paper, have demonstrated that masonry can be designed to dissipate energy through inelastic straining of reinforcement. If this concept is true, the amount of lateral strength that is provided becomes an issue which is then but secondary to the amount of lateral deflection that can be tolerated. With such a seismic design philosophy, it becomes important to know how to calculate lateral drifts with a good degree of accuracy. Because lateral drift has not been regarded as a constraint in the past, there are no standards for computation of it. The purpose of this paper is to explore the precision of several drift estimation methods. Brief descriptions of a number of calculation methods are given which are followed by comparisons with measured response of two reduced-scale structures that were excited by simulated earthquake motions on a shaking table.

EXPERIMENTAL STUDIES

Two, three-story masonry building structures were constructed for the sole purpose of exciting them to failure with a series of simulated earthquake motions. Each test structure consisted of two perforated shear walls which resisted earthquake motions from a single direction parallel with their plane. One-quarter scale, hollow-core concrete blocks (51 x 51 x 102 mm, or 2 x 2 x 4 inch) were used which were fully grouted, and reinforced with annealed No. 11 gage wire. The percentage of vertical reinforcement was nominally 0.15%. The spacing of horizontal reinforcement varied, but in no case was the percentage less than 0.10%. As shown in Fig. 1, one structure (RM1) had a symmetrical pattern of openings while the other (RM3) had an asymmetrical pattern of openings. Each structure had flanges on both ends of its walls (overhanging width equal to 0.305 m or 12" for RM1, and 0.203 m or 8" for RM3). Both structures were designed so that vertical reinforcement would yield at a specific design basis earthquake. However, lighter and much more intense earthquake motions were input to each
structure to examine all ranges of possible response. The weight per story was 13.1 kN (2950 lbs.) for RM1 and 12.7 kN (2850 lbs.) for RM3.

Properties of the reduced-scale masonry were obtained from conventional prism tests. Compressive strengths were 8.38 MPa (1215 psi) for RM1, and 8.47 MPa (1228 psi) for RM3. Elastic moduli, $E_m$, were inferred from initial tangents of the measured stress-strain curves, and were 4970 MPa (720 ksi), and 6280 MPa (910 ksi) respectively for the two structures. The shear modulus, $G$, was taken to be 40% of $E_m$. The wire used as model reinforcement had a mean yield stress equal to 325 MPa (47.1 ksi), and a diameter equal to 3.1 mm (0.121 inch). Detailed descriptions of modeling procedures are in References 1 and 3.

Fig. 1 Description of Test Structures

Fig. 2 Displacement response spectra (1 in. = 25.4 mm)
Each structure was subjected to a base motion that resembled in sequence the ground motion measured at El Centro, California during the 1940 event (NS direction). The time scale was compressed by a factor of 2.5 so that frequencies of the model base motions would be consistent with that of the natural frequencies of the reduced scale structures. Amplitudes of base accelerations were also scaled to result in a variety of intensities. Displacement response spectra were computed from measured base accelerations. As a sample, spectral curves are shown in Fig. 2 for the last test runs of each structure. Curves have been plotted for damping percentages equal to 2%, 5% and 10%.

Measurements consisted of relative deflection of each of the three levels with respect to the base, and absolute accelerations. In addition, distortions of each pier were measured with vertical and diagonal displacement transducers. Lateral drift percentages were determined by dividing the deflection measured at the top level by the overall structure height which was equal to 2.36 m (93 inch). Further information on the experimental results can be found in References 2-5.

**CALCULATION MODELS FOR LATERAL DRIFT**

A number of various computational methods are tested for their accuracies in estimating lateral drifts of the two test structures. The methods that are studied include fairly simple and conventional approaches as well as relatively new and sophisticated ones. Results of linear elastic methods are correlated with low-amplitude dynamic response measurements. In addition, results of two approximate methods for determining nonlinear dynamic response are correlated with large-amplitude drifts which were measured during intense shaking.

**Static Linear Models for Small-Amplitude Response**

Three common analytical models were studied for calculation of lateral drift: (a) a simple pier model, (b) a more elaborate frame model and (c) an extensive finite element model. Each model was used to calculate overall drift of both structures for each test run. A set of equivalent static lateral forces were applied in an inverted triangular distribution. Forces were scaled so that the base shear was equal to the peak base shear measured during a particular test run.

For the **pier model** the flexibility of the system is defined by flexural and shear distortions of each pier in a story. The rotations at the top and bottom of each pier are restrained and the story stiffness is calculated as the sum of the stiffnesses of each pier in the story:

\[
k_{\text{tot}} = \sum k_i = \sum \left( \frac{12E_mI}{h^3} + \frac{GAv}{1.2h} \right)
\]

where \(E_m\) and \(G\) are Young's modulus and the shear modulus of the masonry and \(I\), \(A_v\) and \(h\) are the moment of inertia, shear area and height of the pier, respectively.

The **frame model** is an adaptation of what is commonly used for concrete frames. The ETABS program was used which assumes that the intersections between the spandrels and the piers are rigid. The piers were modeled as columns with heights equal to their clear heights. The "rigid zones" encompassed the masonry and portion of the floor slab directly above and below each pier. Spandrel beams above and below the openings were modeled with portions of the floor slabs with beam elements.

For the **finite element model**, a coarse mesh of linear shell elements was used to depict the lateral stiffness of each story level as shown in Fig. 2 (see Ref. 6). The walls, flanges and
slabs were modeled with the same four-noded rectangular shell element which had six degrees of freedom per node. The "FINITE" computer program was used on an Apollo workstation.

![Fig. 2 Finite element models](image)

The stiffness of each model is reflected by the calculated initial frequency (based on gross-section properties) which is given in Table 1. The initial frequency estimated by the analytical models always exceeded the frequency of the structures measured in free vibration before the first test run. The finite element model provided the best representation of the initial frequency for both test structures. Some difference in frequency may be attributable to slight amounts of shrinkage cracking, and the flexibility of the simulator platform.

The stiffness of each element was "cracked" in accordance with crack patterns observed before each test run. For the pier and frame models this was done directly by using the cracked section properties for all runs after the first one. The finite elements were "cracked" by assigning an equivalent modulus of elasticity based on the cracked moment of inertia \( (E_{cr} = (l_{cr}/l_0)E_m) \). Deflection estimates are correlated with experimental values in Table 2.

It is not reasonable to expect the three linear models to provide reliable estimates of lateral drift for post-yield response (Run 4 for RM1, and Run 6 for RM3). For these runs, measured drifts exceeded 2.5 times the FEM drifts and 10 times the drifts calculated with the pier model. For later test runs, a portion of the measured drift was attributable to sliding along
Table 2: Correlation of measured and calculated drift maxima

<table>
<thead>
<tr>
<th>Structure</th>
<th>Run</th>
<th>( V_b/W )</th>
<th>Lateral Drift, %</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Measured</td>
<td>Pier Model</td>
</tr>
<tr>
<td>RM1</td>
<td>1</td>
<td>0.41</td>
<td>0.03</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>0.72</td>
<td>0.09</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>0.97</td>
<td>0.19</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>1.65</td>
<td>1.06</td>
</tr>
<tr>
<td>RM3</td>
<td>1</td>
<td>0.38</td>
<td>0.04</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>0.72</td>
<td>0.17</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>0.92</td>
<td>0.41</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>0.99</td>
<td>0.49</td>
</tr>
<tr>
<td></td>
<td>6</td>
<td>1.47</td>
<td>1.32</td>
</tr>
</tbody>
</table>

bed joints which was not included with any of these models. For smaller amplitude test runs, however, correlations were also poor. For example, the pier model predicted lateral drifts during Run 1 which were less than 1/7 of those measured. The frame model also underestimated the measured drifts for the early test runs. The FEM model had the best correlation, but measured drifts still exceeded estimates by as much as 75% for the these low-amplitude test runs. All three models were inherently too stiff.

Equivalent Linear Stiffness Method for Large-Amplitude Response

Although response in latter test runs was obviously inelastic, and thus nonlinear, response waveforms could be characterized in terms of modal parameters such as natural frequencies and modal shapes which are usually only identified with linear systems. To illustrate this correlation, measured deflection at the top level is compared with response calculated on the basis of a linear single-degree-of-freedom oscillator with an identical frequency to that measured of structure RM3 during the last test run (Fig. 3). The good correlation in sequence for the two waveforms suggests that lateral drift can be estimated using a linear model if an accurate method is available for calculating the frequency of the nonlinear system.

Fundamental frequencies of each test structure were observed to decrease with progressive damage as shown in Fig. 4. In the figure, frequencies are plotted versus the peak lateral drift experienced during a particular test run.
The trend of the decrease in frequencies was similar to the trend in the reduction in square roots of average stiffnesses. From measured hysteresis relations of base moment versus top-level deflection, average stiffness was taken as the slope of a line drawn between deflection extremes for any particular test run. For Structure RM1, the square root of the ratio between final average stiffness and initial stiffness was 0.24. The ratio of final and initial measured frequencies was 0.27. For Structure RM3, the respective ratios were 0.26 and 0.27. This close correlation infers that fundamental frequency of a nonlinear system can be approximated by taking an average stiffness based on the slope of a line drawn between peaks of the largest hysteresis loop.

On the basis of the last two findings, it may be plausible that drift can be estimated based on an equivalent linear stiffness. To check this, drift maxima have been estimated on the basis of linear displacement response spectra (Fig. 2) and measured frequencies (taken from Fourier Spectra of top-level measured accelerations), and are correlated in Table 3 with measured drifts. Because measured deflected shapes were nearly constant for all amplitudes of motion, a constant modal participation factor equal to 1.29 was used to relate displacements of the oscillator to that measured at the top level. Damping has been arbitrarily assumed at 10%.

<table>
<thead>
<tr>
<th>Structure</th>
<th>Run</th>
<th>Frequency (Hertz)</th>
<th>$S_d$ (mm)</th>
<th>Estimated Drift, %</th>
<th>Measured Drift, %</th>
<th>Estimated Measured</th>
</tr>
</thead>
<tbody>
<tr>
<td>RM1</td>
<td>2</td>
<td>9.5</td>
<td>2.5</td>
<td>0.14</td>
<td>0.09</td>
<td>1.54</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>7.9</td>
<td>5.1</td>
<td>0.28</td>
<td>0.19</td>
<td>1.46</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>3.6</td>
<td>38.1</td>
<td>2.09</td>
<td>1.06</td>
<td>1.97</td>
</tr>
<tr>
<td>RM3</td>
<td>2</td>
<td>6.6</td>
<td>5.1</td>
<td>0.28</td>
<td>0.17</td>
<td>1.63</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>5.4</td>
<td>11.4</td>
<td>0.62</td>
<td>0.41</td>
<td>1.52</td>
</tr>
<tr>
<td></td>
<td>6</td>
<td>2.9</td>
<td>40.6</td>
<td>2.21</td>
<td>1.32</td>
<td>1.67</td>
</tr>
</tbody>
</table>

It is clear that the method of equivalent linear stiffness is an approximate one, and is dependent on the amount of equivalent viscous damping that is considered. For the cracked
masonry structures, it is apparent that perhaps better correlation would be gained if damping were assumed in the range of 15 to 20%. Because the method substitutes hysteresis damping with viscous damping, such percentages are not excessive. Despite the crudeness of the method, it appears to have potential for quick and simple estimates of drift for structures within the nonlinear range of response.

Nonlinear Drift History Method

Measured response waveforms suggested that displacements at each of the three levels could be represented with a single generalized coordinate. This finding implies that response of the multistory structures could be calculated using a single-degree-of-freedom model. For small-amplitude shaking, this finding suggests that the structures responded in essentially the first mode. For large-amplitude motions, the finding suggests that all nonlinear deformations could be associated with one time-dependent displacement variable.

Nonlinear drift histories were calculated using a time-step integration computer program. Nonlinear behavior of the overall structure was defined in terms of the relation between the base shear and the top-level deflection. Measured base accelerations were input to the computer program so that direct correlations with measured drift histories could be made. A personal computer based program known as Nonlinear Earthquake Resistant Design of Systems (Ref. 7) was used. Hysteresis rules are formulated in the program using a set of cubic splines which are assigned properties based on input of the lateral base shear strength, the drift at first yield, and slopes of the unloading and load reversal curves. Parameters describing the lateral strength of each structure and the general hysteresis relations were taken from measurements. The "mechanism" strength was taken to be the peak base shear measured for each direction of loading. The lateral drift corresponding to this strength was approximately 0.2% of the structure height for both structures. The unloading slope was assumed to be the same as the initial loading slope. The reversal slope was assigned a low value (2% of the loading slope for RM1; 15% of the loading slope for RM3) to account for sliding along bed joints which was observed during the final test runs.

Calculated displacements are compared with the experimental values in Fig. 5 for the last test run of each structure. The period and sequence of the calculated response was remarkably...
similar to that measured for both structures. The correspondence between the amplitudes of
the peaks was also impressive considering the simplicity of the method. Results indicate that
a SDOF nonlinear analysis may be capable of providing a reliable depiction of displacement
response.

SUMMARY AND CONCLUSIONS

A variety of methods for calculating lateral drift of reinforced masonry building structures
have been assessed. Three static methods were studied for estimating small-amplitude drifts. The conventional pier model proved to be overly stiff as was a frame model. An elastic finite
element model also underestimated lateral drifts.

Estimates of peak nonlinear drift were obtained using an approximate method that was
based on an equivalent linear stiffness. Using the average slope of a hysteresis loop proved to
be a good technique for calculating first-mode frequencies and drift maxima. Another
approximate method based on a single generalized coordinate proved to produce a faithful
replication of nonlinear drift histories.

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