1. ABSTRACT

A macro-model is developed to predict the in-plane behaviour of grouted concrete masonry. In this model, the masonry assemblage is replaced by an "equivalent material" which consists of a homogenous medium intersected by two sets of planes of weakness (along the head joint and bed joint planes) and two sets of reinforcement. The macro-behaviour of the "equivalent material" is determined by smearing the influence of these sets, which introduces the inherent part of the anisotropic characteristics of masonry assemblages. The behaviour of the homogeneous medium is described by an orthotropic model to account for the induced part of the anisotropic characteristics. The ability of the proposed model to predict the shear response of reinforced masonry is confirmed by comparison with the experimental results of three panels tested under a pure shear state of stress.

Keywords: Masonry; Reinforcement; Shear; Anisotropy; Macroscopic-model.

---

1Post-doctoral Fellow, Department of Civil Engineering, McMaster University, Hamilton, Ontario, Canada, L8S 4L7.

2Professor, Department of Civil Engineering, McMaster University, Hamilton, Ontario, Canada, L8S 4L7.


2. INTRODUCTION

The behaviour of grouted concrete masonry under in-plane stresses, in general, or under shear stress, in particular, is complicated by the presence of the mortar joints acting as planes of weakness. It has been well established that brick masonry\(^1\) and ungrouted concrete masonry\(^2\) exhibit distinct directional properties depending on the bed joint orientation with respect to the principal stresses. This "inherent" anisotropic characteristic has also been observed in recent tests of full scale grouted concrete masonry panels, both unreinforced and reinforced, under biaxial tension-compression\(^3\). This inherent characteristic was found to dominate when the failure occurs along the mortar joints, whereas the formation of cracks along planes, other than the mortar joints, introduces "induced" anisotropy. The interaction of these anisotropic characteristics and the determination of which one dominates impose additional difficulties in predicting the in-plane behaviour of grouted concrete masonry.

Several attempts have been made, at the macroscopic level, to predict the modes of failure and strengths of masonry assemblages considering their inherent anisotropic characteristics\(^4-6\). Others have considered only the induced anisotropic characteristics for prediction of both strength and deformation\(^7,8\). More sophisticated models\(^8,10\), which have the potentials of simulating the two degrees of anisotropy, have also been reported, but their accuracies in predicting the in-plane behaviour of grouted concrete masonry have not been examined. It is also worth noting here that direct implement of the methods developed for modelling the shear response of reinforced concrete structures\(^11\) in predicting the behaviour of masonry structures is not acceptable. Unlike masonry, concrete exhibits induced anisotropic characteristics only. In addition, the amount of reinforcement that can be placed in a grouted concrete wall is limited by the size of the cells of hollow blocks, which typically leads to lower percentages of reinforcement than those used to define the parameters of the reinforced concrete models.

This paper reports on the theoretical part of a research program\(^3\) which explored the in-plane behaviour of grouted concrete masonry under biaxial states of stress. In the following sections, the model is briefly described, followed by comparisons of the predicted and observed results for three panels tested under a direct shear state of stress along the mortar joints.

3. MACRO-BEHAVIOUR MODEL

Although modelling masonry at a microscopic level facilitates study of the influences of different parameters on the global behaviour, there are serious limitations on its general usefulness. First, the geometric characteristics of the masonry units and the mortar joints can lead to ill-conditioning of the algebraic system and/or instability of the numerical solution. Second, very sophisticated and detailed information is required to describe the characteristics of all of the materials and their interactions under various stress states and stress histories. Third, this kind of analysis becomes impractical for masonry shear walls and even large test assemblages. The alternative of modelling masonry at a macroscopic level can still be used to rationally explain the
different aspects of masonry behaviour, having the advantage of being a relatively simple approach which facilitates modelling large masonry structures.

A macro-behaviour model is proposed to simulate the in-plane behaviour of grouted concrete masonry. Although the model is formulated at a macroscopic level, certain considerations are also given to the micro-behaviour along the planes of weakness (mortar joints and crack planes) which yield a general and more realistic simulation of the anisotropic characteristics (inherent and induced). The macro-behaviour model is based on the multi-laminate model, developed by Zienkiewicz and Pande\textsuperscript{12} for jointed rock masses, with several modifications to accommodate the differences associated with the behaviour of grouted concrete masonry. The multi-laminate model has been used extensively in research and design of jointed rock masses\textsuperscript{13}. It was also used by Gerrard and Macindoe\textsuperscript{14} in a qualitative investigation on the strength of reinforced brickwork under in-plane tensile loading.

3.1 Idealization of The Masonry Assemblage

As shown in Fig. 1, the reinforced masonry assemblage is replaced by an "equivalent material" which consists of a homogeneous medium intersected by two sets of planes of weakness (along the head joint and bed joint planes) and two sets of reinforcement. The macro-behaviour of the "equivalent material" is determined by smearing the influence of these sets throughout the respective volumes that they occupy. This idealization provides a means of predicting highly anisotropic behaviour and it is utilized to realistically model the inherent part of the anisotropic characteristics of masonry assemblages. The behaviour of the homogeneous medium is also described by an orthotropic model, originally developed by Darwin and Pecknold\textsuperscript{15}, to account for the induced part of the anisotropic characteristics.

![Fig. 1 Macroscopic idealization of a reinforced masonry assemblage.](image-url)
3.2 Assumptions and the Rheological Analogue

The main assumptions adopted in the development of the macro-behaviour model are:

1. The planes of weakness and the reinforcement are uniformly distributed with spacings that are much smaller than the dominant dimensions describing the external geometry of a particular masonry structure.

2. Each set of planes of weakness and each set of reinforcement is modelled as an elastic-viscoplastic material which can be presented by the rheological analogue shown in Fig. 2. It consists of a spring connected in series with a viscoplastic unit formed by a viscous dashpot and a slider connected in parallel. Symbolized by the spring, the instantaneous response of this material is elastic. When the stresses exceed the yield values, plastic strains, symbolized by the slider, take place with time as the dashpot does not respond instantaneously.

3. The homogenous medium, modelled as stress-induced orthotropic material according to the model proposed by Darwin and Pecknold\textsuperscript{15}, is treated as an incrementally linear elastic material with the stiffness and strains being corrected during each load increment to reflect the latest changes. To be distinguished from the linear elastic material, the homogeneous medium is represented by a dashed spring (see Fig. 3).

4. Following the work of Zienkiewicz and Pande\textsuperscript{12}, the mortar joints comprising each set of planes of weakness are considered to occupy a negligible volumetric portion of the masonry assemblage (5\% in the masonry assemblage under consideration). Consequently, the same global stress vector is assumed to be experienced by the homogeneous medium and each set of planes of weakness.

5. The reinforcement is assumed to be perfectly bonded to the masonry assemblage at the macroscopic level. This assumption, however, does not exclude the possibility of bond failure at a microscopic level.

![Fig. 2 Rheological analogue for an elastic-viscoplastic material.](image)

According to assumptions 1 to 4, the homogeneous medium and the two sets of planes of weakness can be considered, in rheological terms, to be connected in series as shown in the dashed box "A" in Fig. 3. Referenced to the global coordinates, the series connection ensures that they are subjected to the same stress increment and the resultant incremental strains are additive to produce the macro-strain increment. Adoption of the fifth assumption implies that the masonry assemblage and each set of reinforcement undergo the same global strain increment. In rheological terms, a reinforced masonry assemblage can be represented by three strings connected in parallel as illustrated in Fig. 3. The string on the right represents the masonry assemblage and the other two strings represent the two sets of reinforcement normal and parallel to the bed joints. The parallel connection, in this case, means that the same global strain increment is experienced by the three strings upon which the applied stress increment is distributed according to their stiffnesses.
3.3 Properties of Homogeneous Medium, Planes of Weakness, and Reinforcement

Homogeneous Medium: The homogeneous medium is modelled, as indicated before, as a stress-induced orthotropic material according to the model proposed by Darwin and Pecknold\textsuperscript{15} for plain concrete. The failure of the homogeneous medium is described by the biaxial envelope developed by Kupfer and Grestel\textsuperscript{16}. This failure envelope is defined in terms of the uniaxial compressive strength, $f'_m$, of the masonry assemblage at $\theta=0^\circ$ (i.e., compression is normal to the bed joints), rather than the separate strengths of grout, mortar, and masonry units. In this manner, the failure envelope accounts for the influence of mortar and grout on the compressive strengths of the masonry assemblage.

The homogeneous medium is considered to be cracked once the tensile stress reaches the ultimate tensile strength defined by the failure envelope. The cracks in the homogeneous medium are assumed to occur normal to the principal tensile stress. Cracking is modelled here in a smeared manner as the cracked homogeneous medium is treated as a continuum, but with different average stress-average strain relationships. The results from biaxial test of reinforced panels\textsuperscript{3} were used to establish average stress-average strain relationships for cracked masonry. The main features of the relationships are presented here. More details on the experimental results and the regression analyses involved in defining these relationships are described in Reference 3. The stress-strain relationship in the principal compressive stress direction is:

\[
\sigma_{hm2} = f'_m \left[ 2 \left( \frac{\varepsilon_{hm2}}{\varepsilon_o} \right) - \lambda \left( \frac{\varepsilon_{hm2}}{\varepsilon_o} \right)^2 \right] \quad ; \quad |\varepsilon_{hm2}| \leq |\varepsilon_{hm2p}|
\]

\[
\sigma_{hm2} = 2 \left[ 1 - \left( \frac{\varepsilon_{hm2} - \varepsilon_{hm2p}}{2 \varepsilon_o - \varepsilon_{hm2p}} \right)^2 \right] \quad ; \quad |\varepsilon_{hm2p}| < |\varepsilon_{hm2}| \leq 2 \varepsilon_o \quad \ldots (1)
\]

\[
\sigma_{hm2} = 0 \quad ; \quad |\varepsilon_{hm2}| > 2 \varepsilon_o
\]
where

\[
\sigma_{hm2p} = \frac{f_m'}{\lambda}, \quad \epsilon_{hm2p} = \frac{\epsilon_o}{\lambda}, \quad \lambda = \sqrt{\frac{\epsilon_{hm1}}{\epsilon_{hm2}}} - 0.3,
\]

\(\epsilon_o\) = the strain associated with the peak uniaxial compressive stress \(f_m'\) taken equal to 0.002,
\(\sigma_{hm2}\) and \(\epsilon_{hm2}\) = the principal compressive stress and strain, respectively, in the homogeneous medium,
\(\sigma_{hm2p}\) and \(\epsilon_{hm2p}\) = the peak principal compressive stress and the associated strains, and
\(\lambda\) = Damage parameter that accounts for the effect of cracking on the strength and stiffness of masonry in the principal compressive stress direction\(^3\).

The stress-strain relationship obtained for the principal tensile stress direction\(^3\) is given by

\[
\sigma_{hm1} = \frac{f_{cr}}{1 + 400 \epsilon_{hm1}} \tag{2}
\]

where

\(f_{cr}\) = the cracking stress defined by the failure envelope according to the principal stress ratio \(\sigma_{hm1}/\sigma_{hm2}\), and
\(\sigma_{hm1}\) and \(\epsilon_{hm1}\) = the principal tensile stress and strain, respectively, in the homogeneous medium.

Planes of Weakness: Two sets of planes of weakness are used to simulate the bed and head joint planes. The deformations and the failure conditions for these planes are described in terms of the average normal and average shear stresses (\(\sigma\) and \(\tau\)) acting on them. The behaviours of the different material components across the planes of weakness are modelled as elastic-viscoplastic material. Figure 4 shows the failure surface and the potential function adopted in the analyses. As indicated by the solid line, the failure in zone I, dominated by the shear stress, is expressed as a function of stresses only which implies an elastic-perfectly plastic response. This assumption is in agreement with the experimental results obtained in direct shear tests\(^3,17\). For

![Fig. 4 Yield and plastic potential surfaces of a typical component material along the planes of weakness.](image-url)
zone II, where the failure is dominated by the tensile stress, a circular tension cut-off is used, but having the tensile strength at point "A" defined as a function of the normal tensile strain according the strain softening model proposed by Massicotte et al. Accordingly, the yield surface of zone II contracts with increase in the normal tensile strain.

Reinforcement: At the macroscopic level, each set of reinforcement is assumed to resist the average axial stress acting parallel to its direction. This assumption does not account for the local dowel action of reinforcement crossing the cracks and consequently implies that the total average shear stress has to be resisted by the masonry assemblage.

4. MODEL PREDICTIONS

The macro-behaviour model was used to predict the behaviours of a large number of specimen tested by various investigators. The predicted as well as the observed behaviours of three panels tested under a pure shear state of stress along the mortar joints (Fig. 5) are presented herein. The ability to predict the masonry response under pure shear states of stress forms an essential step towards understanding and modelling the behaviour of shear walls under moment, shear, and normal forces. The three panels considered are identical except for the amounts of reinforcement used normal and parallel to bed joints. The main parameters used in the analyses are included in Table 1, whereas a complete list of the parameters can be found in Reference 3.

Table 1 Panel characteristics and model predictions

<table>
<thead>
<tr>
<th>Panel</th>
<th>Normal reinforcement</th>
<th>Parallel reinforcement</th>
<th>$f'_m$, MPa</th>
<th>E, GPa</th>
<th>Experimental results</th>
<th>Predicted results</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\rho_n$, %</td>
<td>Yield stress, MPa</td>
<td>$\rho_p$, %</td>
<td>Yield stress, MPa</td>
<td>Ultimate stress*, MPa</td>
<td>Mode of failure</td>
</tr>
<tr>
<td>RP8</td>
<td>0.26</td>
<td>568</td>
<td>0.26</td>
<td>568</td>
<td>17.0</td>
<td>24.1</td>
</tr>
<tr>
<td>RP13</td>
<td>0.53</td>
<td>415</td>
<td>--</td>
<td>--</td>
<td>19.7</td>
<td>26.0</td>
</tr>
<tr>
<td>RP15</td>
<td>0.53</td>
<td>415</td>
<td>0.26</td>
<td>568</td>
<td>15.1</td>
<td>22.8</td>
</tr>
</tbody>
</table>

YP, YN = Yield of the reinforcement normal and parallel, respectively, to the bed joints,
L = Local failure along one crack, and
E = Young's modulus.

* Ultimate stress is force divided by the gross area of the masonry cross-section.
In panel RP8, reinforced equally normal and parallel to the bed joints ($\rho_n=\rho_p=0.26\%$), the cracking was predicted, similar to the observed behaviour, to occur normal to the principal tensile stress rather than following the mortar joints. After cracking, the panel sustained increasing stresses until the failure took place by a simultaneous yielding of reinforcement in the two directions. A comparison between the predicted and observed stress-strain relationships in the principal stress directions is also shown in Fig. 6(a). These curves provide evidences on the ability of the model to predict the shear response of masonry and the effects of reinforcement on improving both shear strength and ductility.

Having the same total percentage of reinforcement in panel RP8, panel RP13 is only reinforced normal to the bedjoints ($\rho_n=0.53\%$ and $\rho_p=0\%$). However, the behaviour of panel RP13 was quite different. Once the principal tensile stress reached the cracking value, the panel could not sustain any more stresses, whereas the deformation increased significantly as shown in Fig. 6(b). The predicted behaviour

![Predicted and observed stress-strain relationships](image-url)

Fig. 6 Predicted and observed stress-strain relationships.
is in agreement with the test results as the failure took place immediately after the formation of the first crack. It was hard to predict a local failure along a single crack, since the model is developed at a macroscopic level. However, the large size of the observed crack implies local yielding of reinforcement across this crack which agrees with the model prediction. The predicted ultimate stresses are also in good agreement with the measured values.

Panel RP15 was chosen to check the ability of the macro-behaviour model to simulate the shear response of an unequally reinforced masonry assemblage ($\rho_\text{e}=0.53\%$ and $\rho_\text{p}=0.26\%$). In good agreement with the observed behaviour, the panel is predicted to fail by yielding of reinforcement normal and parallel to the bed joints. This behaviour is indicated clearly in the calculated and measured stress-strain relationships in Fig. 6(c). A comparison between the ultimate stress and ultimate strain values at point A or A₁ to the corresponding values at cracking at point B or B₁ reveals how the shear reinforcement is beneficial for both strength and ductility, if it is properly detailed and distributed.

5. CONCLUSIONS

Because of its anisotropic characteristics, realistic modelling of grouted concrete masonry must account for the effects of the mortar joints as well as cracking. The proposed macro-behaviour model provides an accurate, yet reasonably simple, approach for modelling the behaviour of grouted concrete masonry under in-plane stresses. This model accounts for both the inherent and induced anisotropic characteristics of masonry. The good agreement between the model predictions and the test results confirms the potential of the model to realistically predict the shear behaviour of grouted and reinforced concrete masonry.

Properly detailed and distributed shear reinforcement helps avoid brittle shear failure and improves both the shear strength and ductility of the assemblage.

6. ACKNOWLEDGMENTS

This research was funded through operating grants from the Natural Science and Engineering Research Council of Canada. The authors appreciate the contribution of the mason’s time made available through the Ontario Masonry Contractors Association and the Ontario Masonry Industry Promotion Fund, and we thank the Ontario Concrete Block Association for their donation of the blocks.

7. REFERENCES