THE SHEAR STRENGTH OF UNREINFORCED MASONRY WALL

Liu Guiqiu 1  Shi Chuxian 2  Bin Jinlin 3

1. ABSTRACT
This paper proposes a formula for calculating the shear strength of unreinforced masonry wall under the action of vertical compressive and shear stress in common. And furthermore corresponding restriction is presented. The formula may be used in design for the shear load-bearing capacity of unreinforced masonry wall.

2. INTRODUCTION
All sections of masonry wall in actual engineering generally bear not only vertical permanent and variable load transmitted by floor and roof system, but also horizontal wind load and seismic action. Therefore they are in the composite stress condition. It is very significant to research the shear load-bearing capacity of masonry wall under combined action, especially in the seismic region, necessary basis may be provided for the evaluation of the aseismic behaviour of buildings.

It is shown from the existing experimental datas at home that the failure patterns of wall under the action of vertical compressive and shear stress in common are related to the ratio of $a/\tau$ ($a$: vertical compressive stress of the cross section of masonry wall, $\tau$: shear stress of the cross section of masonry wall). Maybe there are three failure patterns (shown in Fig. 1). When the ratio of $a/\tau$ is less, shear-friction failure is produced along the cross section of the whole bed joint of mortar (see Fig. 1 (a)). When the ratio of $a/\tau$ is greater, shear-compression failure along the stepped cross section is caused (see Fig. 1(b)). When the ratio of $a/\tau$ is much greater, diagonal compression failure along the direction of compressive stress is formed (Fig. 1 (c)).

Keywords: Unreinforced Masonry Wall, Vertical Compressive Stress, Shear Strength

1 Lecture, Doctor Postgraduate, Dept of Civil Engineering, Hunan University, China, Changsha 410082
2 Professor
3 Engineer
At present, the calculation of shear strength of masonry wall under the action of vertical compressive and shear stress in common is chiefly based on the theories of the principal tensile stress failure and Coulomb failure. Their expressions are as follows:

\[ f_v = f_{vo} \sqrt{1 + \frac{\sigma_y}{f_{vo}}} \]  \hspace{1cm} (1)

\[ f_v = f_{vo} + \mu \sigma_y \]  \hspace{1cm} (2)

where 
- \( f_v \) = shear strength of masonry;
- \( f_{vo} \) = shear strength of masonry when \( \sigma_y = 0 \), to be taken according to the stipulation of literature [3];
- \( \sigma_y \) = average compressive stress of cross section of masonry corresponding to the representative value of gravity load;
- \( \mu \) = coefficient of friction of masonry.

Based on the experimental datas at home, average value of shear strength of masonry may be calculated by the following formula:

\[ f_{v,m} = f_{vo,m} + 0.4 \sigma_y \]  \hspace{1cm} (3)

where 
- \( f_{v,m} \) = average value of shear strength of masonry;
- \( f_{vo,m} \) = average value of shear strength of masonry when \( \sigma_y = 0 \).

When \( \sigma_y \) is less, the formulas as mentioned above both reflect the effect of vertical compressive stress on the shear strength of masonry wall better. The values obtained from the formulas above are close to each other. The shortage of these formulas lies in failing to reflect different influences of \( \sigma_y \) in different patterns on the shear strength of masonry wall. Moreover, the influence of the ratio of the height to the width of masonry wall on the shear strength of masonry wall isn’t reflected, either. Nor is the strength of the block.

Based on the existing experimental datas [1][2], this paper proposes the calculation formula of
shear strength of unreinforced masonry wall. This formula reveals comprehensively the main factors influencing the shear strength of unreinforced masonry wall.

3. **CALCULATION OF THE SHEAR STRENGTH OF UNREINFORCED MASONRY WALL**

There are three main factors influencing the shear strength of unreinforced masonry wall as follows:

1. **Influence of strength of materials**

   The strength of block and mortar both influence on the shear strength of masonry wall. The higher the grade of the strength of block or mortar, the greater the shear strength of masonry wall. The compressive strength of masonry may embody comprehensively the effects of the strength of block and mortar and quality of masonry on the shear strength of masonry wall.

2. **Influence of dimension of masonry wall.**

   It is shown from the experimental data at home and abroad that the ratio of the height to the width of masonry wall \( \lambda_h = \frac{H}{B} \) (where \( H \) is the height of masonry wall, \( B \) is the width of masonry wall) greatly influences the shear strength of masonry wall. The larger \( \lambda_h \), the larger the bending moment of masonry wall caused by horizontal force, the lower the shear strength of masonry wall. According to the experimental results of 60 pieces of masonry wall, the factor of the effect of the ratio of height to width of masonry wall \( \psi \) may be calculated by the following formula:

\[
\psi = 0.96 - 0.68 \log \lambda_h
\]

3. **Influence of vertical compressive stress \( \sigma_y \).**

   When the ratio of \( \frac{\sigma_y}{f_m} \) (where \( f_m \) is average value of compressive strength of masonry wall) is less than 0.5 or so, friction of the section of shear caused by vertical compressive stress \( \sigma_y \) can reduce or prevent the horizontal slip of the section of shear. In addition, it can decrease bending moment of masonry wall caused by horizontal force and weaken the bad effect of \( \lambda_h \) on the shear strength of masonry wall. This moment, vertical compressive stress \( \sigma_y \) is beneficial to the shear strength of masonry wall. The shear strength of masonry wall goes up with the increase of \( \sigma_y \).

   When the ratio of \( \frac{\sigma_y}{f_m} \) is greater than 0.5 or so, longitudinal cracks caused by \( \sigma_y \) may weaken the rigidity of masonry wall. At last, diagonal-compression failure of masonry wall is caused under the action of vertical compression and horizontal force in common. At this moment, \( \sigma_y \) is not beneficial to the shear strength of masonry wall in contrast. The shear strength of masonry wall goes down with the increase of \( \sigma_y \).

   According to the experimental results of the seismogenic behaviour of twenty-one pieces of brick masonry wall \[1\][2] (Table 1), average value of shear strength of masonry wall may be calculated by the following formula:

\[
f_{v,m} = f_m [0.02 + 0.88(\frac{\sigma_y}{f_m}) - 0.9(\frac{\sigma_y}{f_m})^2] \psi
\]

where \( f_{v,m} = \text{average value of shear strength of unreinforced masonry wall} \);

\( f_m = \text{average value of compressive strength of masonry, to be taken according to the stipulation of literature [3]} \).

Average value of the shear load-bearing capacity of unreinforced wall may be calculated by the
following formula:

\[ V_{n,m} = f_m \left[ 0.02 + 0.88 \left( \frac{\sigma_z}{f_m} \right) - 0.9 \left( \frac{\sigma_z}{f_m} \right)^2 \right] \psi A \]  \hspace{1cm} (6)

where \( A \) = area of cross section of masonry wall.

The mean value of the ratio of the calculated value \( (f_{n,m}) \) by the formula (6) to the experimental value \( (f'_{n,m}) \) is 1.056 and the coefficient of variation is 0.185. The comparison between the test results and calculated values is shown in Fig. 2.

\[ \frac{f}{f_m} \]

\[ 0 \hspace{1cm} 0.1 \hspace{1cm} 0.2 \hspace{1cm} 0.3 \hspace{1cm} 0.4 \hspace{1cm} 0.5 \hspace{1cm} 0.6 \hspace{1cm} 0.7 \hspace{1cm} 0.8 \hspace{1cm} 0.9 \hspace{1cm} 1.0 \]

\[ \frac{\sigma_z}{f_m} \]

\[ 0 \hspace{1cm} 0.2 \hspace{1cm} 0.4 \hspace{1cm} 0.6 \hspace{1cm} 0.8 \hspace{1cm} 1.0 \]

Fig. 2

The mean value of the ratio of the calculated value \( (f'_{n,m}) \) by the formula (3) to the experimental value is 1.212 and the coefficient of variation is 0.235.

In accordance with the stipulation of literature [4], the design values of strength of materials are adopted, the shear load – bearing capacity of cross section of unreinforced masonry wall may be calculated by the following formula:

\[ V \leq f \left[ 0.02 + 0.45 \left( \frac{\sigma_z}{f} \right) - 0.22 \left( \frac{\sigma_z}{f} \right)^2 \right] \psi A \] \hspace{1cm} (7)

where \( V \) = design value of shear of masonry wall;

\( f \) = design value of compressive strength of masonry.

4. CONDITION OF APPLICATION

In order to ensure the load – bearing capacity of masonry wall under compression and restrict the width of diagonal cracks, the shear stress of masonry wall should be controlled not too great. The condition of application of formula (7) is \( \sigma_z \leq f \), in other words,

\[ V \leq 0.25f \psi A \] \hspace{1cm} (8)

5. CONCLUSION

The shear strength of unreinforced masonry wall is mainly related to compressive strength of masonry, vertical compressive stress, ratio of height to width of masonry wall and so on. The shear load – bearing capacity of masonry wall may be calculated according to the formula (7). Meanwhile, in order to avoid shear stress of the cross section of masonry wall too great, the dimensions of the cross section of masonry wall should meet the demand of the formula (8).
Table 1. The experimental results of shear strength of brick masonry wall

<table>
<thead>
<tr>
<th>NO. of Wall</th>
<th>Dimensions of Wall $(B \times H \times b)$(mm)</th>
<th>Ratio of Height to Width $(H/B)$</th>
<th>$f_a$ (MPa)</th>
<th>$\sigma_a$ (MPa)</th>
<th>$\sigma_a/f_m$</th>
<th>$V_a$ (MPa)</th>
<th>$\phi$</th>
<th>$f_{v,w} / f_{v,m}$</th>
<th>$f_{v,w} / f_{v,m}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>PJ – 1</td>
<td>5400 $\times$ 1350 $\times$ 240</td>
<td>0.25</td>
<td>3.24</td>
<td>0.74</td>
<td>0.228</td>
<td>960.0</td>
<td>1.369</td>
<td>1.039</td>
<td>0.858</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.675</td>
<td>3.78</td>
<td>0.6</td>
<td>0.159</td>
<td>240.0</td>
<td>1.076</td>
<td>1.116</td>
<td>1.310</td>
</tr>
<tr>
<td>PJ – 2</td>
<td>2000 $\times$ 1350 $\times$ 240</td>
<td>0.25</td>
<td>3.33</td>
<td>0.6</td>
<td>0.180</td>
<td>322.0</td>
<td>1.076</td>
<td>0.797</td>
<td>0.885</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.675</td>
<td>3.49</td>
<td>0.6</td>
<td>0.172</td>
<td>237.0</td>
<td>1.076</td>
<td>1.101</td>
<td>1.242</td>
</tr>
<tr>
<td>PJ – 3</td>
<td>1250 $\times$ 1500 $\times$ 240</td>
<td>1.25</td>
<td>3.85</td>
<td>0.8</td>
<td>0.208</td>
<td>152.5</td>
<td>0.894</td>
<td>1.109</td>
<td>1.459</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1.25</td>
<td>4.30</td>
<td>0.8</td>
<td>0.186</td>
<td>152.5</td>
<td>0.894</td>
<td>1.156</td>
<td>1.570</td>
</tr>
<tr>
<td>SW – 6</td>
<td>2250 $\times$ 1460 $\times$ 240</td>
<td>0.65</td>
<td>2.59</td>
<td>0.74</td>
<td>0.286</td>
<td>342.6</td>
<td>1.087</td>
<td>0.879</td>
<td>0.811</td>
</tr>
<tr>
<td>SO – 1</td>
<td>2250 $\times$ 1460 $\times$ 240</td>
<td>1.0</td>
<td>2.62</td>
<td>0.8</td>
<td>0.305</td>
<td>125.0</td>
<td>0.96</td>
<td>0.990</td>
<td>1.009</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1.0</td>
<td>2.49</td>
<td>0.4</td>
<td>0.161</td>
<td>108.0</td>
<td>0.96</td>
<td>0.733</td>
<td>0.729</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1.0</td>
<td>2.77</td>
<td>0.8</td>
<td>0.289</td>
<td>108.0</td>
<td>0.96</td>
<td>1.177</td>
<td>1.243</td>
</tr>
<tr>
<td>A – 1 – 1</td>
<td>1000 $\times$ 1000 $\times$ 240</td>
<td>1.0</td>
<td>3.11</td>
<td>0.57</td>
<td>0.183</td>
<td>78.4</td>
<td>0.96</td>
<td>1.380</td>
<td>1.409</td>
</tr>
<tr>
<td>C – 1 – 1</td>
<td>1000 $\times$ 1000 $\times$ 240</td>
<td>1.0</td>
<td>3.80</td>
<td>0.57</td>
<td>0.150</td>
<td>78.4</td>
<td>0.96</td>
<td>1.471</td>
<td>1.737</td>
</tr>
<tr>
<td>D – 1 – 1</td>
<td>1000 $\times$ 1000 $\times$ 240</td>
<td>1.0</td>
<td>4.19</td>
<td>0.58</td>
<td>0.138</td>
<td>117.6</td>
<td>0.96</td>
<td>1.022</td>
<td>1.263</td>
</tr>
<tr>
<td>D – 1 – 2</td>
<td>1000 $\times$ 1000 $\times$ 240</td>
<td>1.0</td>
<td>4.19</td>
<td>0.56</td>
<td>0.134</td>
<td>98.0</td>
<td>0.96</td>
<td>1.198</td>
<td>1.498</td>
</tr>
</tbody>
</table>
REFERENCES