INVESTIGATION ON THE LATERAL STIFFNESS OF SHEAR WALLS WITH OPENINGS

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1. ABSTRACT

Three methods have been in use for estimating the in-plane stiffness of shear walls with openings. These methods assume fixity at the pier-spandrel junction of the wall piers to estimate their stiffness. In this paper, a new method is proposed to determine the in-plane stiffness of the shear walls with openings, in which the spandrels are assumed flexible, and can translate and rotate under lateral load. The in-plane stiffness of the shear walls obtained by using the proposed method are generally much different from the results for the same walls obtained by the well known three methods. However, the in-plane stiffness of the walls computed by employing the linear elastic finite element method are generally in good agreement with the corresponding results determined by the proposed method. It also turns out from the present study that the effect of gravity load on the in-plane stiffness of the walls is insignificant. Finally, it is established that the new method for estimating the in-plane stiffness of the shear walls with openings is a simple, accurate and reliable alternative over the three known methods.

2. KEYWORDS: New method; In-plane Stiffness; Shear Walls with Openings; Piers; Spandrel.

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3. INTRODUCTION

The present concept of modern masonry structure utilizes floors and/or roofs as diaphragm acting as horizontal flanged girders to distribute the lateral forces to walls, which in turn provide the shear resistances needed, in addition to carry the gravity loads. This type of structure is defined as a shear wall system or a box system (1). The lateral load shared by each shear wall in such a system depends on its relative stiffness compared to the other walls in the same direction. Many methods are employed to evaluate the lateral stiffness of the shear wall with openings. The most common approach is pier analysis (2,3,4), which is known as method I in subsequent sections. In this method, the stiffness of a wall is determined by considering only the sum of the stiffness of individual piers within the wall.

Another method includes the deformation of the spandrel by dividing the wall into a number of rectangular panels which constitute the piers and spandrel. Assuming the restrained end conditions, the stiffness of each of these panels is calculated. Then, all the parallel panel stiffness are added resulting in a series of equivalent panels. These series of panels are eliminated by adding the reciprocal of their stiffness to get the reciprocal of the total stiffness (4). This approach is known as method II in this investigation.

To determine the in-plane stiffness of a shear wall with several connected piers, one of the methods appeared in the Tri-services publication (5), and is known as method III in the present study. In this method (valid for one- or two-story structure) firstly, the deflection of the wall, considering it as a solid cantilever, is determined. Secondly, the cantilever deflection of an interior strip, having a height equal to that of the highest opening, is computed and then subtracted from the deflection of the solid wall. Thirdly, the deflections of all the piers within the interior strip are determined from their individual stiffness and added together to get the total deflection of the wall with openings. Finally, the reciprocal of this value gives the relative stiffness of that wall.

The values of absolute stiffness of twelve shear wall models each with one-opening have been computed by using the above three methods. Comparison of these values method-wise, indicates a wide variation among them depending on the wall configuration. Therefore, it turns out that the method used in the determination of wall stiffness with openings is governing the overall result. Finally, this will result in a large variation in the calculated lateral forces carried by each wall. Therefore, none of the above methods are much reliable.

In view of this, a simple new method which has been developed before (6) is presented here to determine the in-plane stiffness of a shear wall with opening. The stiffness of the shear walls obtained by the proposed method are generally in good agreement with the results determined by the linear elastic finite element unlike the corresponding values obtained by the methods I, II, and III.
4. MATHEMATICAL MODEL

An unreinforced masonry shear wall can be subdivided into two types of elements, namely, pier and spandrel. In the three methods, the piers of a shear wall are assumed to be rigidly fixed to the spandrels against rotation at their ends. But, in the new method, a partial fixity is assumed at the junction of the pier and spandrel.

4.1 In-plane Stiffness of a Pier

The derivation of a formula for estimating the in-plane stiffness of a pier in the shear wall is given elsewhere (6), but summarized in the following paragraphs:

(a) Bending Deformation

Equation for the bending deformation, $\Delta_b$, is written as follows:

$$\Delta_b = \frac{H h^3 [k_t k_b + 2 k_t + 2 k_b + 3]}{[(k_t + k_b + 2 k_t k_b) 6 E I]}$$

in which $E$ is modulus of elasticity of masonry, $I$ is the second moment of area of the pier cross-section, $h_t$ is depth of the top spandrel, $k_t$ is a constant and equal to $h/h_t$, $h$ is height of the pier, $k_b$ represents $h/h_b$, $h_b$ is height of the bottom spandrel and $H$ is applied lateral force at top of the wall.

(b) Shear Deformation

The shear deformation, $\Delta_s$, due to the lateral force can be expressed as:

$$\Delta_s = \frac{12 H h (1 + \nu)}{5 A E}$$

where $\nu$ is Poisson’s ratio of the wall material.

(c) Total Lateral Deformation

To estimate the in-plane stiffness, $K$, of the pier, the total deformation, $\Delta$, is given by:

$$\Delta = \frac{H h^3 [k_t k_b + 2 k_t + 2 k_b + 3]}{[(k_t + k_b + 2 k_t k_b) 6 E I]} + \frac{12 H h (1 + \nu)}{5 A E}$$

4.2 In-plane Stiffness of a Shear Wall

The in-plane stiffness of a pier is determined by using Equation (3). Finally, the in-plane stiffness of a shear wall with openings may be estimated by adding the stiffness of the various piers in the wall.

5. NUMERICAL MODEL OF SHEAR WALLS

5.1 Configuration and Dimension of Shear Wall Models

The computation of the in-plane stiffness of shear walls with one-opening is considered in this study to evaluate the accuracy of the results obtained by using the new method. Also, the lateral stiffness of the same shear walls are determined with the
methods I, II, and III and the finite element method using IMAGES 3-D program (7). For all the wall models, the modulus of elasticity, \( E = 10 \) GPa, the Poisson’s ratio, \( \nu = 0.20 \) and the uniform wall thickness, \( t = 0.30 \) m. The configuration and dimension of the shear wall numerical models are described through Figure 1 and Table 1.

Twelve numerical models of a shear wall with single opening are considered in the analysis. A door opening (4-h) m high and 1.25 m wide is placed at different locations in the wall as shown in Figure 1. For a parametric study, a set of variable such as the width \( W_1 \) and \( W_2 \) together with the height of the top spandrel is shown in Table 1.

![Fig. 1 A Typical Shear Wall with One Opening](image.png)

5.2 Parametric Study

To study the effect of gravity load (V) on the in-plane stiffness of the shear wall models, their stiffness have been computed by employing the finite element method. For this investigation, \( V/H \) ratios have been varied in the range of 0 to 2. Then, the stiffness of the twelve wall models subjected to lateral load (H) are obtained using all the four methods. Further, the corresponding stiffness computation have been made by the finite element method but the walls have been subject to gravity loads in addition to the lateral load. For such a study, the values of \( V/H \) are considered as 0.2, 0.8, 1.4 and 2.0.

6. DISCUSSION OF RESULTS

The in-plane stiffness of the twelve wall models ha been computed employing the five methods of analysis as discussed before. The results thus obtained are presented through various plots as shown in representative Figures 2 to 10. The stiffness of the shear walls have been non - dimensionlized and then plotted against the top spandrel.
depth to wall height ratio. Such plots have been made for these parameters using the five different methods of estimating the lateral stiffness of the wall. Different observations made by studying these plots are discussed in the following paragraphs.

### Table 1 Shear Wall Configurations With Single Opening

<table>
<thead>
<tr>
<th>Model Number</th>
<th>Wall Designation</th>
<th>$W_1$ (m)</th>
<th>$W_2$ (m)</th>
<th>$h_t$ (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>WAL11H8</td>
<td>0.75</td>
<td>2.00</td>
<td>0.80</td>
</tr>
<tr>
<td>2.</td>
<td>WAL11H12</td>
<td>0.75</td>
<td>2.00</td>
<td>1.20</td>
</tr>
<tr>
<td>3.</td>
<td>WAL11H16</td>
<td>0.75</td>
<td>2.00</td>
<td>1.60</td>
</tr>
<tr>
<td>4.</td>
<td>WAL11H20</td>
<td>0.75</td>
<td>2.00</td>
<td>2.00</td>
</tr>
<tr>
<td>5.</td>
<td>WAL12H8</td>
<td>1.00</td>
<td>1.75</td>
<td>0.80</td>
</tr>
<tr>
<td>6.</td>
<td>WAL12H12</td>
<td>1.00</td>
<td>1.75</td>
<td>1.20</td>
</tr>
<tr>
<td>7.</td>
<td>WAL12H16</td>
<td>1.00</td>
<td>1.75</td>
<td>1.60</td>
</tr>
<tr>
<td>8.</td>
<td>WAL12H20</td>
<td>1.00</td>
<td>1.75</td>
<td>2.00</td>
</tr>
<tr>
<td>9.</td>
<td>WAL13H8</td>
<td>1.25</td>
<td>1.5</td>
<td>0.80</td>
</tr>
<tr>
<td>10.</td>
<td>WAL13H12</td>
<td>1.25</td>
<td>1.5</td>
<td>1.20</td>
</tr>
<tr>
<td>11.</td>
<td>WAL13H16</td>
<td>1.25</td>
<td>1.5</td>
<td>1.60</td>
</tr>
<tr>
<td>12.</td>
<td>WAL13H20</td>
<td>1.25</td>
<td>1.5</td>
<td>2.00</td>
</tr>
</tbody>
</table>

6.1 Effect of Gravity Loads on In-plane Stiffness of Wall Models

The in-plane stiffness of different numerical models have been computed as described in section 5.2. Typical Figures 2 to 4 show the relationship between the in-plane stiffness and the gravity and lateral load ratio ($V/H$). From these figures, it is seen that the in-plane stiffness of the shear wall with one opening obtained by the finite element method have insignificant effect due to the gravity loads for $V/H$ ratio ranging between 0 and 2. Such observation is valid for all the parametric combinations of the shear wall models.

6.2 Variation of Wall Stiffness with Spandrel Depth

Representative Figures 5 to 10 show the plot of $K/E_t$ against the ratio of the height of top spandrel to the wall height ($S$). The variation of $K/E_t$ with $S$ for the shear wall models WAL11H8 to WAL11H20 for $V/H = 0.20$ and 2.0 is shown in typical Figures 5 and 6. It is observed from Fig. 5 that values of $K/E_t$ computed with the new and the finite element method are in close agreement with one another for all values of $S$. But, from Fig. 6, it is seen that the above trend is valid for $S$ between 0.2 and 0.4, whereas method I gives the result closer to that obtained by the finite element method for $S$ equal to 0.5.

Figures 7 and 8 show the variation of $K/E_t$ for the shear walls WAL12H8 to WAL12H20 with $V/H = 0.8$ and 1.4. From these figures, it is clear that the new method gives close variation of $K/E_t$ with the result determined by the finite element method for $S = 0.2$ to 0.4. But, the in-plane stiffness values obtained by method I are in good agreement with result of the finite element method for $S = 0.5$.  

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Fig. 2: Lateral Stiffness of Shear Wall With \( W_1 = 0.75 \) m and \( W_2 = 2.0 \) m

Fig. 3: Lateral Stiffness of Shear Wall With \( W_1 = 1.0 \) m and \( W_2 = 1.75 \) m

Fig. 4: Lateral Stiffness of Shear Wall With \( W_1 = 1.25 \) m and \( W_2 = 1.5 \) m (For \( V/H > 1 \))
Fig. 5: Variation of Wall Stiffness With Spandrel Height for WAL11H8 to WAL11H20 with V/H=0.2

Fig. 6: Variation of Wall Stiffness With Spandrel Height for WAL11H8 to WAL11H20 With V/H=2.0

Fig. 7: Variation of Wall Stiffness With Spandrel Height for WAL12H8 to WAL12H20 With V/H=0.8
Fig. 8: Variation of Wall Stiffness With Spandrel Height for WAL12H8 to WAL12H20 With V/H=1.4

Fig. 9: Variation of Wall Stiffness With Spandrel Height for WAL13H8 to WAL13H20 With V/H=0.2

Fig. 10: Variation of Wall Stiffness With Spandrel Height for WAL13H8 to WAL13H20 With V/H=2.0
Variation of K/Et against S for the walls WAL13H8 to WAL13H20 with V/H = 0.2 and 2.0 is depicted in Figures 9 and 10. It is observed from these figures that the in-plane stiffness of the walls computed by the finite element and the new method are in close agreement for S = 0.2 to 0.3. On the other hand, the corresponding results obtained by method I and the finite element method give closer variation of K/Et for S = 0.4 to 0.5.

It is established from these representative figures that the values of in-plane stiffness estimated by methods II and III are higher than their corresponding values as determined by the new method, the finite element method and method I.

7. CONCLUSIONS

Based on the results obtained from the present investigation, following conclusions can be drawn:

1. Generally, the values of the in-plane stiffness of the shear walls with one opening obtained by the method I, II and III are relatively higher in comparison with the corresponding values of the in-plane stiffness estimated by the finite element and the new method.

2. From the present analytical study of the shear walls with one opening, it seems that the effect of gravity loads on the in-plane stiffness is insignificant.

3. It seems that the new method is simple, accurate and reliable for estimating the in-plane stiffness of shear walls with one opening mainly for the design office use.

8. REFERENCES


