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LOSS ASSESSMENT AND OPTIMUM ANALYSIS OF MASONRY STRUCTURE

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Abstract

A kind of method for seismic optimising design of masonry is discussed in this paper. An expression of optimization design is presented, in which economy loss of masonry's crack is considered firstly. In this expression, the sum of initial investment of structures and loss of earthquake are considered as the object function, and the seismic reliability is considered as the constrain. This study can supply reference value for the seismic optimising design.

Key Words

Optimization, reliability, masonry structure.

1 Introduction

Masonry is a kind of building material with a long history. In China, masonry structure is a very popular structure form and over ninety percent of the walls are still built of masonry presently. In latest twenty years, masonry structure has developed greatly in many countries, particularly in Europe and America. More and more attention has been paid to this structure system in the world. Because it is a sort of brittle material structure made of building block and mortar, its seismic performance and its deformation are less than structures built of reinforced concrete. So it is very important to take reasonable measures in seismic design.

Today, when the earthquake happened, reducing losses in the damage is the design thought, and this reflects "Non-collapse under severe earthquakes" which is part of the seismic design principle "Undamaged under frequent earthquakes; Non-collapse under severe earthquakes." While masonry element's tensile is weak, and it is easy to crack. Many researches on earthquake destruction show that economic loss caused by the masonry's crack hasn't been thought of the secondary factor. According to " no damage from frequent earthquake shock ", it should reflect economic influence on this fortified level. Literature [2] put forward dual rules that is crack-resisted and collapse resisted.

Based on the dynamic reliability, the influence of the masonry's cracks is considered completely, and the whole building is optimised. It reflects the thought of seismic design, which is not only " no collapse from severe earthquake shock ", but also " no damage from slight earthquake shock ".

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2 Estimation of the earthquake loss

Based on dynamic reliability, several object functions about structure optimization have been put forward to make the total cost of structure C minimum.

$$C = C_0 + C_f P_f \tag{1}$$

C_0 is initial investment of building
C is the loss caused by the masonry's destruction
P_f is the masonry's destruction probability.

Obviously, Eq. (1) doesn't consider the influence of the economic loss which caused by the masonry's crack. Suppose C_f' is the loss caused by the masonry's crack, and P_f' is the probability of the masonry crack. Obviously, P_f' includes P_f, so the desired value of the loss caused by the masonry's crack is C_f' (P_f' -P_f).

Now, a lot of seismic measures for security have been provided. The masonry structure, which is designed according to some seismic rule, can endure some seismic behaviour without collapse. Here, according to the principle of " no damage from freqent earthquake shock ", and different destructive degree, the following object function was puts forward.

$$C = C_0 + C_f P_f + C_f' \left(P_f' - P_f \right) \tag{2}$$

Let $P_f'/P=\gamma$, therefore,

$$C = C_0 + \left(C_f - C_f' \right) P_f + \gamma C_f' P_f = C_0 + C_F P_f = C_0 + C_p \tag{3}$$

where

$$C_F = C_f + (\gamma - 1) C_f'$$

$$C_P = C_F P_f$$

Obviously, C_0 and C_p are both relate to design intensity I_0, and I_0 is the principal variable.

$$C = C_0(I_0) + C_P(I_0) \tag{4}$$

The initial cost (or investment) can be expressed:

$$C_0(I_0) = A_F(F + C_a) = A_F F(1 + C_a / F) = A_F F(1 + \eta) \tag{5}$$

A_F is building area
F is the cost of unit area
$A_F F$ is the fixed cost of building
C_a is the seismic cost of unit area

Apparently, C_a is the function of I_0 , and it depends on the structure types and the design method, what's more, it is always influenced by the designer's judgment.η is the ratio of the seismic investment and the investment in building, and it can be called the rate of seismic design. In the section of V□, η is 1%~3%; in the section of V□, η is 3%~6%, however, in the section of IX, η is 6%~11%.

In order to confirm $C_p(I_0)$, it is assumed that $n(t)$ is the times for earthquake in the period of t. When k=1, 2, …, n(t), P_k is the invalid probability of the structure in the No.k earthquake, and e_k is the random variable which relates to the loss in No.k earthquake. Therefore the total cost of earthquake destruction z is

$$z = \sum_{k=1}^{n(t)} e_k p_k \tag{6}$$

where , assume that e_k has the same distribution when k is different .

Obviously,

$$Z = N(t) E\overline{P} \tag{7}$$

Z is the expectation of z

$N(t)$ is the expectation of random earthquake frequency whose intensity is not less than I_0 in the period of t

E is the expectation of the random earthquake loss

$\overline{P}$ is the mean loss probability of the structure when earthquake happened.

That Z multiplies the depreciation coefficient $g(t)$ is the earthquake loss value $C_p(I_0)$:

$$C_P(I_0) = Zg(t) = N(t)E\overline{P}g(t) \tag{8}$$

Assuming $g(t)$ is a constant, and Literature [5] can obtain the analytic expression of $g(t)$.

3 Deduction of the average earthquake frequency and the average loss probability of structure

Assuming $\{t_n\}$ represents the time series of earthquake, which is a Poisson's course. Therefore,

$$P[n(t) = n] = \frac{(\alpha t)^n}{n!}e^{-\alpha t} \tag{9}$$

α is the earthquake frequency in unit time

Therefore,

$$N(t) = \alpha t$$

Assume

$$Y(t) = \{\max|y(t)|; t \in (0, t_0)\}$$

t_0 is the duration of structural vibration, and let it equal to earthquake duration

$y(t)$ is the response parameter (displacement ,speed, acceleration , stress and so on).

Obviously, for an earthquake, the structural invalid probability is

$$P(I) = P\,[Y(t) \geq \lambda \;|\text{(the earthquake intensity } I\text{)}] \tag{10}$$

λ is the allowable maximum value of response

I is the earthquake intensity; its probability density function $f(I)$ can be attained by the hypocenter quality and regional earthquake analyses.

It is show from Literature [6] that, in the design reference period, earthquake intensity in China accords with the distribution of extremum □:

$$F(I) = \exp\left[-\left(\frac{\omega - I}{\omega - \varepsilon}\right)^K\right] \tag{11}$$

ω is the max of intensity, that is $\omega=12$

ε is the frequency intensity

K is the shape parameter.

To make derivate I, Eq.(11) will be transformed as follows

$$f(I) = \frac{K}{\omega - I}\left(\frac{\omega - I}{\omega - \varepsilon}\right)^K F(I) \tag{12}$$

The structural invalid probability $P(I)$ that defined by the Eq. (10), needs to confirm the structural destroy mechanism, and at the same time, needs to set up the statistic mode which includes response parameter $y(t)$ and the structure reactance R.

Eq.(10) can also be expressed as

$$P(I) = P[Y(t) \geq \lambda; 0 \leq t \leq t_0] \tag{13}$$

Literature [7][8] offers a method to calculate the seismic reliability of the masonry structure.

Assume the masonry's crack is caused by the principal tensile stress, when the principal tensile stress σ_L is stronger than the tensile strength f_t, the masonry can be thought beginning to crack. Therefore, the function is set up as

$$G_Z = \sigma_L - f_t \tag{14}$$

adopting the JCSS method, the crack reliability is obtained.
Because of $P_f'/P_f = \gamma$, we can obtain γ.

4 The statistics of the ground moving

Assume tensile is I , its corresponding ground acceleration of the earthquake is *a(t)*. *a(t)* is the filtrated white noise, and its power spectral density functions adopts Jin Jingqing 's spectrum,

$$S_a(\omega) = \frac{\omega_g^4 + 4\xi_g^2\omega_g^2\omega^2}{(\omega - \omega_g)^2 + 4\xi_g^2\omega_g^2\omega^2} S_0 \tag{15}$$

S, W_g, ξ_y are the parameters that reflect the ground moving character.
The statistic relation of the peak acceleration on the flat ground and the spectral intensity can be expressed as follows.

$$A = \gamma_a \sigma_a \tag{16}$$

γ_α is the peak factor

$$\sigma_a^2 = \int_{-\infty}^{\infty} S_a(\omega) d\omega = \frac{S_0^2 \omega_g \pi}{2\xi_g}\left(1 + 4\xi_g^2\right) \tag{17}$$

Based on research of Prof. Huixian Liu, the relation of the ground's peak acceleration and the intensity is

$$A = 10^{(I \lg 2 - 0.01)} \tag{18}$$

Appling Eq. (17), (18) into Eq.(16), the relation of the spectral intensity and intensity is obtained.

$$S_a = \frac{2\xi_g \omega^2}{\pi \gamma_a^2 \omega_g \left(1 + 4\xi_g^2\right)} \tag{19}$$

For a structure whose frequency of self-vibration is ω_0 and damping is ξ_0 , the earthquake intensity is the measurement of potential destruction, and it can be expressed by the corresponding response spectrum $Y_{\max} = \max|Y(t)|$. According to the suppositions of the ground moving character, adopting the numerical integration, *Y(t)* and Y_{max} *can be attained*, however, immediately integration also can obtain *Y(t)*.
E is assumed as follows

$$E = BS_y(I, \omega_0, \xi_0, t_0) \tag{20}$$

B is a constant
S_y is the mean expectation of $Y_{\max}$ on different intensities, that is

$$S_y = \sum_{I=1}^{12} E[Y_{\max}(I)] P(I) \tag{21}$$

and

$$E[Y_{\max}(I)] = \gamma_y \sigma_y(I)$$

γ_y is peak factor of *y*

$$\left.\begin{aligned} &\gamma_y = [2\ln(\gamma_0 t_0)]^{1/2} + 0.577[2\ln(\gamma_0 t_0)]^{-1/2} \\ &\gamma_0 \approx \omega_0 / \pi \end{aligned}\right\} \tag{22}$$

γ_0 is the frequency of letting $y = 0$.

It can be seen that γ_y is independent of *I*. Therefore

$$S_y = \gamma_Y \sum_{I=1}^{12} \sigma_y(I)P(I) \tag{23}$$

Let

$$B = \beta A_F F$$

β is the loss corresponding with the unit displacement response of building investment, and we can adopt $\beta = 1/[S_{\max}]$, where $[S_{\max}]$ is the structural allowable maximum displacement. Therefore

$$E = \beta A_F F S_y$$

thereby Eq.(8) can be expressed as follows

$$C_P(I_0) = at\beta A_F F\overline{P} g(t) S_y \cdots \tag{24}$$

5 The optimal design expression

According to above analysis, the optimal design expression based on reliability can be obtained.

$$\left.\begin{array}{l} C = A_F F(1+\eta) + at\beta A_F FPg(t)S_y ....(object.function) \\ P \le P_0 ....(reliability..construction) \end{array}\right\} \tag{25}$$

P_0 is the structural allowable invalid probability.

Seeing from the optimal design equation, we can come to the conclusion that the object function is the function of site intensity I and structural parameter. When design intensity is confirmed, the optimization of structure type can be obtained. Contrariwise, when the structure type is confirmed, the optimum earthquake intensity also can be obtained.

Considering the masonry structure's damage in different degrees, the corresponding expectation of the earthquake loss is:

$$E = E_1 + \gamma E_2 \tag{26}$$

E_1 is the expectation of the earthquake loss when masonry structure is destroyed

E_2 is the expectation of the economic influence caused by the masonry structure crack.

In the same way, assume

$$E_1 = B_1 S_y(I, \omega_0, \xi_0, t_0)$$

$$E_2 = B_2 S_y(I, \omega_0, \xi_0, t_0)$$

$$B_1 = \beta_1 A_F F$$

$$B_2 = \beta_2 A_F F$$

in which

$$\beta_1 = 1/[S_{\max 1}]$$

$$\beta_2 = 1/[S_{\max 2}]$$

$[S_{\max 1}]$ is the allowable maximum displacement of masonry structure destroyed

$[S_{\max 2}]$ is the allowable maximum displacement of masonry structure crack

$$E = E_1 + \gamma E_2 = (\beta_1 + \gamma\beta_2) A_F F S_y \tag{27}$$

Therefore, the optimal design equation is

$$
\left.\begin{aligned}
&C = A_F F(1+\eta) + atg(t)(\beta_1 + \gamma\beta_2)A_F F\overline{P}S_y \\
&P \le P_0
\end{aligned}\right\} \tag{28}
$$

the economic influence, which caused by masonry structure crack ,is considered for the first time in the optimal design expression (28). It not only reflects the principal of " no collapse from severe earthquake shock ", but also reflects the seismic design thought, " no damage from frence earthquake shock ". This equation has widely adaptability. As long as the reliability of seismic and crack can be obtained, object function is the single variable function with intensity.

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