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DESIGN OF SLENDER UNREINFORCED MASONRY WALLS

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Abstract

Common design methods for slender unreinforced masonry walls often do not consider the realistic stress-strain relationship of different masonry types. For this reason the design is either uneconomic or unsafe, depending on the masonry unit and mortar type. In contrast the presented new design method makes possible an economic and save wall design by considering the realistic material behavior of any masonry type with simplified design equations. It takes into account the nonlinear stress-strain relationship for compression as well as the flexural strength perpendicular to the bed joints by a few characterizing input parameters. Because of its easy and general use the design method can be applied for unreinforced masonry and for concrete.

Key Words

simplified non-linear design, slender walls, theory of 2nd order, unreinforced masonry

1 Introduction

All over the word masonry buildings have a very long tradition. From the past up to now different masonry products have been developed and optimized for specific fields of application. Figure 1 exemplarily shows the use of different masonry types for residential buildings in Germany. Due to the great variety of masonry units and mortar types many different combinations of units and mortar are possible. Accordingly the material behavior cannot be simplified uniformly for all masonry types.

Today the design of slender, unreinforced masonry walls is based on rough approximations concerning the material properties and the structural system. Consequently the design of walls turns out to be uneconomic or unsafe, depending on the provided masonry type. Usually a linear, parabolic, or rectangular shape simplifies the stress-strain relationship for compression, while the tensile strength perpendicular to the bed joints is being neglected. Although the existing standards for masonry design, e.g. DIN 1053-1:1996 and Eurocode 6 (DIN V ENV 1996-1-1:1996 and

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prEN 1996-1-1: 2003) are based on similar approximations concerning material properties and structural systems, different design models are proposed.
The authors (Graubner and Glock 2003, Graubner et al 2002, Glock and Graubner 2001) therefore have developed new design methods for general use, which are based on realistic assumptions for the material behavior and the effects of lateral wall deformation. These new design methods take into account the non-linear stress-strain relationship of any type of compressed masonry by the standardized relations formulated in DIN 1045-1:2001 and Eurocode 2 (DIN V ENV 1992-1-1:1992) for concrete. Furthermore the tensile strength perpendicular to the bed joints is being regarded. Simplified easy-to-use design equations are proposed for application in engineering practice and masonry standardization. Based on analytical calculations and verified by extensive numerical simulations and experimental data the simplified design equations are more realistic than commonly used equations. Due to the general formulation of the load-bearing capacity as a function of the non-linear material behavior they can be used for the design of slender unreinforced concrete and steel fiber concrete walls, too.

Figure 1 Residential building in Germany with common masonry types (medium sized clay masonry units, large and small sized calcium silicate bricks)

2 Material behavior of masonry

2.1 General

Due to very limited tensile strength and missing reinforcement the load bearing capacity of unreinforced masonry walls is determined mainly by the shape of the stress distribution in the compression zone of the cross section. Taking into account the great influence of varying material behaviors to the resistance of the cross section, and to additional bending moments according to theory of 2^{nd} order, the new design concept is based on realistic assumptions for the stress-strain relationship of different masonry types.
The load-deformation behavior of masonry depends on the material properties of the masonry units and the mortar type. The great variety of different shapes of masonry bricks, masonry materials, and mortar types make it difficult to generalize the stress-strain relationship of masonry. Exemplary Figure 2 shows the load-deformation behavior of different masonry materials according to experiments, carried out by Meyer and Schubert (1992).

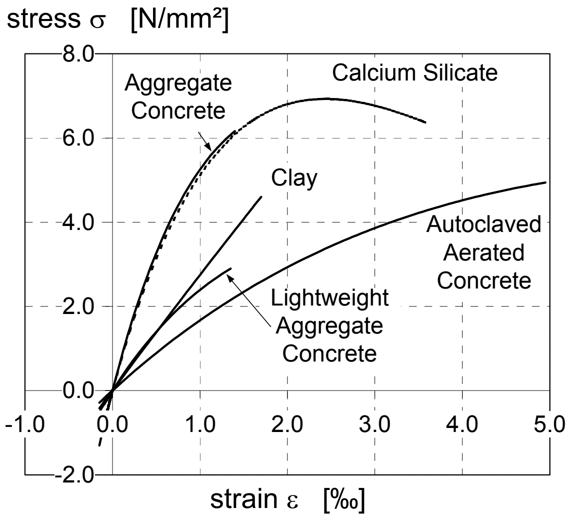

Figure 2 Load-deformation behavior of different masonry materials according to Meyer and Schubert (1992)

2.2 Modeling the stress-strain relationship

The new non-linear design concept for slender masonry walls is based on the stress-strain relationships determined in DIN 1045-1:2001 for compression forces of concrete. These relationships have been verified in various research projects and are established in international standards, e.g. Eurocode 2 and Model Code 1990 for concrete. The general formulation makes possible to adjust the curvature of the stress-strain relationship to any non-linear material behavior between the border cases linear and constant by changing the parameter k_0 (Fig. 3). Consequently it can be used to describe the non-linear stress-strain relationship of any masonry type, too. Due to the limited tensile strength of masonry perpendicular to the bed joints the stress-strain relationship for tension is assumed to be linear.

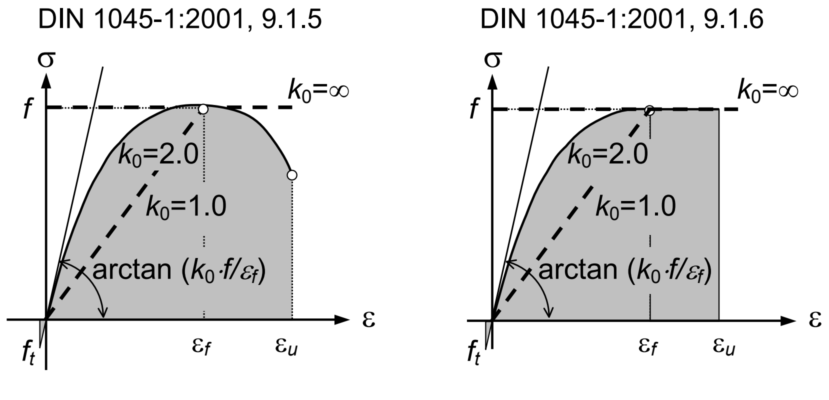

Figure 3 Non-linear stress-strain relationships according to DIN 1045-1:2001

DIN 1045-1:2001 defines two different equations of the stress-strain relationship: one for structural analysis and another for the design of the cross section. The non-linear Equation 1 for structural analysis describes realistic material behavior for strains smaller and greater than the strain ε_f at the peak stress $\sigma=f$ (Fig. 3-Left). In contrast, the stress-strain relationship for the design of the cross section (Eq. 2) simplifies the

descending part of the curve by assuming a constant stress $\sigma = f$ for $\varepsilon_f < \varepsilon \leq \varepsilon_u$ (Fig. 3-Right). The ascending part of both curves is roughly the same.

According to DIN 1045-1, 9.1.5 the stress-strain relationship for structural analysis is:

$$\sigma / f = \frac{k_0 \cdot \eta - \eta^2}{1 + (k_0 - 2) \cdot \eta} \qquad (1)$$

According to DIN 1045-1, 9.1.6 the stress-strain relationship for the cross section design is:

$$\sigma / f = \begin{cases} 1 - (1 - \eta)^{k_0} & 0 \leq \eta \leq 1.0 \\ 1 & 1.0 < \eta \leq \eta_u \end{cases} \qquad (2)$$

σ/f	standardized stress
$\eta = (\varepsilon/\varepsilon_f)$	standardized strain
$\eta_u = (\varepsilon_u/\varepsilon_f)$	standardized ultimate strain
$k_0 = E_0 \cdot \varepsilon_f / f$	curvature factor, i.e. standardized modulus of elasticity

Exemplarily Figure. 4 compares the stress-strain relationship according to Equation (1) to experimental data from Schubert and Meyer (1989) for clay and calcium silicate masonry.

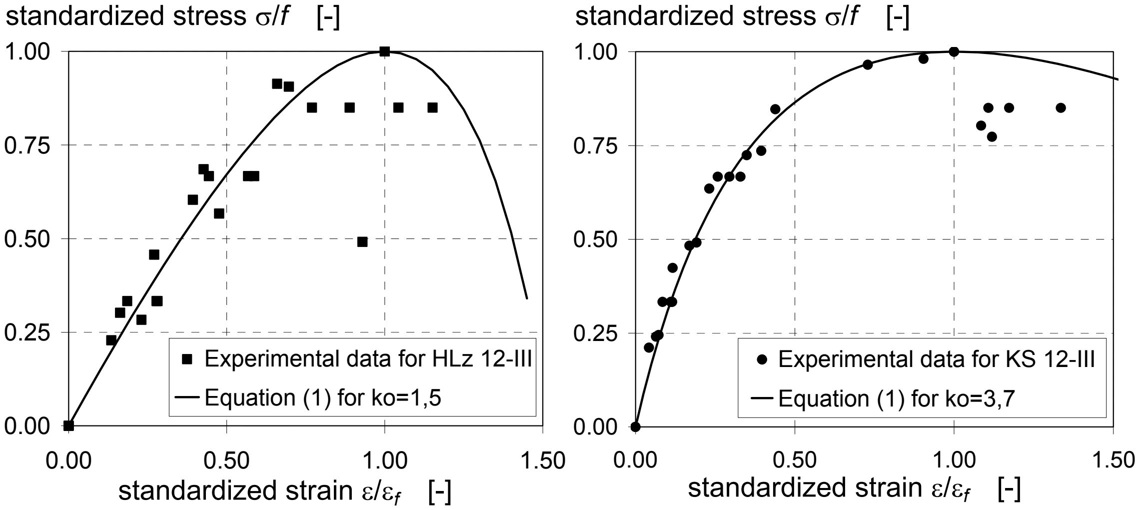

Figure 4 Stress-strain relationship of clay and calcium silicate masonry according to Schubert and Meyer (1989) and Equation (1)

2.3 Material parameters of different masonry materials

The application of Equation 1 and 2 requires four specific material parameters: the compression strength f, the strain ε_f at the peak stress $\sigma = f$, the ultimate compression strain ε_u, and the curvature factor k_0. Due to linearly elastic material behavior for small tension forces the tensile strength f_t of masonry perpendicular to the bed joints is needed, only. In DIN 1053-1:1996 these material parameters are not determined completely. For this reason the German standardization committee dealing with the revision of DIN 1053-1 is currently discussing about the material parameters of different masonry types.

Meyer and Schubert (1992) propose four different masonry categories, each with a specific stress-strain relationship governed by the parameters as shown in Table 1 without differentiating between the design of the cross section and the structural analysis. Different material parameters are also proposed in the latest redraft of Eurocode 6 (prEN 1996-1-1:2003) as shown in Table 2. The tensile strength of masonry is defined in Eurocode 6 and Schmidt and Schubert (2004), and varies between $0 \le |f_t/f| \le 0.10$ as a function of the masonry's compression strength, it's unit type, and mortar type.

Table 1 Material parameters for masonry according to Meyer and Schubert (1992)

Masonry Material	Lightweight Aggregate Concrete and Concrete	Clay and Autoclaved Aerated Concrete	Calcium Silicate (vertically perforated brick)	Calcium Silicate (solid brick)
k_0	1.0	1.0	2.0	2.0
ε_f	1.2 ‰	2.0 ‰	2.0 ‰	2.0 ‰
ε_u	1.2 ‰	2.0 ‰	2.5 ‰	3.5 ‰
$\eta_u=\varepsilon_u/\varepsilon_f$	1.0	1.0	1.25	1.75

Table 2 Material parameters for masonry according to prEN 1996-1-1:2003

Masonry Type	a)	b)	c)	d) (constant stress)
k_0	1.0	2.0	2.0	∞
ε_f	2.0 ‰	2.0 ‰	2.0 ‰	-
ε_u	2.0 ‰	2.0 ‰	3.5 ‰	-
$\eta_u=\varepsilon_u/\varepsilon_f$	1.0	1.0	1.75	-

3 Design of compact walls - cross section design

3.1 General

The load-bearing capacity N_{RI} of rectangular cross sections made of unreinforced masonry with vertical loading mainly usually is determined as a function of the load eccentricity e_I according to theory of 1st order. Hereby the resistance of the cross section represents the maximum load of very compact walls. Due to the non-linear stress-strain relationship of compression forces and the limited tensile strength perpendicular to the bed joints, the resistance of the cross section to axial loads generally has to be calculated by an iterative process. In order to enable an analytical solution the stress-strain relationship is simplified linear in DIN 1053-1:1996 and constant in Eurocode 6 for the design of the cross-section.

3.2 Design equations

If the stress distribution in the compression zone is modeled by Equation 2 and the maximum compression strain is limited to $\varepsilon_f=\varepsilon_u$, i.e. $\eta_u=1$, the resistance of the cross section to mainly vertical loading can be calculated without iteration, because the equilibrium conditions can be solved analytically. The tensile strength f_t of masonry perpendicular to the bed joints is regarded on the basis of linearly elastic material behavior by a simplified equation independent of k_0. Also strains greater than ε_f can be regarded by simplified equations if $\varepsilon_u \le 2 \cdot \varepsilon_f$, i.e. $\eta_u \le 2$ and by accepting a small underestimation of the cross section's resistance N_{RI} to vertical loads. This approach results in:

$$N_{RI} = \Phi_I \cdot b \cdot t \cdot f \tag{3}$$

$$\Phi_{I,un} = \frac{1}{1 + 2 \cdot \frac{\eta_u \cdot (k_0+1)+1}{\eta_u \cdot (k_0+1)-1} \cdot {e_I}/{t}} \quad \text{for } {e_I}/{t} \leq \left({e}/{t}\right)_{lim} \tag{4}$$

$$\Phi_{I,cr} = \left(1 - \frac{1}{\eta_u^{\,2} \cdot (k_0+1)^2}\right) \cdot \left(1 - 2 \cdot {e_I}/{t}\right) \quad \text{for } \left({e}/{t}\right)_{lim} < {e_I}/{t} \leq \left({e}/{t}\right)_{lim,b} \tag{5}$$

$$\Phi_{I,b} = \frac{\left|{f_t}/{f}\right|}{6 \cdot {e_I}/{t} - 1} \quad \text{for } {e_I}/{t} > \left({e}/{t}\right)_{lim,b} \tag{6}$$

$$\left({e}/{t}\right)_{lim} = \frac{1}{2 \cdot \eta_u \cdot (k_0+1) + 2} \quad \text{and} \quad \left({e}/{t}\right)_{lim,b} = \frac{1}{3} + \frac{1}{6} \cdot \sqrt{1 - \left|{f_t}/{f}\right| \cdot \frac{3}{1 - \frac{1}{\eta_u^{\,2} \cdot (k_0+1)^2}}} \tag{7}$$

b	width of the cross section
t	height of the cross section
f	compressive strength
k_0	curvature factor according to Equation 2 for $1.0 \leq k_0 \leq \infty$
e_I	load eccentricity according to theory of 1st order

The simplified non-linear design Equations 3-7 are based on the stress-strain relationships according to Equation 2 and a linear stress distribution for tension forces. Due to the small deviations of Equation 1 and 2 they also can be used to approximate the load-bearing capacity on the basis of Equation 1. Figure 5 shows the standardized load-bearing capacity Φ_I as a function of the eccentricity e_I according to theory of 1st order for different curvature factors k_0 and tensile strengths f_t. Figure 6 illustrates the influence of ultimate strains ε_u greater than the strain ε_f at the peak stress (η_u>1) for three curvature factors k_0 and f_t=0.

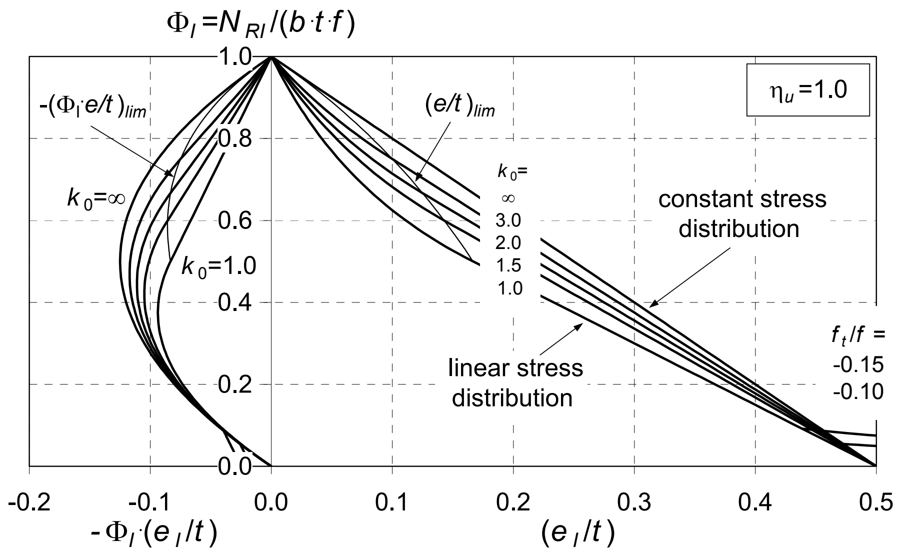

Figure 5 Standardized load-bearing capacity Φ_I of compact walls for η_u=1

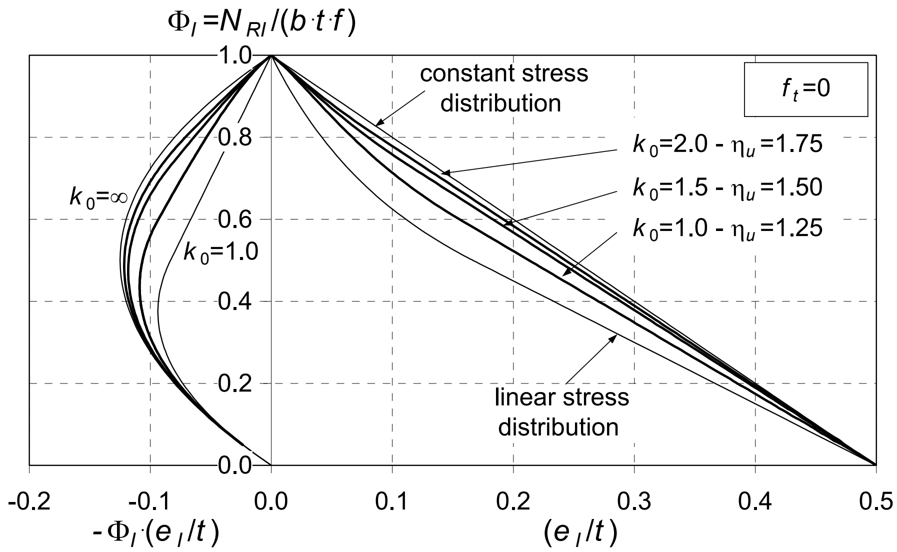

Figure 6 Standardized load-bearing capacity Φ_I of compact walls for η_u>1

4 Design of slender walls

4.1 General

For the design of slender unreinforced masonry walls, lateral wall deformation has to be considered by a structural analysis according to theory of 2nd order. Due to the non-linear stress distribution in the compression zone and cracks caused by tensile stresses, this analysis usually requires iterative calculation methods. For this reason a new software tool has been developed in Darmstadt that makes it possible to calculate the load bearing capacity of walls on the basis of any stress-strain relationship for compression stresses and a limited tensile strength. The wall deformation is hereby calculated on the basis of the bending moment-curvature relationship of the cross section for a single two-hinged column.
Furthermore, two analytical design methods have been developed for the approximation of the load-bearing capacity N_{RII} of slender walls with vertical loading mainly. These simplified design methods make it possible to consider realistic material behavior of different concrete and masonry walls by analytical equations. While one simplified design concept (see 4.3) has been developed for an easy use for common buildings, another has been developed for an optimized approximation of the real load-bearing capacity.

4.1 Numeric analyses

The objective of the numeric analyses is to identify the effect of varying material behaviors to the load-bearing capacity N_{RII} of slender masonry walls, and then to determine simplified design equations. The numerical evaluation for different non-linear stress-strain equations and the comparison to experimental results show that Equation 1 and 2 allow a very realistic modeling of masonry. Besides the dominant influence of the compressive strength f to the ultimate load N_{RII} of the wall, significant influences of the strain ε_f at the peak stress σ=f and the curvature factor k_0 have been detected (Fig. 7). These three material parameters determine the modulus of elasticity $E_0=k_0 \cdot f/\varepsilon_f$. If the tensile strength f_t of masonry perpendicular to the bed joints is being considered for wall design the load bearing-capacity can be increased significantly for very slender walls with a large load eccentricity e_I according to theory of 1st order.

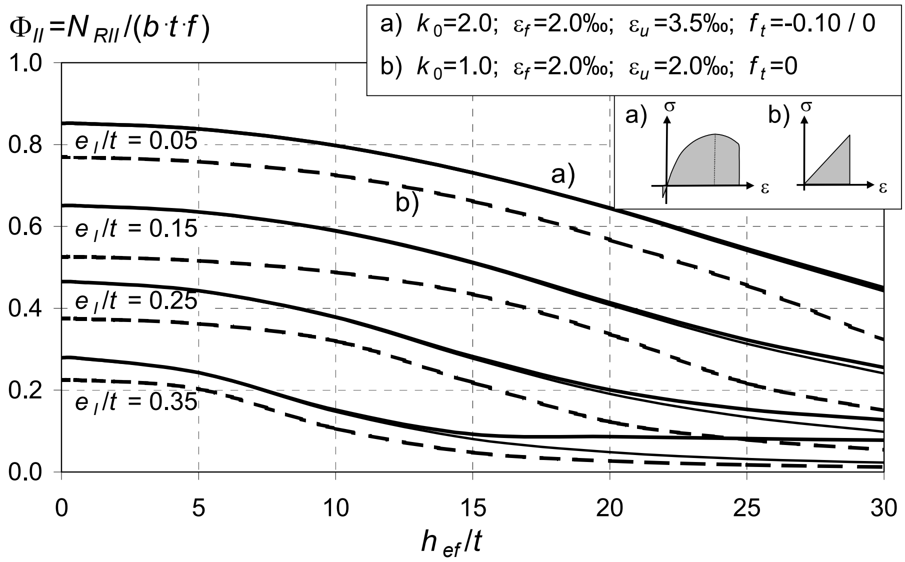

Figure 7 Standardized load-bearing capacity Φ_{II} of slender walls

4.2 Design equations

The new simplified design method for the approximation of the load-bearing capacity N_{RII} of slender, unreinforced masonry walls has been developed with the goal of easy-to-use equations and good acceptance in structural engineering. Therefore the general design equations are based on the design method of DIN 1045-1:2001 and Eurocode 2 for unreinforced concrete (DIN V ENV 1992-1-6:1994) and DIN 1053-100:2003 for masonry. The tensile strength of masonry perpendicular to the bed joints is being neglected. In accordance to the concept of DIN 1045-1 and Eurocode 2 for reinforced concrete, the lateral wall deformation is regarded on the basis of material behavior according to Equation 1, and the resistance of the cross section is calculated on the basis of Equation 2. Accordingly, the resistance N_{RII} of slender walls can be approximated by:

$$N_{RII} = \Phi_{II} \cdot b \cdot t \cdot f \tag{8}$$

$$\Phi_{II} = k_1 \cdot \left(1 - 2 \cdot {}^{e_I}\!/_{t}\right) - k_2 \cdot \frac{h_{ef}}{t} \leq \Phi_I \tag{9}$$

$$k_1 = 0.95 \cdot \frac{\ln\left(0.2 \cdot k_0 + 0.8\right)}{\left(0.2 \cdot k_0 + 0.8\right)^4} + 1.05 \quad \text{and} \quad k_2 = \left(0.5 - 0.03 \cdot k_0\right) \cdot \sqrt{\varepsilon_f} \tag{10}$$

b	width of the cross section
t	height of the cross section
h_{ef}	effective height of the wall
f	compressive strength
ε_f	strain at compressive strength according to Equation 1
k_0	curvature factor according to Equation 1 for $1.0 \leq k_0 \leq 5.0$
e_I	load eccentricity according to theory of 1st order
Φ_I	standardized load-bearing capacity according to Equation 4-5

In Figure 8-9 the design Equations 8-9 are compared to the exact numeric results for different stress-strain relationships according to Table 1 and 2. The small deviations between numeric and approximated standardized load-bearing capacities Φ_{II} illustrate

the accuracy of the new design concept. In case of unreinforced concrete C 25/30 Equation 9 is equal to DIN 1045-1:2001 and Eurocode 2, part 1-6.

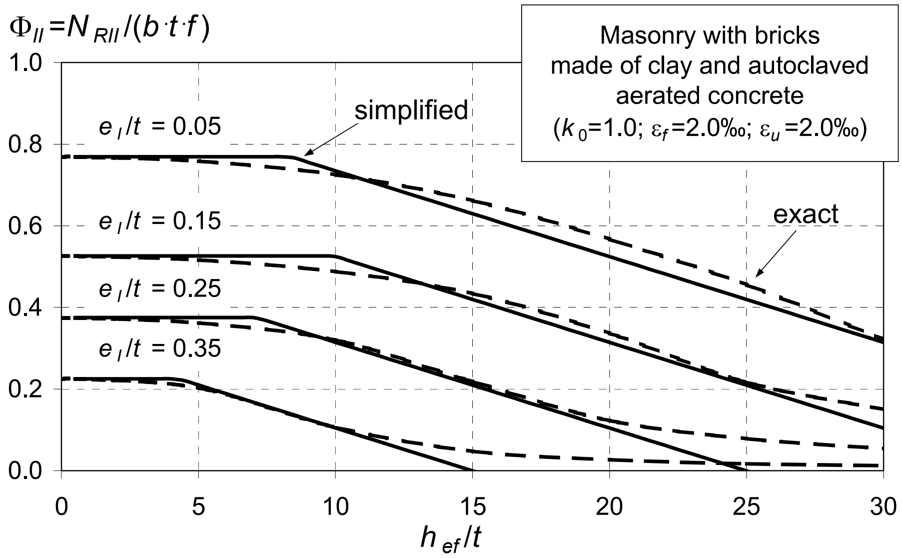

Figure 8 Standardized load-bearing capacity Φ_{II} of slender walls for masonry with bricks made of clay or autoclaved aerated concrete

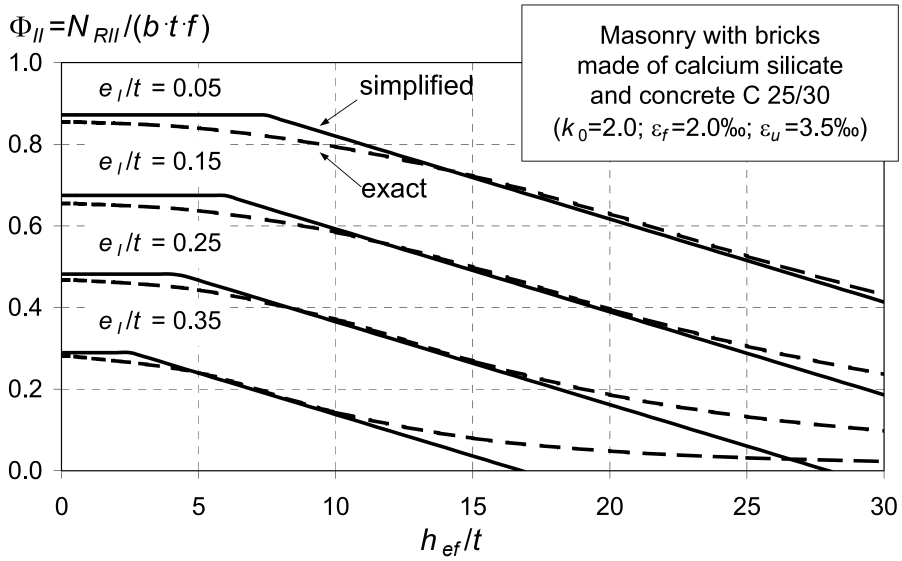

Figure 9 Standardized load-bearing capacity Φ_{II} of slender walls for masonry with bricks made of calcium silicate and for concrete C 25/30

5 Conclusion

The design of unreinforced masonry walls is currently based on various building standards and different design methods, although the load-deformation behavior and the structural system is modeled similar. The existing design methods do not consider the varying non-linear material behavior of different masonry types. In contrast the new simplified non-linear design models for unreinforced masonry walls allow realistic material behavior to be modeled by easy-to-use equations. The realistic stress-strain relationship of different masonry materials can be modeled by a few characterizing input parameters without a time-consuming non-linear structural analysis. Due to more realistic assumptions about material behavior, a reduction of the safety factors is possible for wall design. The general approaches of the new concepts allow application to other structural materials as well, e.g. unreinforced concrete or fiber concrete, and guarantees the consistency of the design methods for different mineral building materials.

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