



## PREDICTING THE BEHAVIOUR OF Laterally LOADED MASONRY PANELS

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### Abstract

The paper aims to compare predictions by different methods of analysis of the lateral load capacity of unreinforced masonry walls, and to determine whether a relationship can be drawn between theory and experiments. Several analysis methods were employed to predict failure loads of a number of wall panels, and the results were compared with test values. It is concluded that boundary conditions play a major role in the accuracy of analytical methods. When the boundary conditions assumed in the analysis most nearly match the real conditions of the test panels, predictions are most accurate. The authors conclude that finding a rational analysis and design procedure for laterally loaded masonry walls is possible.

### Key Words

analytical expressions, flexural strength, lateral loads, testing.

### 1 Introduction

Intensive research has been directed towards lateral strength of masonry wall panels in recent decades, with the main objective being the derivation of analytical equations for the prediction of strength properties. The results of these researches widely vary, making it difficult to draw any reliable conclusions about the behaviour of the material in flexure. The various design approaches adopted by different codes of practice is testimony to the designers' limited understanding of the behaviour of the material. The UK adopted the yield line theory as the basis for design of masonry panels subjected to lateral loading, while some European countries employ equations that are based on elastic plate theory. Because predictions using these theories have not produced consistent results, researchers have also looked at the development of empirical relationships.

This paper reviews and compares the theories on which some of the codified methods of design are based. Results of a series of tests performed by Lawrence (1983) are used to compare the ability of these theories to match experimental data. It has been assumed in

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this review that material properties, such as mortar types, water absorption rate of units, etc., were kept constant during the tests, and do not influence the comparisons.

## **2 The need for design formulae**

Traditionally, structural masonry was not treated as engineered material in the same sense as steel, concrete and timber. This resulted in rule of thumb procedures being applied for masonry construction, which in turn produced excessively thick walls with consequent cost penalties and wastage of space within a building. Haseltine et al (1977) note that, with the massive walls, the ability to resist such small loads as wind was never in question. As walls became thinner, and lower strength materials such as aerated concrete blocks were introduced, the need to carry out structural calculations could no-longer be avoided. Moreover, Haseltine et al (1977) observe that the narrowing of walls and introduction of lower strength materials were accompanied by an increase in the pressure used to represent the effect of wind in design.

The need to find a simple strength-to-size relationship of panels subjected to lateral loads has been a topic of discussion since the inception of masonry-design codes. In December 1978, a discussion group of The Institution of Structural Engineers, considered a paper by Haseltine et al (1978), and declared that design data on this subject was sorely lacking. A year earlier, Haseltine et al (1977) had reported that very little information was available to enable engineers and checking authorities to design walls for lateral loads, and that, what there was had to allow for a wide range of dissimilar materials and take into account variations in workmanship from well controlled to totally neglected. The code of practice of the time, CP121 (Haseltine et al, 1977), had given rule of thumb methods for the sizing of panel walls.

According to Lovegrove (1988), the lack of theoretical basis for the design of masonry walls subjected to lateral loads made it impossible to extend the design rules beyond what was currently contained in the design codes. He cites, as an example, the design of walls containing openings.

## **3 Existing theories**

There are a number of theories on which prediction of lateral loading capacities of masonry walls can be based, which include: (a) elastic plate methods, (b) yield line theory, (c) finite elements, (d) strip method, and (e) fracture line theory. Analyses using these methods give quite different results when compared with experimental data, thus prompting researchers to focus on finding new techniques, or a rationale for existing methods. While there is some indication that each of these theories does give reasonable results at certain times, the results are random and dependant on the specific testing programme undertaken. That is to say, there is no consistent evidence, so far, linking theory with all types of masonry. Consistent results only occur over specific sectors of the subject.

### **3.1 Elastic Plate Theory**

Elastic plate theory would appear to be the most promising analytical technique since, in most of the tests recorded in literature, the load-deflection relationship for laterally loaded panels is nearly linear. Allowance for the orthotropic properties of brickwork is also made without any difficulty.

The European code (Eurocode 6, 1996) allows designers to choose between using the moment coefficients derived by the yield-line method and those derived by the elastic plate theory. The moment coefficients in Table 1 of this review were calculated using plate-bending equations derived by Timoshenko (1959).

### **3.2 Yield Line Theory**

Yield line theory was developed for use with reinforced concrete, and assumes that the bending moment along a line or lines reaches a yield value, and stays constant until other parts of the line reach that yield value. Thus, a pattern of yield lines develops with constant moment along each line, when failure occurs. It has been argued by several researchers (Sinha 1978, Lovegrove 1988, and Lawrence 1995) that, with masonry, this is theoretically unsound as it assumes the existence of plastic hinges which cannot exist in a brittle material.

The method of design of unreinforced masonry panels given in the British Code of practice, BS 5628: Part 1, is based on the yield line theory. To the best of the author's knowledge, the UK is the only country which uses this method. The yield line analysis used in this review applies the moment coefficients taken from Table 9 of BS 5628: Part 1.

### **3.3 Finite Elements**

Finite Elements have been applied by many researchers to simulate the behaviour of masonry structures and has often produced very good results when compared with experiments. The method is suitable for the prediction of failure loads, as well as stress distributions in the working stress range. Page (1978) used the method to investigate stress distributions and found that it was able to reproduce these with good accuracy. It has even been shown, by Bouzeghoub and Riddington (1994), that there is no need to use 3-D finite elements since simpler 2-D elements are adequate to simulate the behaviour of masonry structures. The biggest drawback of the method is the effort and time it takes to idealise the structure, input the data and, interpret the results.

Although the finite element method is a good analytical tool, it is not suitable for design purposes. Designers need a simple analytical approach which enables accurate predictions of wall capacity. For this reason, this method is not reviewed here.

### **3.4 Fracture Line Theory**

The Fracture line method was proposed, and applied, by Sinha (1978), for the design of masonry wall panels against lateral loads. In this method, Sinha proposes that the variation of Young's modulus with direction should be taken into account, and that the all deformations take place only along the fracture lines. Besides the variation of  $E$  with direction, the method does not differ from the yield line theory, since the resulting equilibrium equations are based on the cracked pattern, with assumed ultimate (constant) moments along the crack lines. As with the yield line method, the fracture line theory assumes that the individual parts of the failed panel rotate as rigid bodies, and the equations of equilibrium are derived from energy principles.

To the best of the author's knowledge, the fracture line method is not incorporated in any code of practice, but it is reviewed here because it has been available for some time and is widely known in the field of masonry. This method can be found in a number of published texts. The reader is referred to references 3 and 19 for the derivations and discussion of the crack line formulae from which the coefficients in Table 3 were determined.

### **3.5 Strip Method**

In this method, the load is split into parts which are carried by individual systems of strips, designed as beams. Since the static equations are satisfied, a lower bound solution is obtained. The strip method basically assumes that the load is carried by bending only, that is, twist-free moment fields are considered. Since the existence of twisting moments along both bed and perpend joints in masonry panels cannot be avoided, a rather conservative solution can be expected. In reinforced concrete slab design, this is compensated for through savings in reinforcement by providing only the amount necessary to resist the

bending moments. The strip method equations used to calculate the values in Table 3 were derived from crack patterns of the panels tested by Lawrence (1983).

#### **4 Comparison of the theories**

The theories discussed above have been employed to reproduce the results of a series of tests that were carried out by Lawrence (1983). The tests used clay bricks laid in a 1:1:6 mortar (cement: lime: sand by volume). Table 1 shows failure loads of the wall panels as calculated by the different theories, as well as experimental results as recorded by Lawrence. Lawrence recorded three different failure stages: the load at initial cracking; the load at full-crack pattern; and the ultimate failure load. The results have been plotted on Figures 1 to 4.

In Figure 1 the failure loads of the panels as calculated by the different theories are compared to the full-crack-pattern load as obtained by Lawrence. The wall panels are grouped according to the boundary conditions, as can be seen from the figure. It is apparent from the figure that, with exception of very few panels, the yield line method over-estimates the failure load of the panels. The strip method, on the contrary, under-estimates the failure loads of most panels. Elastic plate theory also predicts larger failure loads than obtained by tests, with the exception of simply supported panels. The fracture line method was only applied to panels with simple supports and to those with fixed edges and simply supported along the top and bottom edges (boundary condition 3). For other support conditions of the tested walls, the fracture line equations were not readily available. While it could have been possible to derive such equations from first principles, the available equations were deemed adequate for the purposes of this review. In the simply supported panels, the fracture line method was found to under-estimate Lawrence's test results, while in the other panels, the fracture line method invariably over-estimated the failure loads. Closer observation of Figure 1 also shows that the test values for a few panels (panels 12, 14, 15, 16, 37 and 38) are much higher than the corresponding theoretical predictions. All of these are square panels, i.e., their aspect ratio is 1. This gives some indication that regardless of boundary conditions, theoretical methods used here under-estimate the strength of square panels with the largest error margin.

Figure 2 compares the theoretical results to the failure load at initial crack formation. With the exception of the strip method when applied to panels with boundary conditions 4 and 5, the test results indicate that cracks start forming in the panels at loads lower than those predicted by theory. In the case of panels with all edges fully fixed (boundary condition 2), test results are much lower than the theoretically predicted values. This means that panels will start developing cracks under loads that are much lower than those predicted by theory.

In Figure 3, the theoretical methods are compared with experimental ultimate load capacities of the panels. It is noticed here that results from the yield line method almost coincide with the test results for the majority of panels. Comparing this figure to Figures 1 and 2, it is evident that theoretical predictions are closest to the ultimate failure loads of the wall panels, but still there is considerable variability.

Figures 4 to 8 are extracts from Figure 1. They show the full crack failure loads of the panels grouped according to their support conditions. Figure 5, with 8 wall results, contains the largest number of panels tested. This is a very small number of panels from which meaningful conclusions regarding the behaviour of walls can be made, and more test data is, therefore, required. A lot of other test data does exist, boundary conditions are questionable, and the detail with which Lawrence tested his walls is not always evident.

In general, all of the methods of analysis reviewed here reveal some level of inconsistency and large variability. An important observation from the Figures is that the results for simply supported panels are less scattered than those for panels with any other support conditions

This indicates either that all theoretical methods reviewed here predict the failure loads for simply supported panels with better accuracy than for panels with other support conditions or that it is easier to accurately produce simple supports than other supports when testing walls. The methods also give better approximations for panels with mixed boundary conditions as compared to panels with all supports fixed. It is observed that as the number of built-in edges increases, the theoretical predictions drift further from test results. This trend can be associated with the values of the moment coefficients as they are assigned with the assumption of full continuity at the built-in edges. When the assumed built-in edge is not capable of full moment resistance, the assigned coefficient becomes faulty. With the simple supports or free edges better approximations of the coefficients can be made. This confirms the influence of boundary conditions on the strength of panels, with accurate predictions occurring when the boundary conditions assumed in the analysis nearly match the real conditions of the test panels.

It is very important to note here that the major task of this review was to compare the ability of the different theories to match test results, as opposed to determining the load-carrying capacity of panels with different boundary conditions. In Table 2 the ratios of predicted values to the test results are displayed. These ratios are then plotted in Figure 9. It is clear from the figure that, for most panels, the strip method gives the best results of the walls tested as its values are closest to unity. It is also apparent from the graph that, in general, the yield line method over-estimates the load carrying capacity of these walls more than the other methods. However, the yield line method gives the best results for panels with free top and simple supports on the other edges (far right of graph, panels 28, 26, 10, 36, 17 and 15). Incidentally, almost all of the walls tested by Haseltine et al (1977) for the validation of the yield line method in masonry panels were supported on three edges (free along the top edge).

The foregoing discussion clearly demonstrates the intricacy of any attempt to draw conclusions from the predictions of the existing theories, and illustrates the need to generate more test data. With today's improved testing equipment and methods, we should strive for enhanced simulation of the boundary conditions.

## **5 Conclusions and recommendations**

The following are the conclusions drawn from this review:

1. The methods used to predict lateral strength of wall panels are not consistent.
2. Boundary conditions play a major role in the strength of the walls, and, results of theoretical methods can be greatly improved if a good representation of the boundary conditions can be made.
3. Different methods yield good approximations for different types of panels
4. All methods reviewed here give reasonable results for panels simply supported on all edges.
5. All methods give poor results for panels with all edges built-in.
6. The strip method yields very conservative values of the lateral strength of panels, while the yield line method almost invariably over-estimates the failure load.

It is recommended that:

1. More test data be generated, paying particular attention to boundary conditions.
2. The possibility of applying more than one of the above-reviewed methods to derive a method of design be investigated.

The possibility of combining two or more of the above-reviewed methods with empirical methods be investigated.

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**Table 1: Theoretical and Experimental Failure Loads**

Test #	B.C.	f <sub>lx</sub>	H	t	L	$\mu$	Asp. ratio				moment coefficient, $\alpha$				W <sub>k</sub> (kN)				Tests (Lawrence) (kN)		
								Ex	Ey	$\beta$	YL	elastic	Strip	Fracture Li	YL	Elastic	Strip	Fract. Li	Full crack	Initial crack	Ultimate
8	1	1.61	3	0.11	6	0.638	0.500	16.93	21.29	0.3003	0.024	0.0464	0.0270	0.0394	3.76	1.94	3.35	2.29	3	1.6	3.2
12	1	0.94	2.5	0.112	2.5	0.420	1.000	13.85	19.34	0.4318	0.061	0.0479	0.0881	0.0802	5.15	6.56	3.57	3.92	8.6	7.6	8.6
18	1	1.03	2.5	0.109	3.75	0.393	0.667	21.09	26.09	0.3088	0.043	0.0498	0.0473	0.0670	3.37	2.91	3.07	2.17	4.9	2.9	4.9
22	1	1.16	2.5	0.111	5	0.485	0.500	18.2	24.15	0.2761	0.029	0.0464	0.0279	0.0449	3.29	2.05	3.42	2.12	4.7	3.1	4.7
27	1	1.91	2.5	0.109	6	0.845	0.417	17.23	22.11	0.2932	0.018	0.04292	0.0189	0.0284	5.84	2.45	5.55	3.70	3.1	1.9	3.1
32	1	2.31	3	0.109	6	0.760	0.500	20.95	30.43	0.3374	0.0218	0.0464	0.0263	0.0325	5.83	2.74	4.84	3.92	3.5	1.7	3.5
6	2	1.71	3	0.11	6	0.810	0.500	19.05	20.81		0.01	0.0158	0.0173		9.58	6.06	5.53		4.4	1.9	8
7	2	1.46	3	0.11	6	0.699	0.500	18.46	20.67		0.011	0.0158	0.0177		7.44	5.18	4.61		4.4	2.3	8.1
13	2	1.31	2.5	0.112	2.5	0.413	1.000	14.07	17.35		0.03	0.0231	0.0580		14.61	18.97	7.43		9.1	9.1	12.1
20	2	0.93	2.5	0.109	3.75	0.416	0.667	16.18	22.29		0.021	0.0203	0.0313		6.24	6.45	4.19		5.2	3.6	11.6
23	2	1.54	2.5	0.111	5	0.545	0.500	18.72	23.67		0.015	0.0158	0.0183		8.43	8.01	6.90		5.5	29	9.9
31	2	1.98	2.5	0.109	6	0.768	0.417	18.6	23.93		0.008	0.0125	0.0128		13.61	8.71	8.53		4.2	1.8	6.9
33	2	2.04	3	0.109	6	0.957	0.500	20.63	28.03		0.0095	0.0158	0.0168		11.81	7.10	6.67		3.3	1.6	4.7
37	2	1.48	2.5	0.109	2.5	1.050	1.000	20.12	27.29		0.0205	0.0231	0.0406		22.87	20.30	11.54		10.7	9	24
9	3	1.51	3	0.112	6	0.457	0.500	15.92	21.33	0.6289	0.022	0.0142	0.0267	0.0165	3.99	6.18	3.29	5.32	2.5	1.6	5.5
14	3	1.31	2.5	0.112	2.5	0.519	1.000	15.77	19.93	0.4655	0.0355	0.0244	0.0703	0.0361	12.34	17.96	6.24	12.13	11.3	11.3	20
19	3	0.89	2.5	0.109	3.75	0.432	0.667	18.74	24.63	0.5802	0.028	0.0179	0.0431	0.0249	4.48	7.00	2.91	5.03	4.8	4.3	6.7
24	3	1.6	2.5	0.111	5	0.512	0.500	18.2	22.87	0.6246	0.021	0.0142	0.0262	0.0163	6.26	9.26	5.01	8.09	5	2.9	6.4
30	3	1.82	2.5	0.109	6	0.704	0.417	17.91	22.4	0.6291	0.015	0.0125	0.0183	0.0115	6.67	8.01	5.46	8.74	4.7	2.3	4.7
34	3	2.18	3	0.109	6	0.994	0.500	18.22	25.71	0.5431	0.014	0.0142	0.0228	0.0123	8.57	8.44	5.27	9.76	3	2.2	3.9
38	3	1.34	2.5	0.109	2.5	0.832	1.000	18.22	27.95	0.3805	0.0306	0.0244	0.0556	0.0241	13.87	17.40	7.63	17.60	9	9	18.8
16	4	0.84	2.5	0.109	2.5	0.336	1.000	19.16	24.49		0.0493	0.0972	0.1657		5.40	2.74	1.61		8	8	14
21	4	0.93	2.5	0.109	3.75	0.460	0.667	18.9	24.32		0.0415	0.0558	0.0998		3.16	2.35	1.31		3.9	3.6	4
25	4	1.48	2.5	0.111	5	0.550	0.500	18.28	23.96		0.035	0.0293	0.0685		3.47	4.15	1.78		2.6	2.6	3.9
29	4	1.85	2.5	0.109	6	0.726	0.417	17.08	21.57		0.03	0.01855	0.0494		3.39	5.49	2.06		2.4	2.4	3.5
35	4	1.53	3	0.109	6	0.760	0.500	18.41	24.82		0.0314	0.0293	0.0584		2.68	2.87	1.44		1.7	1.7	2.5
10	5	1.08	3	0.112	6	0.400	0.500	16.04	20.51		0.031	0.022	0.0893		2.02	2.85	0.70		1.7	1.7	1.7
15	5	1.31	2.5	0.112	2.5	0.493	1.000	14.65	20.49		0.083	0.039	0.1683		5.28	11.24	2.60		7.8	7.8	7.8
17	5	1.02	2.5	0.109	3.75	0.451	0.667	19	25.06		0.067	0.03	0.1234		2.14	4.79	1.16		3.4	3.4	3.4
26	5	1.46	2.5	0.111	5	0.503	0.500	19.08	24.25		0.056	0.022	0.0832		2.14	5.45	1.44		2.7	2.7	2.7
28	5	1.83	2.5	0.109	6	0.728	0.417	18.91	23.92		0.043	0.022	0.0576		2.34	4.58	1.75		2.3	2.3	2.3
36	5	1.6	3	0.109	6	0.904	0.500	19.52	25.86		0.047	0.022	0.0656		1.87	4.00	1.34		1.9	1.9	1.9

Boundary Condition reference number (B.C.):

- 1 = all sides simply supported
- 2 = all sides built-in
- 3 = simply supported top and bottom, built-in sides
- 4 = simply supported bottom, free top, built-in sides
- 5 = simply supported bottom and sides, free top

H- Wall height

t - wall thickness

L - length of wall

$\mu$  - orthogonal strength ratio =  $f_{lx} / f_{ly}$

Asp. Ratio = H / L

Ex - horizontal beam elastic modulus

Ey - vertical beam elastic modulus

$\beta$  - a factor derived for fracture line method by Sinha, see reference 3

**Table 2: Ratios of Theoretical to Experimental Failure Loads**

Test no.	B.C.	Full-Crack Pattern Load				Initial-Crack Load				Ultimate Load			
		YL	Elastic	Strip	Fracture Li	YL	Elastic	Strip	Fracture Li	YL	Elastic	Strip	Fracture Li
27	1	1.25	0.65	1.12	0.76	2.35	1.21	2.09	1.43	1.17	0.61	1.05	0.72
8	1	0.60	0.76	0.42	0.46	0.68	0.86	0.47	0.52	0.60	0.76	0.42	0.46
22	1	0.69	0.59	0.63	0.44	1.16	1.00	1.06	0.75	0.69	0.59	0.63	0.44
32	1	0.70	0.44	0.73	0.45	1.06	0.66	1.10	0.68	0.70	0.44	0.73	0.45
18	1	1.88	0.79	1.79	1.19	3.07	1.29	2.92	1.94	1.88	0.79	1.79	1.19
12	1	1.67	0.78	1.38	1.12	3.43	1.61	2.85	2.30	1.67	0.78	1.38	1.12
31	2	2.18	1.38	1.26		5.04	3.19	2.91		1.20	0.76	0.69	
6	2	1.69	1.18	1.05		3.23	2.25	2.01		0.92	0.64	0.57	
7	2	1.61	2.08	0.82		1.61	2.08	0.82		1.21	1.57	0.61	
23	2	1.20	1.24	0.81		1.73	1.79	1.16		0.54	0.56	0.36	
33	2	1.53	1.46	1.25		2.91	2.76	2.38		0.85	0.81	0.70	
20	2	3.24	2.07	2.03		7.56	4.84	4.74		1.97	1.26	1.24	
13	2	3.58	2.15	2.02		7.38	4.44	4.17		2.51	1.51	1.42	
37	2	2.14	1.90	1.08		2.54	2.26	1.28		0.95	0.85	0.48	
30	3	1.59	2.47	1.31	2.13	2.49	3.86	2.05	3.33	0.72	1.12	0.60	0.97
9	3	1.09	1.59	0.55	1.07	1.09	1.59	0.55	1.07	0.62	0.90	0.31	0.61
24	3	0.93	1.46	0.61	1.05	1.04	1.63	0.68	1.17	0.67	1.04	0.43	0.75
34	3	1.25	1.85	1.00	1.62	2.16	3.19	1.73	2.79	0.98	1.45	0.78	1.26
19	3	1.42	1.70	1.16	1.86	2.90	3.48	2.37	3.80	1.42	1.70	1.16	1.86
14	3	2.86	2.81	1.76	3.25	3.89	3.84	2.39	4.43	2.20	2.17	1.35	2.50
38	3	1.54	1.93	0.85	1.96	1.54	1.93	0.85	1.96	0.74	0.93	0.41	0.94
29	4	0.67	0.34	0.20		0.67	0.34	0.20		0.39	0.20	0.11	
25	4	0.81	0.60	0.34		0.88	0.65	0.36		0.79	0.59	0.33	
35	4	1.34	1.60	0.68		1.34	1.60	0.68		0.89	1.06	0.46	
21	4	1.41	2.29	0.86		1.41	2.29	0.86		0.97	1.57	0.59	
16	4	1.58	1.69	0.85		1.58	1.69	0.85		1.07	1.15	0.58	
28	5	1.19	1.68	0.41		1.19	1.68	0.41		1.19	1.68	0.41	
10	5	0.68	1.44	0.33		0.68	1.44	0.33		0.68	1.44	0.33	
26	5	0.63	1.41	0.34		0.63	1.41	0.34		0.63	1.41	0.34	
36	5	0.79	2.02	0.53		0.79	2.02	0.53		0.79	2.02	0.53	
17	5	1.02	1.99	0.76		1.02	1.99	0.76		1.02	1.99	0.76	
15	5	0.99	2.11	0.71		0.99	2.11	0.71		0.99	2.11	0.71	

Figure 1: Comparison with Full-crack load

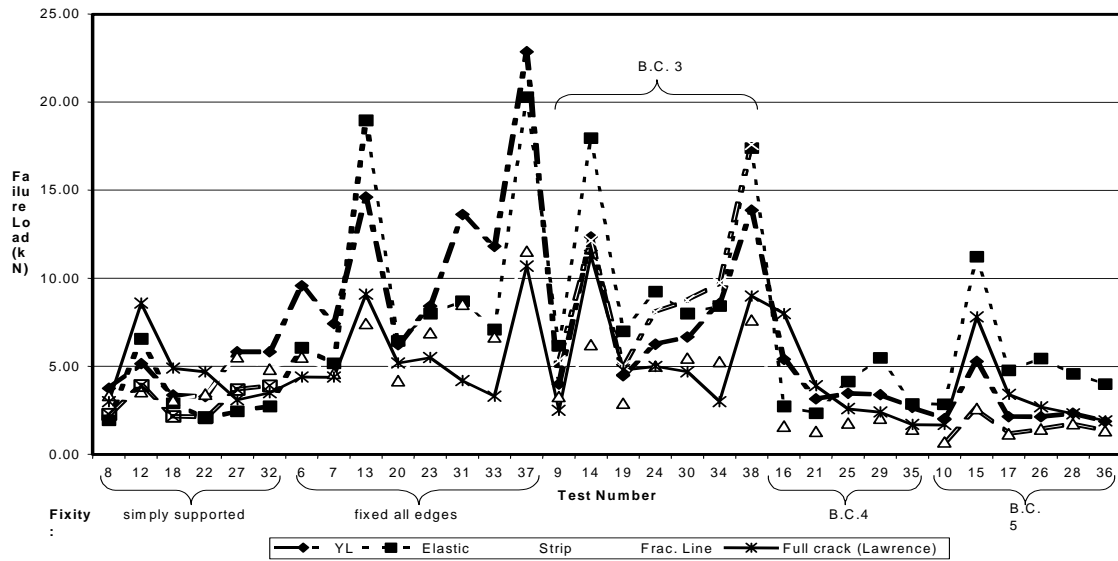


Figure 2: Comparison with Initial Crack Load

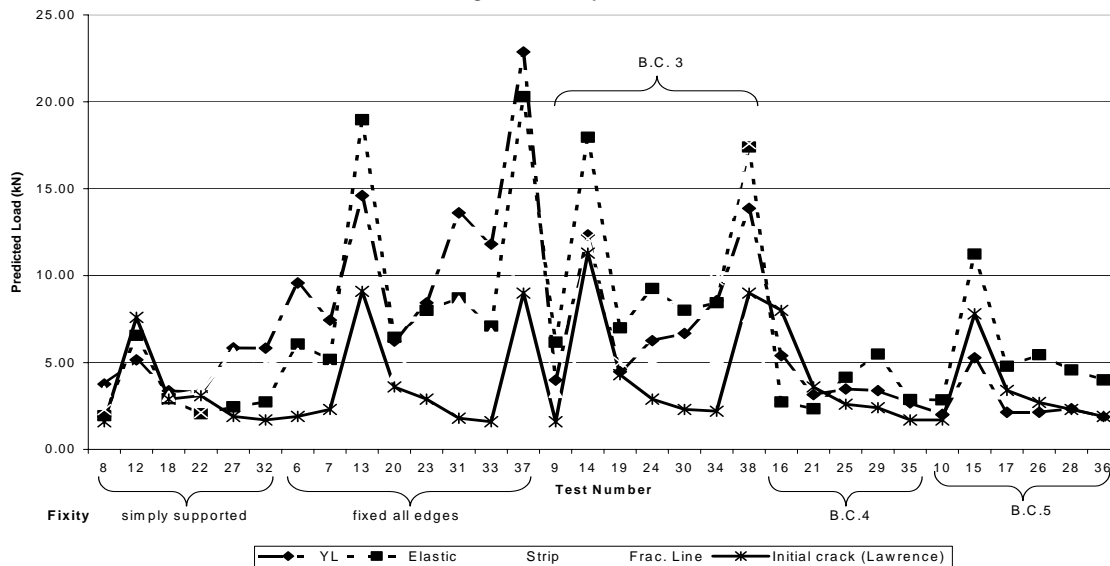


Figure 3: Comparison with Ultimate Load

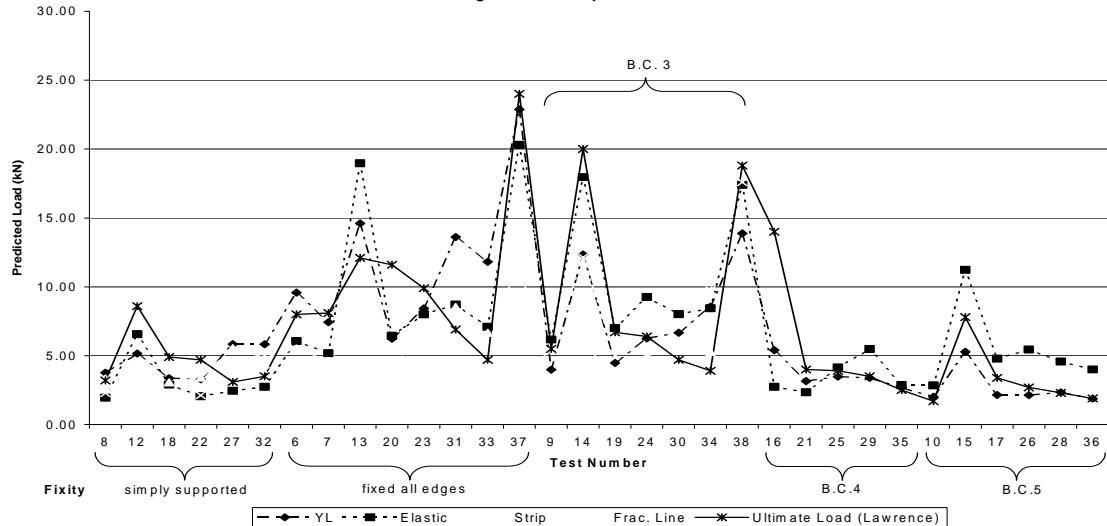




Figure 4: Simply Supported Panels

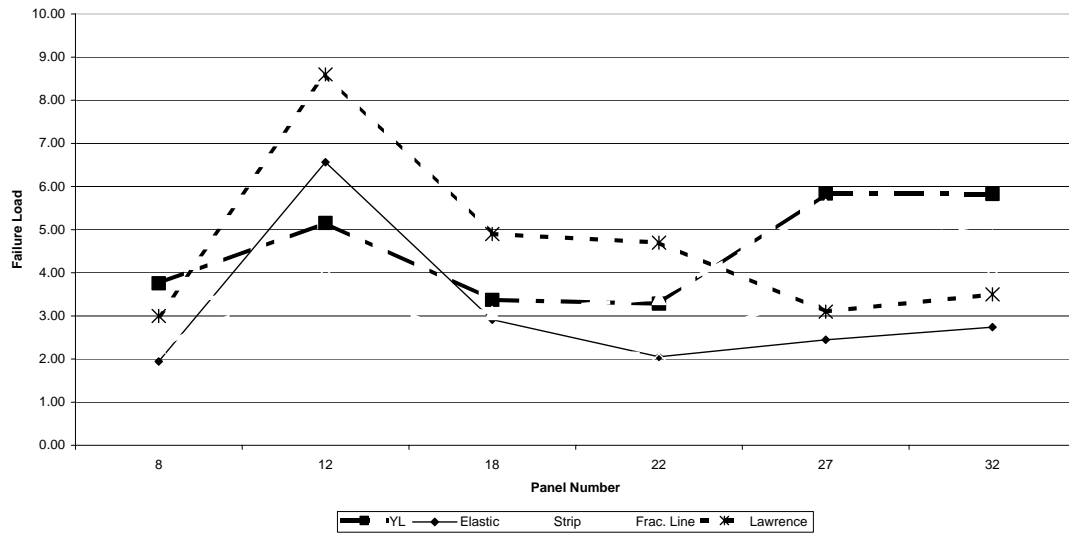


Figure 5: Panels with All Edges Fixed

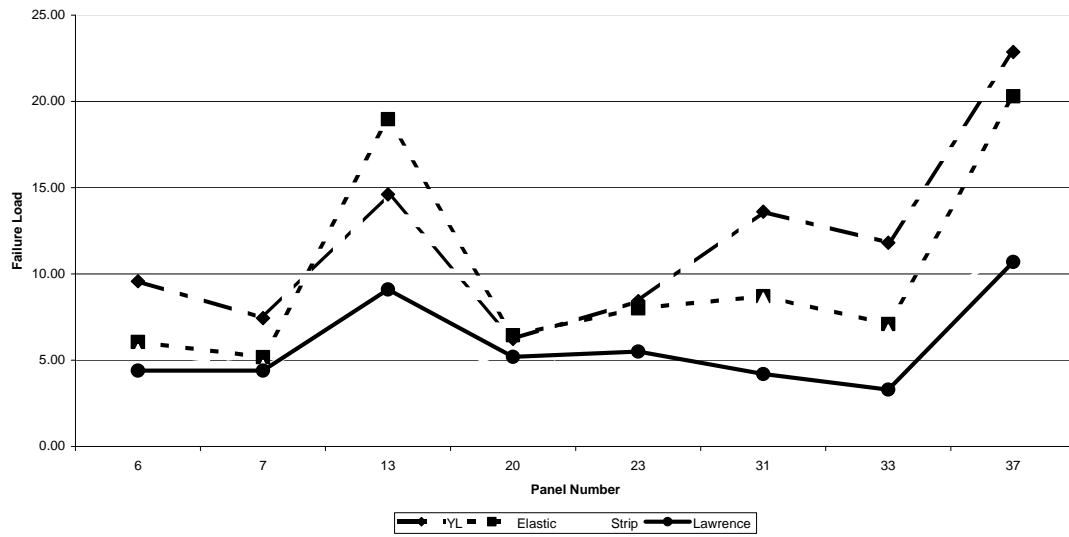


Figure 6: Boundary condition 3

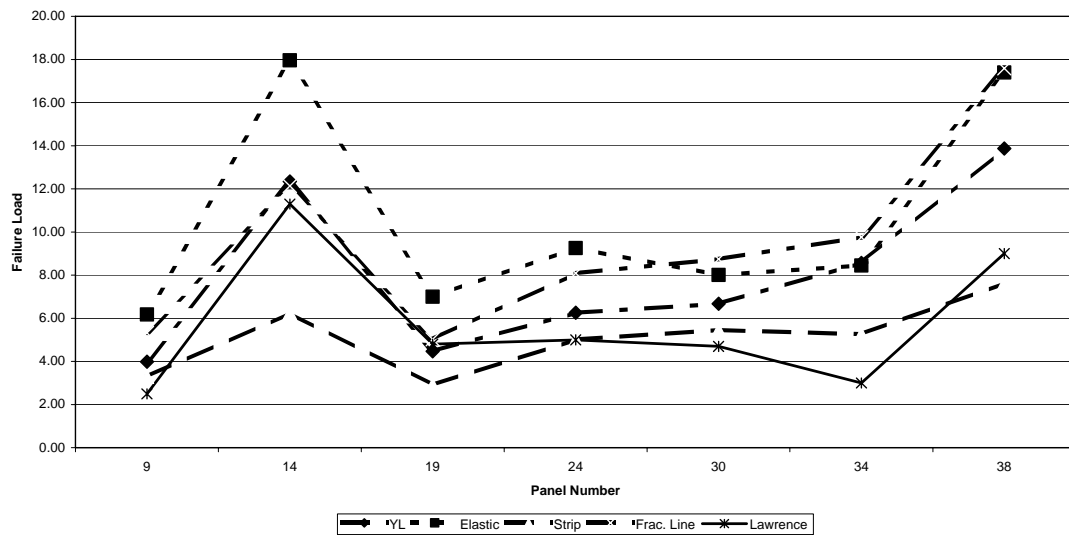


Figure 7: Boundary Condition 4

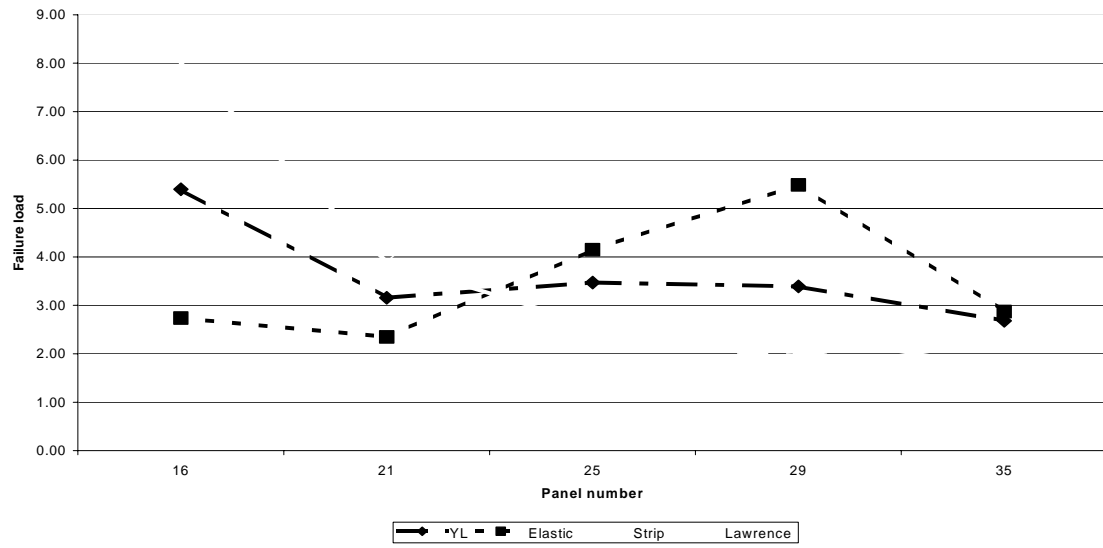


Figure 8: Boundary Condition 5

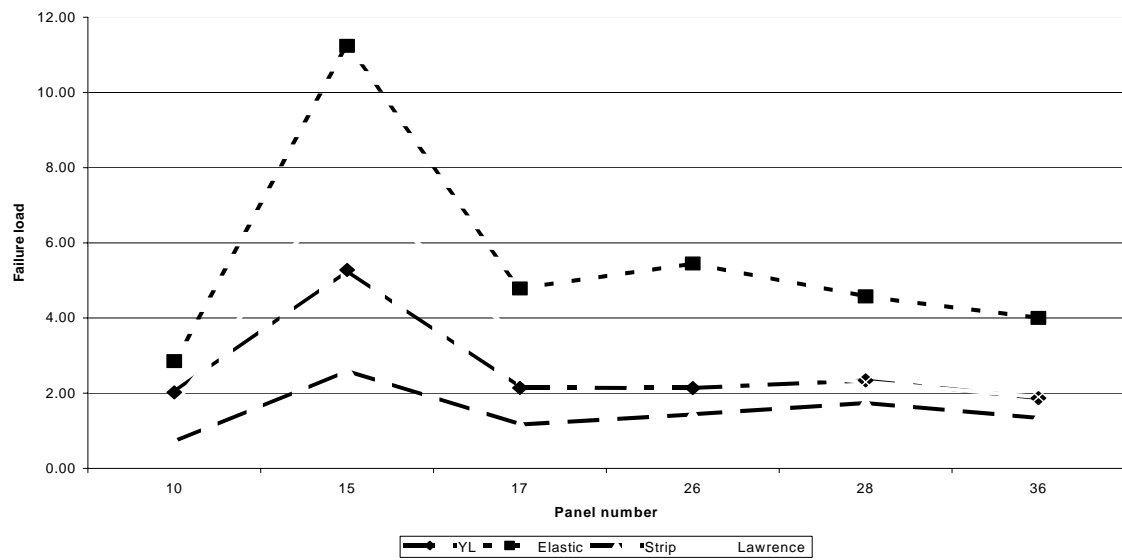


Figure 9: Ratio of theory to Test Results (full crack pattern)

