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Numerical Modelling of Blockwork Prisms Tested in Compression Using Finite Element Method with Interface Behaviour

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Abstract

The aim of this work was the development of a mathematical model in order to numerically simulate the uni-axial compression test of concrete block prisms. The structural response of masonry prisms was obtained using the Finite Element Method. The yield criterion of Drucker-Prager with isotropic hardening was implemented for compression, while the smeared crack model was used for material under tension. The friction criterion of Coulomb was also included in the model to represent the non-linear behaviour of the interface between mortar and block.
The type of prism used to calibrate to model was of un-grouted concrete block prisms, with both shell face and webs filled mortar joints. A non-linear material 3-D analysis, considering small strains and displacements, was performed on the prisms.
The analytical results were compared with the experimentally obtained stress-strain curves, showing very good agreement.

Key Words

Blockwork, compressive strength, numerical modelling, interface.

1 Introduction

The test of full panel in compression is accepted as the best way to evaluate the behaviour of structural masonry. However, the cost of this type of test is very high and it is not easy to find laboratories able to perform it. Therefore, compression test with masonry prisms, either of 2 or 3 courses high have been largely used. Satisfactory mathematical models do not follow the large number of prism test results available. These are generally simplified equations obtained from statistical adjustment of the experimental data.
Models from finite element analysis are generally based on linear elastic analysis.
The non-linear analysis is extremely important when a complete structural response of the prism behaviour is needed, starting from the beginning of the load process up to the failure, including intermediary stages when the first cracks appear. They also allow better understanding of the stress distribution along the prism, and, more important, to establish how these stresses develop during the entire process of loading.

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In order to have these answers a non-linear model was developed using the classic theory of plasticity concepts to simulate the material lost of compression strength and a smeared cracking model to represent the lost of tensile strength at defined points of the prism (Gomes 2001). This paper describes part of the model.

2 Type of prim investigated

The behaviour of concrete block prism with full joint filled with mortar submitted to compression was studied. Figure 1 shows the type of concrete block investigated. The prism type may be seen in Figure 2.

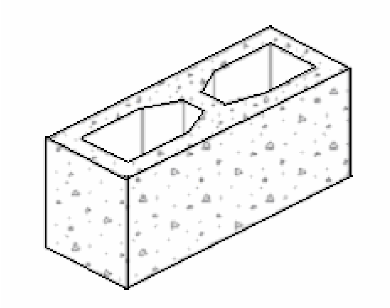

Figure 1 Concrete block type

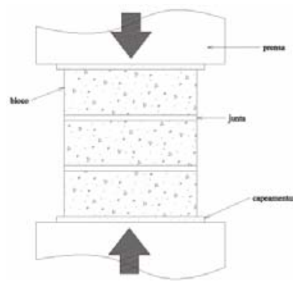

Figure 2 Type of prism modelled

This type of prism had been previously tested in laboratory and presented the failure mode as shown in Figure 3.

Figure 3 Type of failure of prisms with the full joint filled

3 Model description

The plasticity theory and the smeared cracking model were used to numerically simulate the uni-axial compression prism test on the mathematical model. Finite element solid quadratic with 20 nodes were used to discrete both blocks and mortar joints. Discontinuous finite elements with 16 nodes were used to model the interface. These can be seen in Figure 4 and 5 respectively.

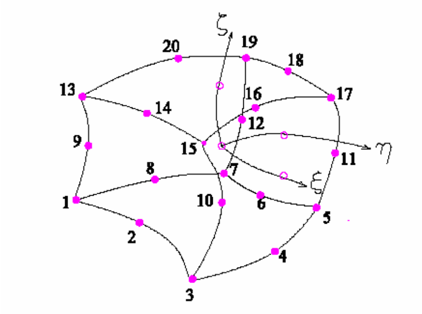

Figure 4 Twenty nodes quadratic finite element used to model block and mortar joint

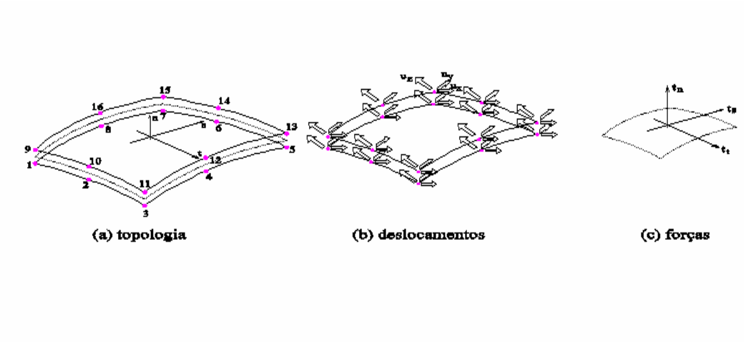

Figure 5 Sixteen nodes quadratic finite element used to model the interface a) topology, b)displacement, c) forces.

The Drucker-Prager criterion was used to the compressed area and the smeared crack model for the tensile area. A crack initialisation criterion was applied to define the cracking beginning as shown in Figure 6. Therefore, on the compressed area the Drucker-Prager criterion is directly applied whilst for the tensioned zone the Figure 6 criterion is first applied. If the integration point satisfies the beginning criteria, the smeared crack model is used to calculate the elastic-plastic stiffness matrix tangent. The latest model allows the occurrence of multiple cracks at the same integration point and may easily to be connected to the plasticity theory.

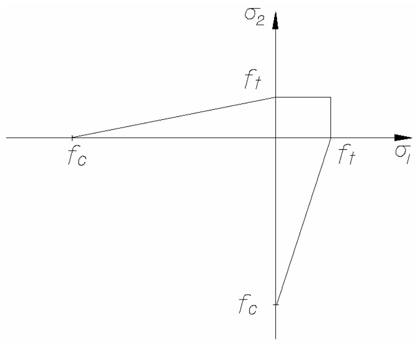

Figure 6 Cracking beginning criterion

The classical theory of plasticity was used in order to model the non-linear behaviour of the interfaces. The Coulomb criterion was adopted to take into consideration the interface shear strength increment due to pre-compression. The interfaces elastic properties (string constants) were initially taken with high values to avoid the interface influence on the structural prism behaviour during the elastic period. Therefore, it is assumed that on the initial stage there are no defects on the interfaces and, as a consequence, there is complete adhesion between block and mortar joint with integral shear stress transmission. Table 1 shows the shear strength taken as reference to the Coulomb criterion.

Table 1 Experimental results for shear strength extracted from Roman e Sinha (1994)

Confinement stress (N/mm^2)	Shear strength (N/mm^2)	Type of failure
0.00	0.55	joint
0.88	0.90	Joint
1.54	0.87	Joint
2.92	1.40	Joint
6.13	1.42	block

To solve the non-linear problem the traditional incremental-iterative method associated to the restricted Newton-Raphson method was used. A severe limitation of the later is not globally convergent, i.e., does not converge for a non-linear equation system solution from any initial point. The strategy was to use the line search method to accelerate the convergence process and make it more stable.
The use of energetic convergence criteria was considered more attractive. It takes into account the effect of loads and strain simultaneously. A tolerance of 10^{-4} was adopted for all analysis. This number was later changed when the convergence was lengthy.

4 Results of the model

4.1 Elastic and plastic properties of materials

Two experimental programs previously done were used to collect the data to evaluate the model (Mohamad 1998, Romagna 2000). Figure 7 shows one type of data used to model the prism.

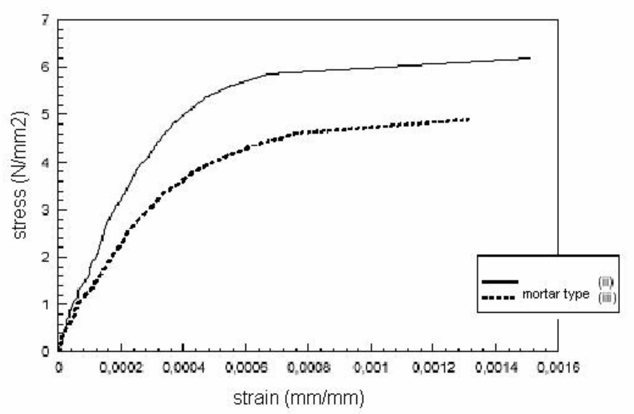

Figure 7 stress x strain relationship for prisms of two different mortars (Mohamad 1998)

The elastic and plastic properties used to run the model can be seen in Tables 2 and 3.

Table 2 Elastic and plastic properties of materials for Em/Eb = 0.34

Material	**Elastic properties (N/mm^2)**	**Plastic properties (N/mm^2)**
Mortar	E = 5781 ν = 0.12	f_c = 3.90 f_t = 0.38 c = 0.48 (initial) c = 0.62 (final)
Concrete block	E = 15540 ν = 0.17	f_c = 17.68 f_t = 1.31 c = 1.35 (initial) c = 5.11 (final)
Interface	$k_n = k_s = k_t = 1x10^6$	c = 0.51 tan ϕ = 0.80

f_c e **f_t** : Compressive and tensile strength, respectively; **c**(initial and final) : initial and final cohesion; **k_n, k_s, k_t** : string constants

The c values were calculated as function of the compressive strength and of the angle of internal friction of materials. The need of an initial and a final value for the cohesion is due to the considered assumption of isotropic hardening adopted for Drucker-Prager criterion. The initial value was established as function of the elastic range of the stress x strain curve. The final value was calculated based on the compressive strength and on the angle of internal friction. The dilatation angle was taken equal to the angle of internal friction, which was considered constant as 25º (traditional for concrete) for all materials. Consequently, the stiffness matrix of the element was symmetric, decreasing the computer processing time. This hypothesis was adopted due to the few number of experiments, which could produce a reliable value for the dilatancy angle. Also, this simplification was taken because the volume expansion was considered small.

Table 3 Elastic and plastic properties of materials for E_m/E_b = 0.67

Material	**Elastic properties (N/mm^2)**	**Plastic properties (N/mm^2)**

Mortar	E = 11055 ν = 0.20	f_c = 19.90 f_t = 1,60 c = 2.32 (initial) c = 5.75 (final)
Concrete block	E = 16400 ν = 0.17	f_c = 18.20 f_t = 1.19 c = 2.02 (initial) c = 5.26 (final)
Interface	$K_n = k_s = k_t = 1x10^6$	c = 0.82 tan ϕ = 0.60

The plastic properties were extracted from experimental stress-strain relationship of materials. To validate the mathematical model two different E_m/E_b relationships were used in the simulation. The properties for E_m/E_b equal to 0,34 were taken from Mohamed (1998). The properties for E_m/E_b equal to 0,67 were taken from Romagna (2000).
It also can be seen in Tables 2 and 3 that the interface elastic constants are very high and, therefore, may be considered as infinite when comparing with the elastic properties of concrete block and mortar. As a consequence, with low level of stress at the beginning of loading the interfaces have no effect on the analysis. After a certain value of stress related to the angle of internal friction, the Coulomb model is responsible for the interfaces behaviour. The interfaces plastic properties, i.e., cohesion and friction angle were taken from Table 1.

4.2 Finite element model

The DIANA computational program was used to perform the numerical simulation. Figure 8 shows the finite element mesh used to model the prism with all joint faces filled with mortar. It may be noticed that the mesh presents neither distortion nor excessively disproportional elements. The block nominal dimensions were (140x190x390) mm (thickness x high x length). The block flange and web dimensions were taken from Figure 1. The joint thickness was 10 mm. Due to symmetry only one forth of the prism was represented. The total number of elements was 210 with 1540 nodes.

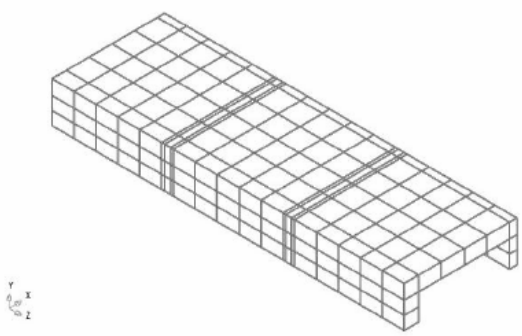

Figure 8 Finite element mesh used

The X and Y-axis are parallel to the joint lay whilst the Z-axis is normal to the joint and parallel to the loading direction. The four interfaces are disconnected from both block and joint as shown in Figure 8 and 5 mm thickness was considered to generate the

finite element mesh. The DIANA program disregards these distances taken as the total prism height as the sum of 3 blocks height and 2 joint thickness.

The loads were applied uniformly on the top of the prism as seen in Figure 9.

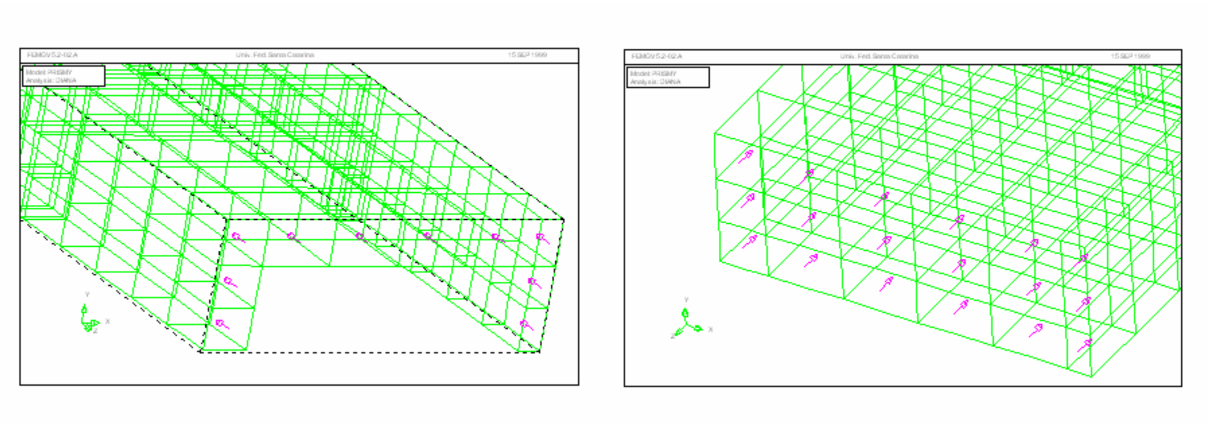

Figure 9 Increasing uniform pressures applied on top of the prism

4.3 Prism strength and mode of failure

Figures 10 and 11 show the stress-strain relationship for prisms with rates E_m/E_b = 0; 34 and E_m/E_b = 0; 67 respectively.
The most efficient way to evaluate the quality of simulation is to compare the experimental and theoretical stress x strain curves. If there is good agreement between both curves it means that the computational program is able to capture the stiffness changes along the whole loading and therefore, the model is able to draw the structural prism response.
In order to evaluate the failure point of the model the general picture of cracks produced was compared with the experimental cracks. With one sole exception, when failure was for compression, the failure was generally characterized by the appearance of tensile cracks. This means that the softening in compression, which was not taking into account due to lack of experimental results, was not influencing the programme results. This fact also allows concluding that the volume expansion is small.
As can be seen in Figure 10, for E_m/E_b = 0,34, the proposed model gives very close results to the experimental ones. The failure stress found by the program is 11,7 N/mm^2 whilst the test result was 12,5 N/mm^2. The difference between the results was of 6,5 %. It is important to say that the experimental curve was not taken up to failure due to safety reasons. The stress-strain curve for E_m/E_b = 0,67 shown in Figure 11 allows to see a good approximation on the beginning, but after a stress of around 5 N/mm^2, there is probably a re-distribution of stresses not given by the theoretical model.

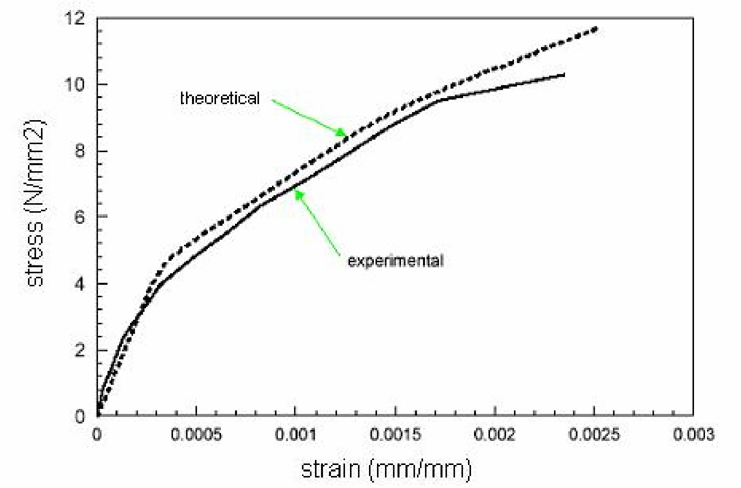

Figure 10 Stress x strain curve for E_m/E_b *= 0,34*

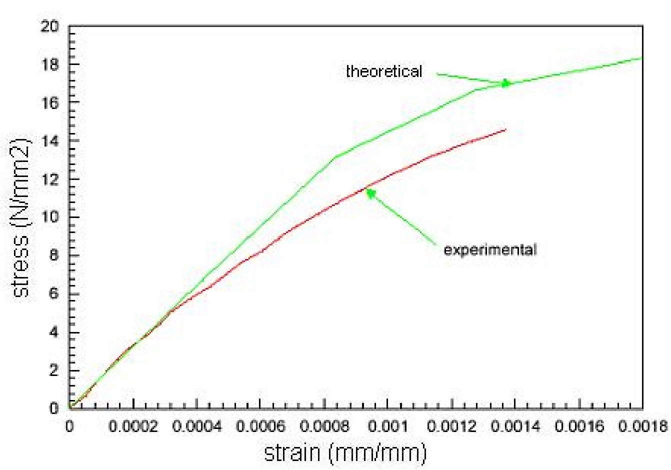

Figure 11 Stress x strain curve for E_m/E_b *= 0; 67*

Figures 12 and 13 show the cracking evolution for both E_m/E_b relationship.

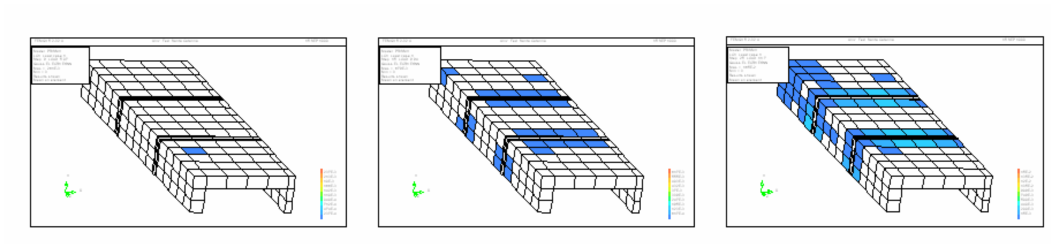

Figure 12 Cracking evolution for prisms with E_m/E_b *=0.34*

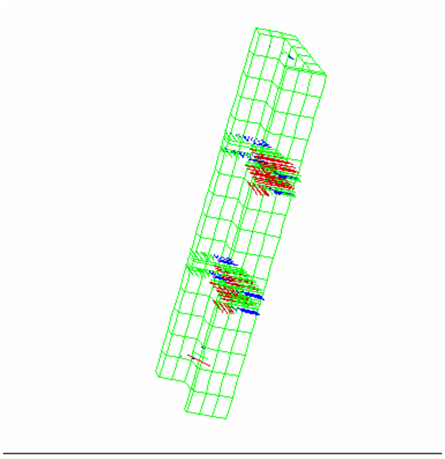

Figure 13 Cracking evolution for prisms with E_m/E_b =0.67

5 Conclusion

The proposed mathematical model proved to be sensitive to the variation of the interfaces plastic properties. In general way very good results were achieved to different materials and stiffness proportions. The model response was able to follow the experimental curve and predict the failure load with good accuracy. For both very low and very high relations of mortar block Young's modulus the model took more time to converge.

The Drucker-Prager criterion used to simulate the material in compression and the smeared crack model used for tensile stress regions seem to be adequate to obtain the structural response of concrete block prisms.

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