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FINITE ELEMENT ANALYSIS AND APPROXIMATE CALCULATION OF MASONRY WALL SUPPORTED ON TWO-SPAN RC FRAME

X. Li[1], S.X. Gong[2], and X.L. Gu[3]

Abstract

In this paper, a finite element model for masonry wall supported on two-span RC frame is established, and the corresponding finite element analysis for the composite structure, which has been proved to be effective by the test, is conducted. Through the extensive theoretical calculation, the distribution of inner forces inside the composite structure and the force transfer between the panel masonry wall and the two-span RC frame are studied. Based on the analysis results, a set of approximate calculation equations to determine the inner forces for critical sections of the masonry wall without opening and the supporting RC frame are proposed.

Key Words

Finite element analysis, masonry wall, supporting RC frame, inner force

Notations

b_b = width of the section of the frame beam
b_c = width of the section of the frame column
b_s = width of the section of the structural concrete column
b_t = width of the section of the top beam
F_n = resultant force of the horizontal loads applied on the top beam
H_c = clear height of the frame column
H_0 = calculation height of the wall-beam in the mid section
h = thickness of the masonry wall
h_b = height of the section of the frame beam
h_c = height of the section of the frame column
h_s = height of the section of structural concrete column
h_t = height of the section of the top beam
h_w = height of the masonry wall
l_1 = length of the longer span
l_2 = length of the shorter span
Q_v = resultant force of vertical loads applied on the top beam
q_h = horizontal uniformly distributed loads applied on the top beam
q_v = vertical uniformly distributed loads applied on the top beam

1. X. Li, Tongji University, Shanghai 200092, China, lixiang-tongji@tom.com
2. S. X. Gong, Tongji University, Shanghai 200092, China
3. X. L. Gu, Tongji University, Shanghai 200092, China, gxiangl@online.sh.cn

1 Introduction

Composite structural members of masonry wall and supporting reinforced concrete frame are widely used in multistory masonry structures when the large space is needed in the first story of the building now in China. There are lots of papers in which the behavior of the masonry wall supported on continuous RC beam has been studied [Gong et al. 1997, Gong et al. 2001, Li et al. 2001, and Page 1978]. Based on the existing research results, the Design Code for Masonry Structures (GBJ3-88) in China presented some regulations for one-span frame under vertical top uniformly distributed loads. But it is still a problem how to design the masonry wall without opening supported on RC frame with more than one span under vertical and horizontal loads in seismic regions.

Among the composite structural members of masonry wall and supporting RC frame, the member of masonry wall supported on two-span RC frame is the most popular in real building structures. The interaction between the reinforced concrete frame and the adjacent upper masonry wall under the vertical and horizontal loads has been proved by the test [Li et al. 2004]. This paper focuses on the theoretical analysis of this kind of the structure. Through the finite element analysis for a set of designed structural members, the behavior of the structure is studied and approximate calculation equations to determine the inner forces of the frame are proposed.

2 Finite element analysis method

2.1 Finite element analysis model

As shown in Figure 1, according to the material character and the dimension of real structural member, finite element analysis model is set up. Vertical and horizontal loads are applied on every node of the element on the top beam uniformly. Plane rectangular element with 4 nodes for the masonry wall or the concrete member is used in the model. And the mechanical behavior of the element is determined according to the test results of masonry or concrete material. Meanwhile, it is presumed that the bond between the masonry wall and the RC structural member is reliable, no any slip happens there. Based on the finite element model, a computer program is developed. The grid coordinates to form the mesh of elements can be created automatically by the program. The inner forces for every section of the structural member can be calculated from the stress distribution in every element deduced by finite element analysis.

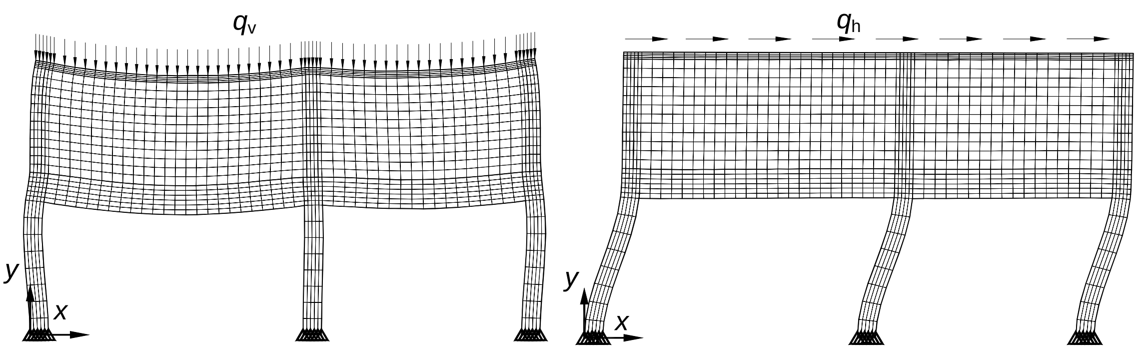

Figure 1 Models of Finite Element Analysis

2.2 Verification by the test

Figure 2 shows the comparison of calculation and testing results of horizontal load-displacement relationship for 3 specimens did by the authors [Li et al. 2004]. To

simulate the elasto-plastic behavior of the specimen, the global rigidity of the composite structural member was adjusted with the increase of the horizontal load according to the deterioration law of the rigidity observed in the test. So, the nonlinear load-displacement relationship for the testing specimen can be calculated using the finite element model mentioned above. From Figure 2, it can be found that the finite element analysis based on the model shown in Figure 1 is an effective way to analyse the mechanical behavior of the composite structure of brick wall and two-span RC frame without opening.

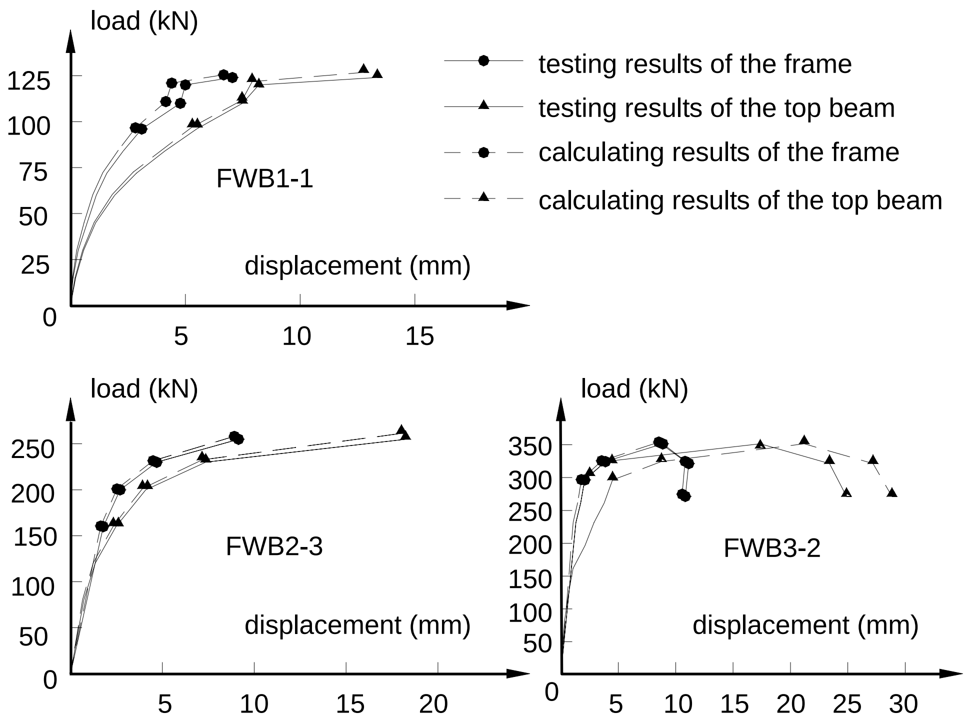

Figure 2   Comparison of calculation and testing results

3   Numerical simulation analysis

3.1   Orthogonal design of analysis members

For the purpose to understand the main factors to affect the mechanical behavior of the composite structure of masonry wall and supporting RC frame, a thorough numerical simulation analysis for different kind of structural members is carried out. To optimize the number of composite structural members, the orthogonal design method is used. As a mathematic analysis method, the orthogonal design is used to solve the multi-factors contrast test, which is used in the design of specimens to accomplish the purpose of reducing analysis works. The orthogonal table of L16 (5^4) is used in the analysis and shown in Table 1, i.e. there are 16 members, 5 factors, and 4 levels. The interlacing action between any couple of factors is not considered.

3.2   Behavior of the composite structure under vertical top uniformly distributed loads

According to the results of finite element analysis, distribution of horizontal normal stress σ_x^v, vertical normal stress σ_y^v and shear stress τ_{xy}^v for specimen 14 in Tab.1 are shown in Figure 3. Under vertical uniformly distributed load q_v applied on the top of

the wall, the wall and the top beam in the mid section of the member are compressed and the frame beam is compressed at the upside of the section and tensioned at the downside of the section in the middle of the span. The situation in the section near the support of the frame beam is reversed. So, the frame beam is an eccentrically tensile member, and the top beam is an eccentrically compressive member. Because the effect of arch action exists in the member, the vertical normal stress in the wall concentrates to the support of wall-beam, especially in the structural concrete column. The maximum vertical stress σ^{v}_{ymax} inside the structural column is about 1.6 times of q_v, the local compression of masonry above the frame column is reduced by the structural concrete column. The distribution of axial force between the frame columns has been affected by the arch action (Figure 4). The axial compressive force in the side-column is more than that of a common frame, and the contraflexure point of the column is located at 0.35~0.39 times of the column height. Considering the interaction between the wall and the frame, the bending moment in the middle of the frame beam is about (1/64 ~ 1/320)$q_v l^2$, the shear force of the beam near the side support is about 0.12~0.17$q_v l$, the inner forces inside the beam of the frame are reduced.

Table 1 Orthogonal design of simulation analysis members

No. of group	factor					parameter (mm)			
	A (l_2/l_1)	B (h_b/l_1)	C (h_w/l_1)	D (H_c/l_1)	E (E_c/E_m)	l_2	h_b	h_w	H_c
1	1 (1.0)	1 (1/ 8)	1 (0.60)	1 (0.80)	1 (12.4)	6000	750	3600	4800
2	1 (1.0)	2 (1/ 9)	2 (0.55)	2 (0.73)	2 (11.4)	6000	670	3300	4400
3	1 (1.0)	3 (1/10)	3 (0.50)	3 (0.67)	3 (10.4)	6000	580	3000	4000
4	1 (1.0)	4 (1/12)	4 (0.45)	4 (0.60)	4 (9.50)	6000	500	2700	3600
5	2 (0.9)	1 (1/ 8)	2 (0.55)	3 (0.67)	4 (9.50)	5400	750	3300	4000
6	2 (0.9)	2 (1/ 9)	1 (0.60)	4 (0.60)	3 (10.4)	5400	670	3600	3600
7	2 (0.9)	3 (1/10)	4 (0.45)	1 (0.80)	2 (11.4)	5400	580	2700	4800
8	2 (0.9)	4 (1/12)	3 (0.50)	2 (0.73)	1 (12.4)	5400	500	3000	4400
9	3 (0.8)	1 (1/ 8)	3 (0.50)	4 (0.60)	2 (11.4)	4800	750	3000	3600
10	3 (0.8)	2 (1/ 9)	4 (0.45)	3 (0.67)	1 (12.4)	4800	670	2700	4000
11	3 (0.8)	3 (1/10)	1 (0.60)	2 (0.73)	4 (9.50)	4800	580	3600	4400
12	3 (0.8)	4 (1/12)	2 (0.55)	1 (0.80)	3 (10.4)	4800	500	3300	4800
13	4 (0.7)	1 (1/ 8)	4 (0.45)	2 (0.73)	3 (10.4)	4200	750	2700	4400
14	4 (0.7)	2 (1/ 9)	3 (0.50)	1 (0.80)	4 (9.50)	4200	670	3000	4800
15	4 (0.7)	3 (1/10)	2 (0.55)	4 (0.60)	1 (12.4)	4200	580	3300	3600
16	4 (0.7)	4 (1/12)	1 (0.60)	3 (0.67)	2 (11.4)	4200	500	3600	4000

3.3 Behavior of the composite structure under horizontal top uniformly distributed loads

Under horizontal uniformly distributed load applied on the top of the wall, the masonry wall works like a shear wall (Figure3), the section of the frame beam near the left support of the frame is eccentrically tensile, and the section near right support is eccentrically compressive at the same time. The left frame column is an eccentrically tensile member, and the right frame column is an eccentrically compressive member. Because of the interaction, the point of contraflexure of the column is located at 0.53~0.55 times of the column height (Figure 4), which is higher than that of a common frame in the first floor.

4 Calculation method of inner forces for critical sections of the frame

4.1 Under vertical top uniformly distributed loads

The distribution of bending moment M, axial tensile or compressive force N and shear

force V of critical sections of two-span frame can be produced under vertical uniformly distributed loads and horizontal top uniformly distributed loads according to the results of finite element analysis (Figure 3). As an example, the variation of moment in the mid section of the frame beam by every above influence factor according to the analysis results for structural members in Tab. 1 is shown in Figure 5. From Figure 5, it can be seen that the parameter (h_b/l_1) and (h_w/l_1) are the main influence factors. According to the least square method, the approximate equation (1) to calculate the moment is given. Accordingly, inner forces of critical sections of the frame shown in Figure 6 can also be determined directly, which are expressed with equation (2) to equation (30).

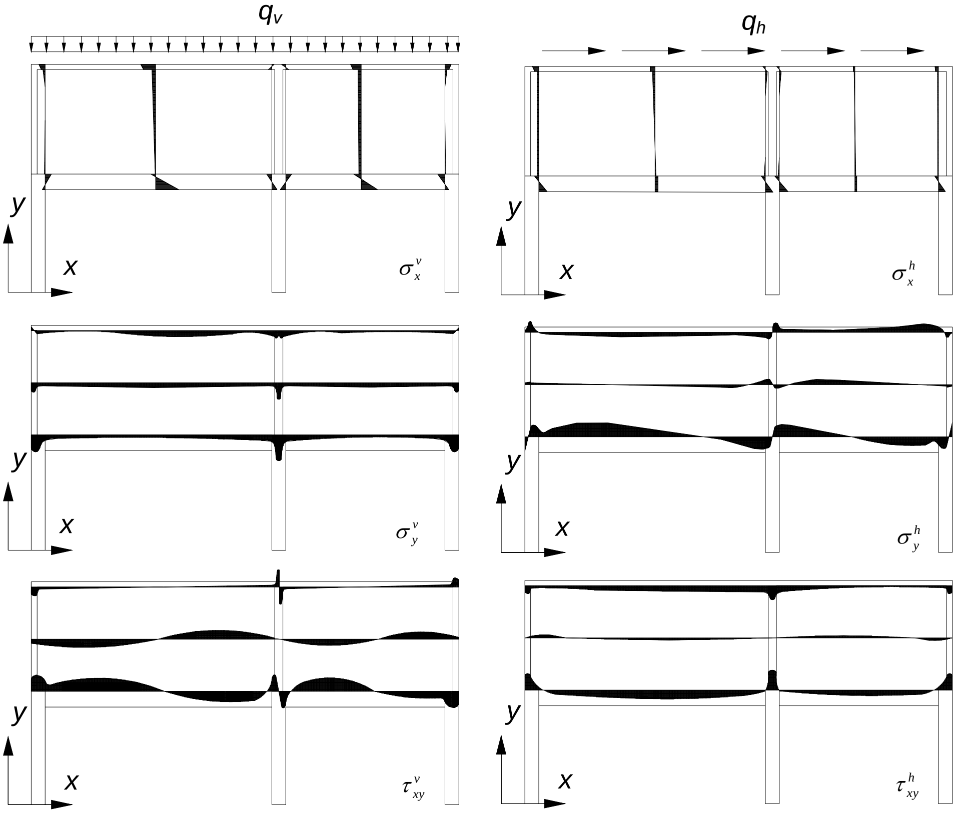

Figure 3　Distribution of stress inside specimen 14 in Table 1

For the section 1 in the middle of the longer span beam,

$$M_1 = (0.15h_b/l_1-0.009)(1.78-1.33h_w/l_1)\, q_v l_1^2 \quad (1)$$

$$N_1 = (0.07+0.14h_w/l_1)(0.77+0.32H_c/l_1)q_v l_1^2 / H_o \quad (2)$$

$$H_o = h_w + 0.5\, h_b \quad (3)$$

For the section 2 in the middle of the shorter span beam,

$$M_2 = (0.178\, h_b/l_1-0.009)\, (2.46-1.72l_2/l_1)\, q_v l_2^2 \quad (4)$$

$$N_2 = (0.06+0.16h_w/l_1)(0.71+0.43H_c/l_1)\, q_v l_2^2 / H_o \quad (5)$$

For the section A,

$$M_A = -\,(0.001+0.0004E_c/E_m)\, (1.46-0.65H_c/l_1)q_v l_1^2 \quad (6)$$

$$N_A = (0.05+0.92h_b/l_1)(0.13+1.24h_w/l_1)\, N_1 \quad (7)$$

$$V_A = (0.08+0.68h_b/l_1)(1.29-0.55h_w/l_1)\,q_v\,l_1 \qquad (8)$$

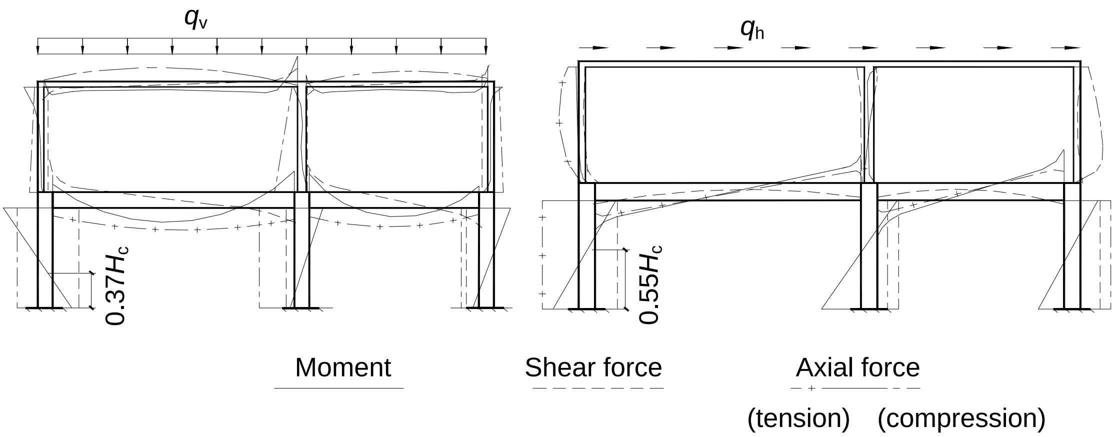

Figure 4 Envelope distribution of inner forces

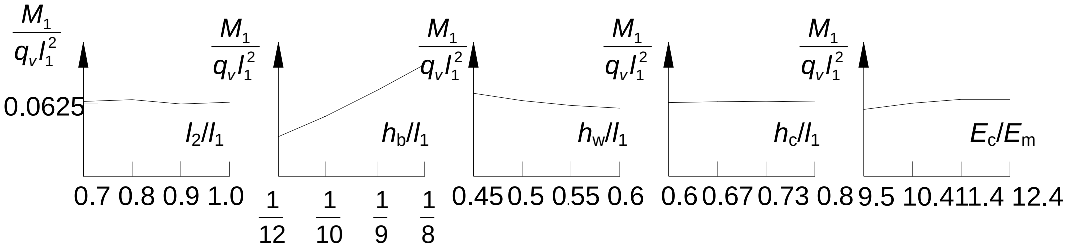

Figure 5 Variation of the moment with different kinds of influence factors

For the section *B*,

$$M_B = -(0.34+0.77l_2/l_1)(-0.006+0.21h_b/l_1)\,q_v\,l_1^2 \qquad (9)$$

$$N_B = (3.25-2.65l_2/l_1)(-0.12+3.83h_b/l_1)(-1.66+5.07h_w/l_1)\,N_1 \qquad (10)$$

$$V_B = -(0.12+1.11h_b/l_1)(0.56+0.52l_2/l_1)\,q_v\,l_1 \qquad (11)$$

For the section *C*,

$$M_C = -(-0.01+0.29h_b/l_1)(2.34-1.58l_2/l_1)\,q_v\,l_2^2 \qquad (12)$$

$$N_C = (0.97-0.66l_2/l_1)(-1.07+19.8h_b/l_1)\,N_2 \qquad (13)$$

$$V_C = (0.12+1.25h_b/l_1)(1.32-0.37l_2/l_1)\,q_v\,l_2 \qquad (14)$$

For the section *D*,

$$M_D = -(0.012-0.007l_2/l_1)(0.25+0.07E_c/E_m)\,q_v\,l_2^2 \qquad (15)$$

$$N_D = (0.05+1.04h_b/l_1)(0.23+1.28H_c/l_1)\,N_2 \qquad (16)$$

$$V_D = -(0.08+0.68h_b/l_1)(1.36-0.42l_2/l_1)\,q_v\,l_2 \qquad (17)$$

For sections of column I ,

$$N_I = (-0.43+0.17l_2/l_1)(0.86+0.27h_w/l_1)\,Q_V \qquad (18)$$

$$M_{I'} = (-0.009+0.0055H_c/l_1)(1.53-0.95h_w/l_1)\,q_v\,l_1^2 \qquad (19)$$

$$M_I = -(0.69-0.14H_c/l_1)\,M_{I'} \qquad (20)$$

$$V_I = (M_{I'} - M_I)/H_c \qquad (21)$$

$$Q_V = q_v\,(l_1 + l_2 + h_c) \qquad (22)$$

For sections of column II,

$$N_{II} = (-0.57+0.16h_w/l_1)(0.88+0.15l_2/l_1)\,Q_V \qquad (23)$$

$$M_{II'} = (0.0055-0.0055l_2/l_1)(-0.23+0.11E_c/E_m)\,q_v\,l_1^2 \qquad (24)$$

$$M_{II} = -(1.04-0.44l_2/l_1)\,M_{II'} \qquad (25)$$

$$V_{II} = (M_{II'} - M_{II})/H_c \qquad (26)$$

For sections of column III,

$$N_{III} = (-0.16-0.10l_2/l_1)(0.80+0.37h_w/l_1)\,Q_V \qquad (27)$$

$$M_{III'} = (0.011-0.005 l_2/l_1)(1.65-0.99 H_c/l_1)\ q_v\ l_2^2 \quad (28)$$
$$M_{III} = (0.72-0.18 H_c/l_1)\ M_{III'} \quad (29)$$
$$V_{III} = (M_{III'} - M_{III}) / H_c \quad (30)$$

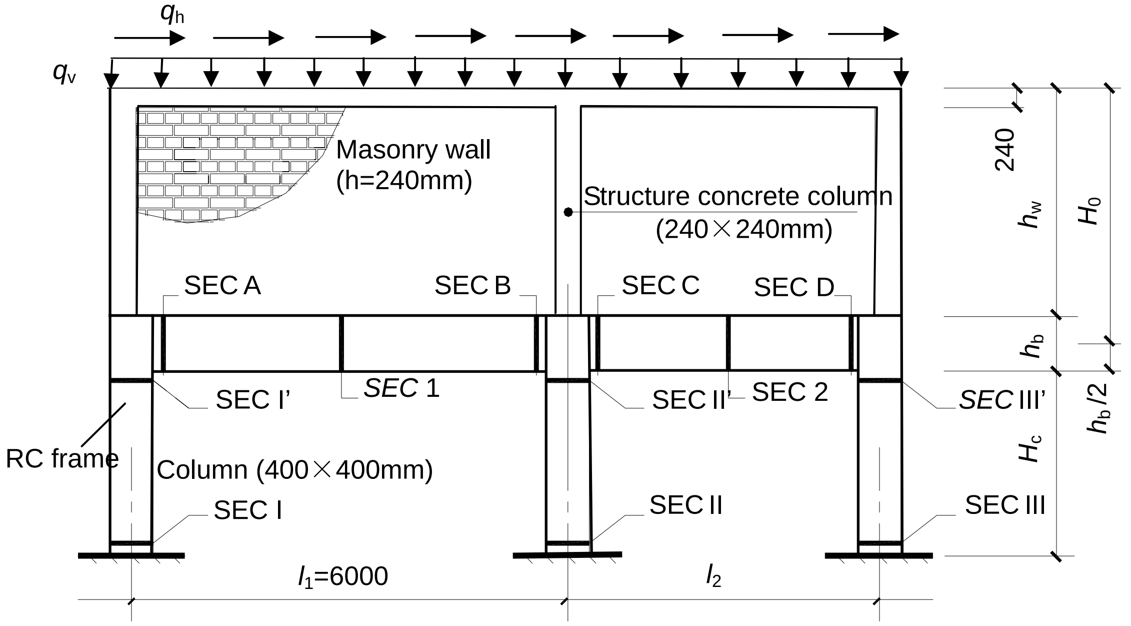

Figure 6 Dimension and critical sections of the composite structural member

4.2 Under horizontal top uniformly distributed loads

Based on the results of finite element analysis under left-side horizontal top uniformly distributed loads q_h, with further intuitive and square deviation analysis, values of inner forces for critical sections of the frame shown in Figure 6 can be calculated with equation (31) to equation (55). The inner forces under right-side horizontal top uniformly distributed loads can be worked out by changing the direction of the force.

For sections of column I,

$$V_I = (0.316+0.01 H_c/l_1)\ F_h \quad (31)$$
$$M_I = (-0.597+0.067 H_c/l_1)\ V_I\ H_c \quad (32)$$
$$M_{I'} = (0.403+0.067 H_c/l_1)\ V_I\ H_c \quad (33)$$
$$N_I = (1.03+0.8 H_c/l_1)\ F_h\ (h_w + h_b) / (l_1 + l_2) \quad (34)$$
$$F_h = q_h\ (l_1 + l_2 + h_c) \quad (35)$$

For sections of column III,

$$V_{III} = (0.306+0.024 H_c/l_1)\ F_h \quad (36)$$
$$M_{III} = (-0.597+0.067 H_c/l_1)\ V_{III}\ H_c \quad (37)$$
$$M_{III'} = (0.403+0.067 H_c/l_1)\ V_{III}\ H_c \quad (38)$$
$$N_{III} = (-1.3 + 0.3 l_2/l_1)\ N_I \quad (39)$$

For sections of column II,

$$V_{II} = (0.373-0.028 H_c/l_1)\ F_h \quad (40)$$
$$M_{II} = (-0.563+0.044 H_c/l_1)\ V_{II}\ H_c \quad (41)$$
$$M_{II'} = (0.437+0.044 H_c/l_1)\ V_{II}\ H_c \quad (42)$$
$$N_{II} = -(N_I + N_{III}) \quad (43)$$

For the section *A*,

$$M_A = (0.043+6.29 h_b/l_1)(1.16-0.24 H_c/l_1)\ M_{I'} \quad (44)$$
$$N_A = (0.83-2.80 h_b/l_1)\ (0.48+0.74 H_c/l_1)\ F_h \quad (45)$$
$$V_A = (-2.40+8.26 h_b/l_1)\ (1.64-1.23 h_w/l_1)\ (0.42+0.82 H_c/l_1) \times (0.72+0.32 l_2/l_1)\ (M_A - M_B + q_h\ l_1\ h_w) / l_1 \quad (46)$$

For the section *B*,

$$M_B = (0.061+2.86h_b/l_1)(-1.13+0.19H_c/l_1)\,M_{II}, \quad (47)$$

$$N_B = (-0.58+0.31l_2/l_1)(1.59-5.70h_b/l_1)\,F_h \quad (48)$$

$$V_B = (-1.59+6.13h_b/l_1)(1.76-1.45h_w/l_1)(0.60+0.47l_2/l_1)(0.48+0.74H_c/l_1) \times (M_A - M_B + q_h\, l_1\, h_w)/l_1 \quad (49)$$

For the section *C*,

$$M_C = (0.042+3.06h_b/l_1)(1.40-0.47l_2/l_1)(1.17-0.25H_c/l_1)M_{II} \quad (50)$$

$$N_C = (-0.09+0.36l_2/l_1)(1.95-9.13h_b/l_1)\,F_h \quad (51)$$

$$V_C = (-1.53+5.83h_b/l_1)(0.46+0.63l_2/l_1)(1.74-1.41h_w/l_1)(0.53+0.68H_c/l_1) \times (M_C - M_D + q_h\, l_2\, h_w)/l_2 \quad (52)$$

For the section *D*,

$$M_D = (-0.033-6.24h_b/l_1)(1.32-0.38l_2/l_1)(1.17-0.24H_c/l_1)\,M_{III}, \quad (53)$$

$$N_D = (-0.83+2.89h_b/l_1)(0.49+0.73H_c/l_1)\,F_h \quad (54)$$

$$V_D = (-2.33+8.36h_b/l_1)(0.45+0.65l_2/l_1)(1.58-1.10h_w/l_1)(0.47+0.76H_c/l_1) \times (M_C - M_D + q_h\, l_2\, h_w)/l_2 \quad (55)$$

5 Conclusions

The finite element analysis method is effective to analyse the interaction of masonry wall without opening and supporting two-span reinforced concrete frame. Under vertical top uniformly distributed loads, the structure works like a composite deep beam, the frame beam is the eccentrically tensile member, the distribution of axial force between frame columns has been affected by the arch action, contraflexure point of the column is located at 0.35~0.39 times of the column height. The local compression of masonry wall above the frame column is reduced by the structural concrete column. Under horizontal top uniformly distributed loads, the masonry wall works like a cantilevered shear wall, the frame beam is an eccentrically tensile or compressive member according to the direction of the loads, the point of contraflexure in the column is located at 0.53~0.55 times of the column height. Based on the analysis for main influence factors to the inner forces of the critical sections of the reinforced concrete frame, approximate equations to determine inner forces in critical sections of the frame are recommended.

Deep analysis for different kinds of the masonry wall supported on the RC frame, such as two-span member with opening inside the wall or the member with more than two-spans, are going to be done in the future.

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