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PARAMETER STUDY ON COMPUTER SIMULATION OF COLLAPSE RESPONSES FOR MASONRY STRUCTURES

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Abstract

Compared with other types of structures, masonry structures are vulnerable to severe external actions. Intensive studies have been carried out focusing on simulations for the collapse of masonry structures. Studies show that several key parameters, such as time interval, stiffness, and damping coefficient, play major roles in numerical calculations. In this paper, an extended-rigid-body-spring (ERBS) model to simulate collapse responses of masonry structures is proposed. Meanwhile, a thorough parameter study is conducted. The good match between the results of simulation and the shaking table test for a masonry model structure indicates that the proposed ERBS model and the method of parameter determination are suitable for the analysis of masonry structures.

Keywords

Masonry structure, collapse response, computer simulation, parameter study

Notations

b_0 = width of a brick in the model masonry structure

C_c = critical damping coefficient

$c_{xi}, c_{yi}, c_{zi}, c_{\theta xi}, c_{\theta yi}, c_{\theta zi}$ = damping coefficients

$D_{\min}$ = the minimum value of the distance between centers of two connected elements

$FRAC$ = a coefficient to calculate the time interval

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F = resultant of the forces applied on the center of gravity of an element

F_{xi}, F_{yi}, F_{zi} = resultants of the forces applied on the center of gravity of element *i* in a global coordinate system

g = acceleration of gravity

I_{xi}, I_{yi}, I_{zi} = moments of inertia of element *i*

l_0 = length of a brick in the model masonry structure

k = stiffness of a single degree of freedom spring-mass system

k_i = stiffness of element *i*

k_n = normal stiffness of the connecting spring

k_x = tangent stiffness of the connecting spring

M = resultant of the moments applied on an element

M_{xi}, M_{yi}, M_{zi} = resultants of the moments applied on element *i*

m = mass of the particle in a single degree of freedom spring-mass system

m_i = mass of the element *i*

n = number of elements in a masonry structural system

T = natural period of a single degree of freedom spring-mass system

$T_{\min}$ = the minimum of natural periods of elements in a masonry structural system

t_0 = thickness of the mortar bed

u = displacement of an element

$\dot{u}$ = velocity of an element

$\ddot{u}$ = acceleration of an element

$\overline{V}$ = force of the shear spring

$\overline{V}_{x0}$ = bearing capacity of the shear spring

$\overline{V}_z$ = force of the normal spring

$\overline{V}_{zc0}$ = compressive capacity of the normal spring

$\overline{V}_{zt0}$ = tensile capacity of the normal spring

v_p = velocity of the *p* wave

x, y, z = axes in a global coordinate system

$\overline{x}, \overline{y}, \overline{z}$ = axes in a local coordinate system

$\ddot{x}_g, \ddot{y}_g, \ddot{z}_g$ = accelerations of the ground in three directions

$\dot{x}_i, \dot{y}_i, \dot{z}_i$ = velocity responses of element *i* in a global coordinate system

$\dot{x}_N$ = velocity of an element at the beginning of the next time interval

$\overline{\Delta}$ = deformation of the connecting spring

Δ_{x0} = ultimate deformation of the shear spring

$\overline{\Delta}_{z\max}, \overline{\Delta}_{z\min}$ = ultimate deformation of the normal spring

Δt = time interval

θ = angular displacement of an element

$\dot{\theta}$ = angular velocity of an element

$\ddot{\theta}$ = angular acceleration of an element

$\dot{\theta}_{xi}, \dot{\theta}_{yi}, \dot{\theta}_{zi}$ = angular velocity responses of element *i* in a global coordinate system

ω_i = circular frequency of element *i*

1 Introduction

Masonry structures play an important role in the development of structures. They have been used on many memorials, dwellings, infrastructures, and so on. But masonry structures are more vulnerable compared with others and they are easier to collapse under severe external actions. In order to optimize designs of new structures and accurately assess existing structures, it is necessary to analyze collapse responses of this kind of the structure.

The analysis of collapse, in a sense, is a full range simulation including all branches of the stress-strain curve [Gu and Sun 2002]. Because of discontinuous characters of the structure under large displacement, numerical methods which are based on continuum mechanics are evidently not suitable for the analysis of collapse of masonry structures, even though they are effective in the quasi-static analysis under small displacement [Lotfi and Shing 1991] [Lee et al. 1996].

Discrete element method (DEM) [Cundall and Strack 1979] based on the extended-rigid-body-spring (ERBS) model which is to be thoroughly discussed is an alternative. Characters such as discontinuity and large displacement can be well considered in this model.

There are two kinds of iterative patterns of DEM, the static one and the dynamic one. In this paper, a numerical calculation is carried out based on a dynamic relaxation iterative algorithm. When using this algorithm, it is not necessary to store huge sparse matrix in the memory of computer because the iteration is explicit. And the stop of iteration, bought by the ill conditions of the matrix which could be resulted from the irregular shape of the elements or above-mentioned complex mechanical properties of the materials, would be avoided. Along with the advantages, the dynamic relaxation algorithm is fit for the large displacement dynamic calculation and it is a good choice for programming. When using this algorithm, time interval, stiffness of springs, and damping coefficient are the key parameters which reflect the dynamic characters of the structure, and they would affect the accuracy of the result and the stability of the calculation. For example, the elements might keep oscillating and it is greatly time-consuming to converge. Otherwise the element would overlap each other because of a bad choice of these key parameters. It is necessary to determine the value of these parameters carefully according to the theoretical analysis and testing study.

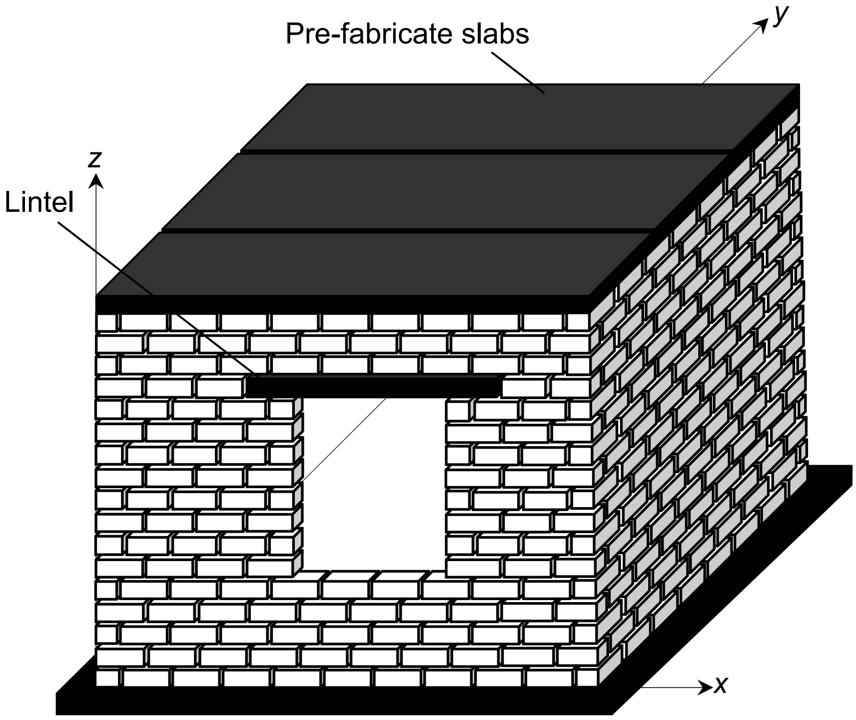
2 Analysis of collapse responses

2.1 ERBS model

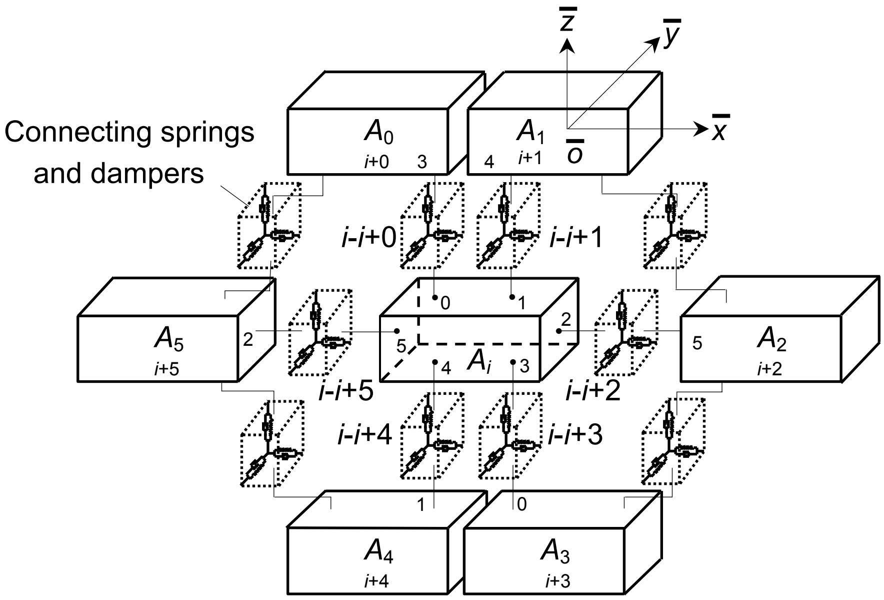
To simulate collapse responses of masonry structures under earthquake, the extended-rigid-body-spring (ERBS) model is used [Miao 2003]. The masonry structure of an actual building is indicated in Figure 1(a). In the calculation model, every brick is taken as a rigid body, and the mortar bed is replaced by a set of springs which connect the rigid elements (Figure 1(b)). It is obvious that cracks were supposed to originate and develop only inside the mortar, which is true under most circumstances because the strength of the mortar is usually far less than the strength of the brick itself, especially in

the old masonry structures.

The constitutive relationships of the springs could be determined by mechanical properties of the mortar. Some concrete members in the structure, for example, the pre-fabricate slabs used for floors and lintels are also taken as rigid elements, but the characters of these elements and the connecting springs are different from those of bricks.



(a) Sketch map of a part of the model masonry structure



(b) The extended-rigid-body-spring model

Figure 1 Discretization of the structure

2.2 Basic process of calculation

Under the action of earthquake, the motion of every rigid element could be described by equation (1) considering the effect of viscosity damping.

$$\begin{cases} m_i\ddot{x}_i + c_{xi}\dot{x}_i = F_{xi} - m_i\ddot{x}_g \\ m_i\ddot{y}_i + c_{yi}\dot{y}_i = F_{yi} - m_i\ddot{y}_g \\ m_i\ddot{z}_i + c_{zi}\dot{z}_i = F_{zi} - m_i\ddot{z}_g - m_i g \\ I_{xi}\ddot{\theta}_{xi} + c_{\theta xi}\dot{\theta}_{xi} = M_{xi} \\ I_{yi}\ddot{\theta}_{yi} + c_{\theta yi}\dot{\theta}_{yi} = M_{yi} \\ I_{zi}\ddot{\theta}_{zi} + c_{\theta zi}\dot{\theta}_{zi} = M_{zi} \end{cases} \tag{1}$$

fasten all of the elements first, then loose the element one by one randomly at the time point t. By solving the equation above with the center difference approach at every time point t, the velocity of the element *i* at $t+\Delta t/2$ is known. Presuming the element *i* moves with a uniform velocity during a time interval Δt, which is small enough, the increment of the element displacement could be worked out. So the position of the element *i* and the spring forces applied on the element could be determined. When the element *i* moves to the new position, fasten it again. Because the time interval is small enough, it is reasonable to consider that the forces produced by loosing the element affect only the neighboring elements. Then move the next rigid body until all of the elements are loosed and fastened at least for one time. Repeating the course in every sequence time interval until the end of the time range, the displacement track of every element could be acquired (Figure 2) [Miao 2003]. It should be pointed out that the approximation could be better if the number of iteration increased within a time interval, but longer calculation time is needed by this way.

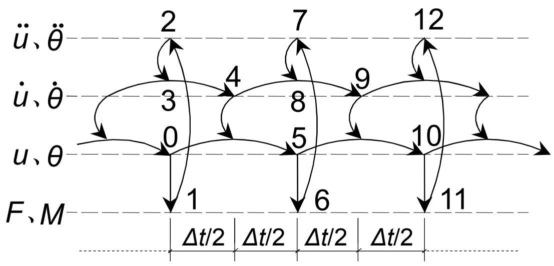


Figure 2 The iteration procedure of DEM

2.3 Verification of calculated results

Results of numerical calculations, including the amplitude and the frequency of the displacement response, the time of collapse, the direction of collapse, and so on, are verified by the record of a shaking table test for a model masonry structure. The dimensions of the model and mechanical behavior of the mortar are listed in Table 1 and Table 2. Furthermore, a post-processing module is developed for the graphic simulation, and the calculating collapse process could be compared directly with the recording video of the test (Figure 3).

Table 1 Dimensions of the model

Dimensions	Value(mm)
Length of the model (in y direction)	1830
Width of the model (in x direction)	1525
Height of the model (in z direction)	4260
Length of the brick (in $\overline{x}$ direction)	300/150
Width of the brick (in $\overline{y}$ direction)	125
Height of the brick (in $\overline{z}$ direction)	100

Table 2 Mechanical behavior of the mortar

Mechanical behavior	Value(Mpa)
Cubic compressive strength of the mortar	9.81
Prism compressive strength of the mortar	6.87
Young's modulus of the mortar	1.07×10^4
Shear modulus of the mortar	7.79×10^6

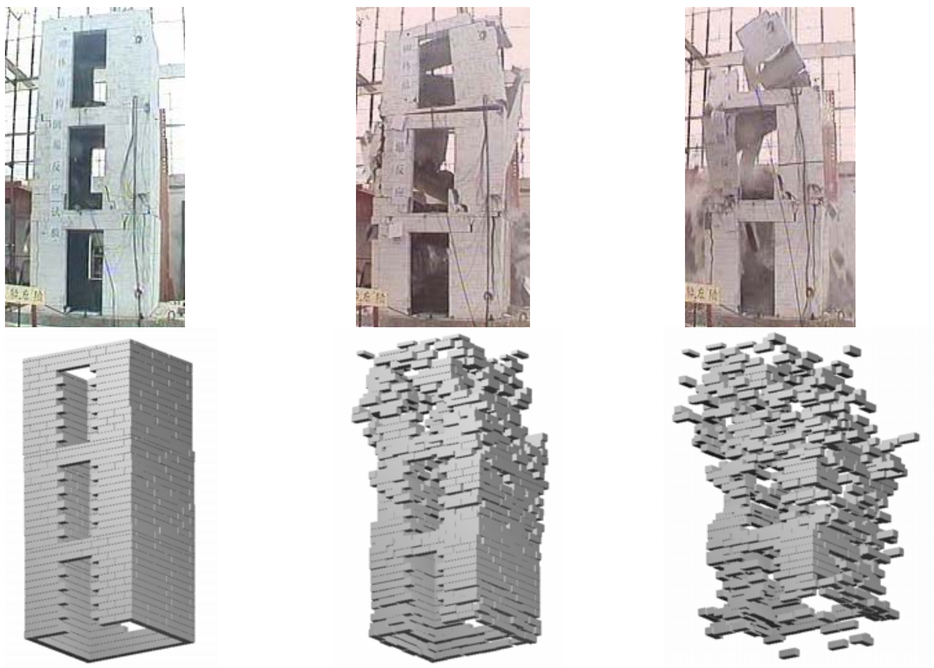


Figure 3 Verification of the numerical simulation results

Because the analysis of collapse is no doubt a very complex process, it is unrealistic and unnecessary to expect every comparison mentioned above to be fully satisfied. A reasonable alternative is to meet the controlling requirements as much as possible. Good matches between the test and the calculation on the time of collapse and the direction of collapse are taken as the main standards to calibrate the calculation parameters in this paper.

3 Parameter study

3.1 Time interval

The convergence and the stability of the dynamic relaxation iterative method depend distinctly on the value of time interval, i.e., Δt. As mentioned above, big Δt would result in a severe motion, including translation and rotation of the element in this time interval. And the element would overlap the neighboring ones. This is one of the reasons that lead to divergence. On the other hand, an extremely small Δt would always make the solution successful, but the time consumed would be unacceptable.

It is necessary to find a limited value of Δt. And with this limit, the time interval can be chosen as large as possible. It has been proved that the minimum natural period of the system is always longer than the natural period of each element in the system. It would be safe to determine the Δt based on the latter. The format of the dynamic equation of an element in a structural system is the same as that for the spring-mass system of single degree of freedom (SDOF), and its natural period is a reasonable standard to determine the limit of time interval. To ensure the spring-mass system of SDOF to vibrate reasonably, equation (2) must be satisfied [Wang and Xing 1991]:

$$\Delta t < \frac{T}{\pi} = 2\sqrt{\frac{m}{k}} \tag{2}$$

So the limit of Δt for DEM could be

$$\Delta t < \frac{T_{\min}}{10}, T_{\min} = 2\pi \min_{1 \le i \le n} \sqrt{\frac{m_i}{k_i}} \tag{3}$$

As for a system composed of blocks, the method above is effective when the damping is absence. To take the damping into account, the limit could be [Hart et al 1988]:

$$\Delta t = \frac{FRAC \bullet \sqrt{2}}{\max_{i=1,n}\{\omega_i\}} \tag{4}$$

Another way to find the limit is to consider the velocity of the load wave. Hakuno proposed the following value [Hakuno and Meguro 1993]:

$$\Delta t < \frac{D_{\min}}{v_p} \tag{5}$$

But the numerical test [Miao 2003] showed that the value above can not ensure the convergence sometimes. In practice, half of the value calculated using equation (5) should be chosen.

The input earthquake wave of the shaking table test in this paper is the Elcentro wave. The time interval for the record is 0.02s, and the similarity coefficient of the model test is 0.707. For the convenience to interpolate the wave linearly, the Δt should be divided exactly by the product of 0.02 and 0.707. The simulation reveals that $\Delta t = 1.414 \times 10^{-6}$ s is an appropriate value for time interval in this case.

3.2 Mechanical properties of the spring

All discrete elements affect each other through the connecting springs, which exist in both normal and tangent direction, as shown in the ERBS model (Figure 1). And the mechanical characters of the springs are significant.

Within a small range of displacement, the stiffness of the spring is very important, and it should reflect the deforming behavior of the element in most circumstances. But as mentioned above, for masonry structures the bricks could be taken as rigid bodies without damaging the accuracy of the result. So the stiffness of the spring is then determined only by the behavior of the mortar.

First, the behavior of the mortar is simplified. Mortar bed on a brick is divided evenly at the midpoint of the long edge (Figure 4), and the strain of the deformed mortar is taken as uniform. By integrating the stress inside the mortar, a set of equivalent forces, including pressure and shear force, can be calculated for each portion.

Based on the simplification above, each portion of the mortar bed is replaced by a set of springs which connect the blocks(Figure 1). The stiffness of the spring is determined by the mechanical test of the mortar.

It should be pointed out that the time interval and the stiffness affect the motion of elements simultaneously. For a given time interval, a large stiffness of the spring would result in large force, and then cause the severe motion of elements.

Different from the stiffness, failure criteria of the spring are crucial when the motion of elements come into large displacement range. The mortar suffers multiaxial action during the full range of the calculation, and the failure surface is rather complex. As an alternative, simple constitutive curves are employed independently for the normal and shear spring (Figure 5). The terminus for the constitutive curve is determined by taking into account only the coupling relationship between the normal and the shear strength of the mortar [Miao 2003]. Meanwhile, a conservative criterion is used to describe the failure behavior of the springs [Gu and Sun 2002].

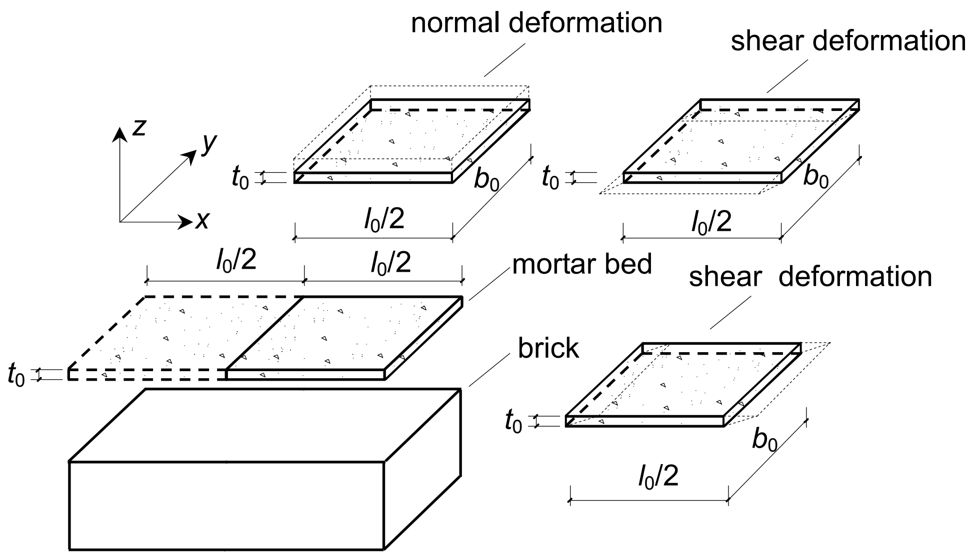
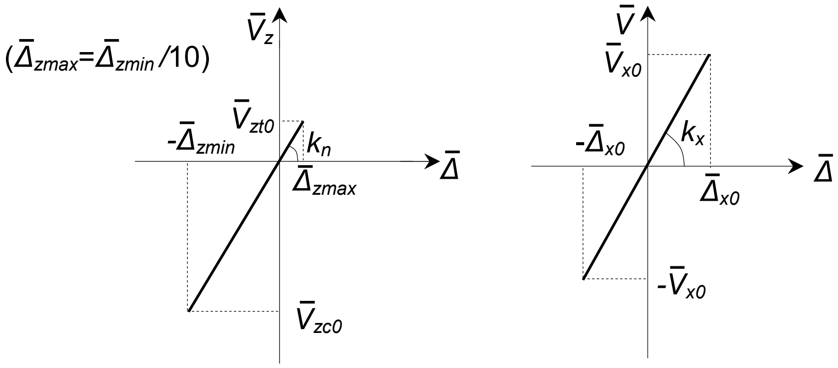


Figure 4 Simplified deformation of the mortar



(a) Normal spring *(b) Tangent spring*

Figure 5 Constitutive relationship of the connecting springs

3.3 Damping coefficient

Numerical tests show that the damping coefficient disturbs the result distinctly. To analyze collapse responses, the damping ratio of the structure could be determined by scanning the model with white noise wave during the shaking table test, or determined theoretically by matrix operations. Unfortunately, acquired damping ratio of the structure is different from that of elements, which are needed in the DEM calculation. Numerical test has proved that it would result in divergence of the calculation to use the damping ratio of the structure in the analysis.

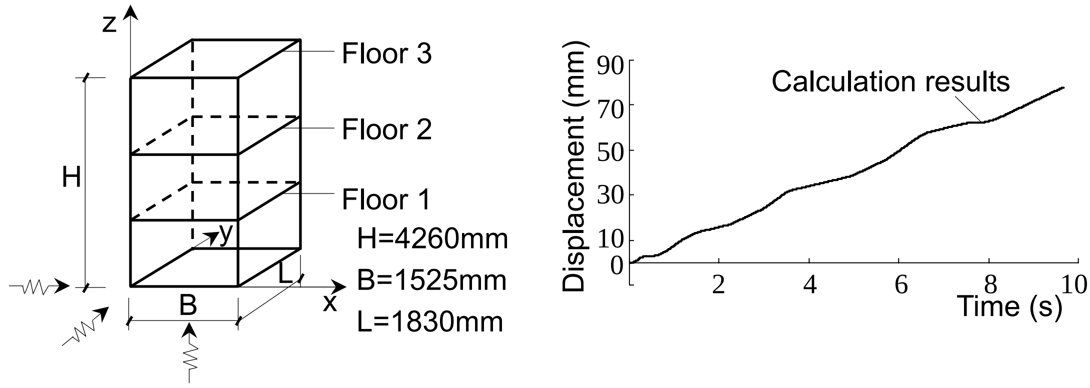
The damping characters of elements could also be determined theoretically, but the results seem not helpful. To keep the element from oscillating, the critical damping coefficient should be employed, which could be decided by equation (6).

$$C_c = 2\sqrt{mk} \tag{6}$$

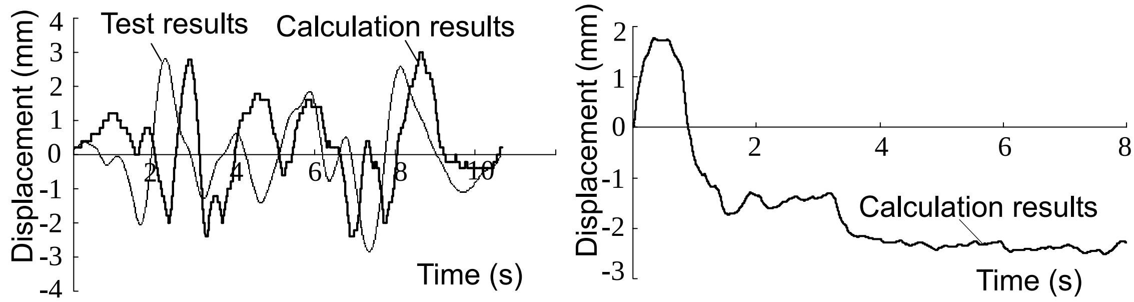
Numerical test shows that the above coefficient could not achieve the convergence also (Figure 6). One of the explanations is that the parameter k in equation (6) is difficult to be determined. Another may come from the truth that the effect of damping is determined by the damping coefficient and the time interval simultaneously [Miao 2003]. Damping coefficient is taken as direct proportion to the mass of the element in this paper. According to employed time interval, the effect of the damping coefficient on the calculation results is indicated in Figure 6. Comparing the numerical simulation results for different values of damping coefficient in Figure 6, it can be concluded that a value greater than the result of equation (6) is proved to be appropriate. A too large damping coefficient, just like a too small one, would lead to divergence.

4 Conclusions

1. Time interval, stiffness of the connecting spring, and damping coefficient affect the calculation results simultaneously. The values of them are influenced by each other. And it complicates the determination of these key parameters.
2. Failure criterion for the connecting spring should be carefully chosen by taking the characters of the mortar suffering multiaxial loads into consideration.
3. A simplified damping coefficient, which is in direct proportion to the mass of the element, could be chosen by numerical test.



(a) Dimension of the model structure *(b) $c_{xi} = C_c = 0.35m_i$ (too small)*



(c) $c_{xi} = 0.35\times10^4 m_i$ (appropriate) *(d) $c_{xi} = 0.35\times10^5 m_i$ (too large)*

Figure 6 Displacement responses of Floor 3 of testing model in direction x under Elcentro wave (Peak acceleration input in direction x is 0.4g)

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