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AN EXPERIMENTAL INVESTIGATION ON MULTIPLE-LEAF STONE MASONRY

A. Anzani[1], L. Binda[2], A. Fontana[3], J. Pina Henriques[4]

Abstract

Most historic centres in Italy are characterised by the presence of monumental and minor architectures, often built by stonework multiple-leaf masonry. Its characteristics need being understood when approaching the safety assessment of many kinds of structures, especially in seismic areas. An experimental research has been carried out on three-leaf walls purposely built using two stones of different characteristics, guaranteeing some degree of connection between leaves, trying to reproduce two masonry typologies frequently used in Italian historic centres. Aim of the research was to understand the stress-strain behaviour of this masonry under compression and shear actions and to evaluate the possibility of interpret it by analytical modelling.

Key Words

Multiple leaf masonry, compression and shear behaviour, elastic modulus.

1 Introduction

The historical-monumental and residential patrimony diffused on the Italian territory, declared seismic by the national Code, is mostly made of masonry constructions often built in multiple leaf-masonry. In the last decades, the collapse of historic structures took place: in the case of the pillars of the Noto Cathedral the collapse was connected with the weak interaction between outer masonry and inner core. Understanding the behaviour of multiple-leaf masonry requires the knowledge of the interface response between different leaves, which mainly depends on the adhesion between them and on the geometry of the surface of reciprocal contact. The purpose of this research is to study the response of this interface when varying its geometry and the stone nature. Limit situations have been reproduced, so to represent (by analogy) the numerous cases typical of the building practice. The experimental results have then been interpreted analytically, modelling the measured masonry stiffness.

2 Material characterisation

The materials, their combination, the morphology and the dimensions of the specimens were chosen in order to reproduce the behaviour of multiple leaf walls and piers, like those present in several churches in Sicily (Binda et al., 2003b).

[1] Anzani Anna, DIS, Politecnico di Milano, p.za Leonardo da Vinci, 32, Milano, Italy
[2] Binda Luigia, DIS, Politecnico di Milano, p.za Leonardo da Vinci, 32, Milano, Italy
[3] Fontana Alberto, DIS, Politecnico di Milano, p.za Leonardo da Vinci, 32, Milano, Italy
[4] Pina-Henriques José, Department of Civil Engineering - University of Minho, Portugal

2.1 Mortar

The same ready-to-mix hydraulic lime mortar was used to build both the external and the internal leaves of all the masonry specimens. During the construction of the masonry walls, mortar specimens of dimensions 40x40x160mm^3 were prepared for testing according to EN 1015-11, CEN 1999 after different curing times (28, 70, 90, 175 days); the values of compressive and tensile strength have been plotted against the curing time and reported in Figure 1. Elastic modulus of the mortar (E_m) was calculated, on three prisms, after respectively 28 and 60 days of curing, giving the following average values:

E_m at 28 days = 2475 N/mm^2
E_m at 60 days = 3389 N/mm^2

The results of phenolphthalein testing carried out at different curing time, by visual evaluation of the carbonated portion, are shown in fig. 2. It may be estimated that, when tested, the walls had presumably reached a carbonation of about 5%.

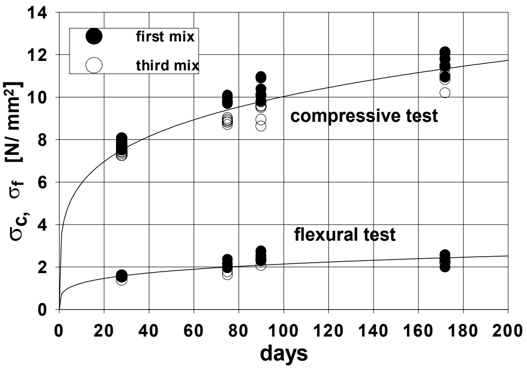


Figure 1 Compressive and tensile strength of mortar vs. curing time.

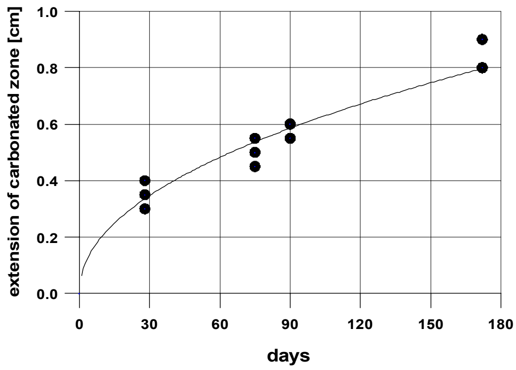


Figure 2 Results of phenolphthalein tests.

2.2 Stones

In order to investigate the influence of stone on the wall behaviour, two types of stones were chosen: the same limestone used for the Noto Cathedral (Noto stone) and a sandstone, frequently used in central and southern Italy (Serena stone). The characteristics of the two stones were obtained on cylindrical specimens of 80mm diameter and 160mm height, cored from regularly cut stones. The compressive strength (σ_c) was determined in both directions, normal and parallel to the bed joints, and the tensile strength (σ_t) in the same direction of the bed joints. The elastic modulus (E_a) and Poisson's ratio (ν_a) as an average over 3 specimens are reported in Table 1.

Table 1 Characteristics of the stones used for the preparation of the samples.

Stone	bed joint direction	σ_c [N/mm^2]	E_a [N/mm^2]	ν_a	σ_t [N/mm^2]
Noto	normal	20,6	9476	0,10	_
Noto	parallel	17,6	8526	0,09	2,06
Serena	normal	104,2	18218	0,19	_
Serena	parallel	89,0	23293	0,21	6,07

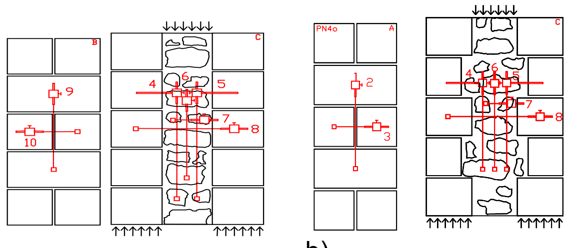
3 Experimental program

Twelve three-leaf wallets, of dimensions 310x510x790 mm^3 were built at DIS - Politecnico di Milano by Spadaro srl Contractor - Rosolini, Sicily. Two types of connection between the leaves (with and without offsets) were adopted (Figure 3, 4).

The specimens were tested after 75 to 172 days of curing according to the following procedures:

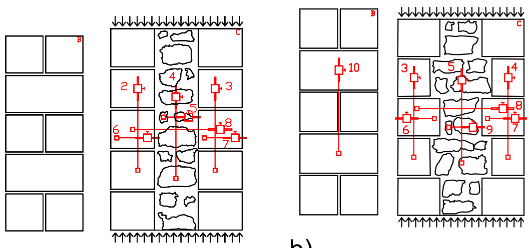
- *Shear tests*. Two specimens of each of the four groups were tested applying to the inner leaf a monotonic compressive load, being the specimens supported by the external leaves (Figure 3).
- *Compression tests*. One prism of each group was tested applying uniformly a monotonic compressive load (Figure 4).

The tests were carried out with a hydraulic servo-controlled MTS press (2,500KN) in displacement control (1μm/s). The position of the transducers is shown in Figures 3, 4.



a) b)

Figure 3 Transducers in shear tests on walls without offset (a) and with offset (b).



a) b)

Figure 4 Transducers in compression tests on walls without offset(a) and with offset(b).

3.1 Compression tests on single leaves

After the shear bond tests on the specimens without offsets had been carried out, and a neat separation in shear between the outer and inner leaves (without any damage to the single leaves) took place, each individual leaf could be tested separately in monotonic compression. A total of four external leaves and two internal leaves for each type of stone were therefore tested. In Figures 5, 6 the stress-strain diagrams obtained for the two stones are presented; the shown diagrams were obtained from the direct displacement readings. Specimen PS2 could not be loaded up to the peak stress because this was beyond the machine limit load (2,500KN): a maximum load of 2,480KN was applied. The higher strength of the external leaves is evident, while the internal leaves presented practically equal strengths in the cases of the two stones. This suggests that, due to the technique of construction, the strength of the inner leaves is mainly conditioned by the characteristics of the mortar.

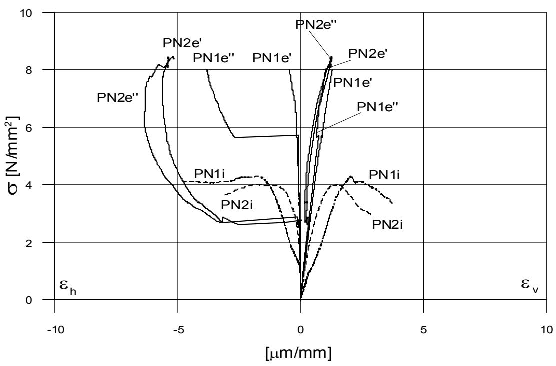


Figure 5 Results on Noto stone: ext. leaves (solid line) and int. ones (dashed line).

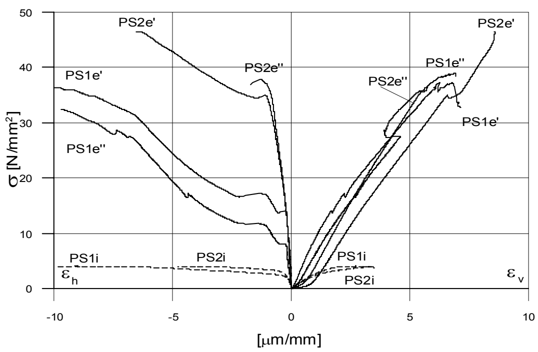


Figure 6 Results on Serena stone: ext. leaves(solid line) and int. ones(dashed line).

In Table 2 the results of compression tests on single leaves are summarised; two values of elastic modulus are reported, based on vertical displacements readings taken respectively between machine plates (E_p) and directly on the specimen (E_d) (average strain values were considered). The rather large differences are due to the adjustments

between specimens and machine plates that result into larger displacements recorded by the transducers placed between the plates.

Table 2 Results obtained from compression tests on single leaves.

Specimen		Strength (N/mm^2)	E_p (N/mm^2) between plates	E_d (N/mm^2) on the wall
PN1	Outer leaves	9.0	3680	-
PN2	Outer leaves	8.3	2615	-
PS1	Outer leaf’	36.5	5145	6736
	Outer leaf”	38.2	4965	6205
PS2	Outer leaf’	39.1	4825	7241
	Outer leaf”	-	4540	6055
PN1	Inner leaf	4.2	1825	2430
PN2	Inner leaf	3.9	1915	4280
PS1	Inner leaf	4.0	1515	2659
PS2	Inner leaf	3.9	1295	1999

In the case of the Noto stone, the outer leaves have shown about the 42% of the strength of the single stone and the inner one about the 20%, while in the case of the Serena stone the values obtained are about 36% for the outer leaves and only 3.7% for the inner leaf. The outer leaves have about twice the strength of the inner ones in the case of the Noto stone, while in the case of the Serena stone this relation is of about ten times. The Serena outer leaves exhibited a more brittle failure and a stiffer behaviour than the Noto ones. The outer leaves failed due to the development of several vertical cracks and spalling on the base and on top of the specimens. In the case of the Noto specimens the cracks started to develop earlier than in the case of the Serena ones, especially in the case of PS2e” specimen, on which cracks started to appear very near the maximum reached load. Relatively to the inner leaves, also here failure was due to the development of vertical cracks, but the cracks followed respectively different paths because of the higher strength and lower adhesion between Serena stones and mortar. In this case, the cracks mainly went around the stone pebbles, whereas in the case of the Noto specimens they cut the stones.

3.2 Compression tests on the three leaf specimens

One specimen for each of the four groups was tested as described in Sec. 3 and in Fig. 4. The failure of the Serena specimens was not achieved being the required load beyond the limits of the testing machine. The experimental results are reported in Figures 7 - 10.

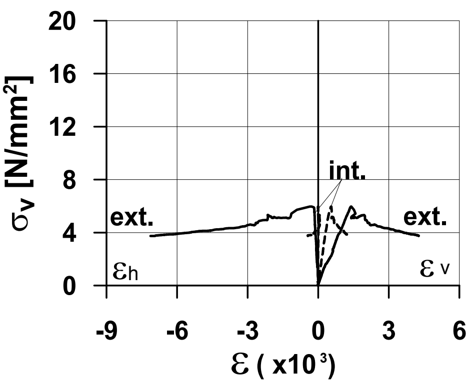


Figure 7 Results of test on PN3.

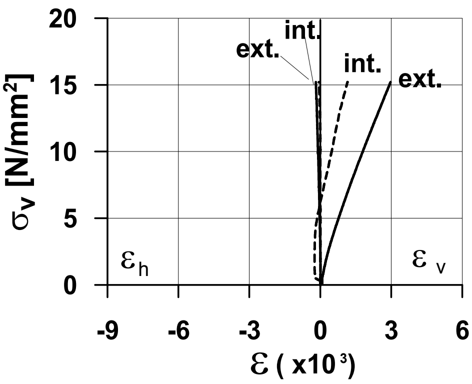


Figure 8 Results of test on PS3.

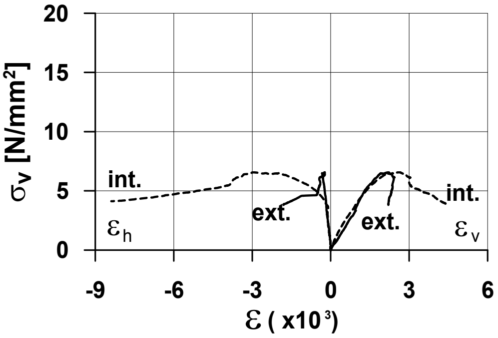


Figure 9 Results of tests on PN5.

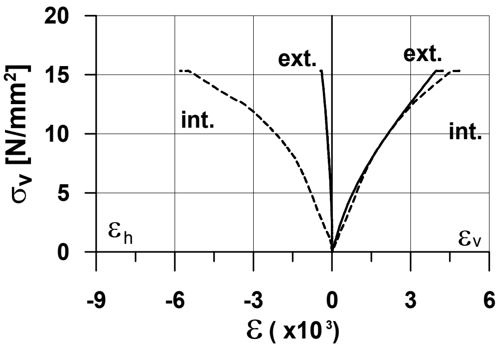


Figure 10 Results of tests on PS5.

In Table 3 the values of the strength and of the elastic modulus are reported. The peak load of the Noto stone specimen with offsets is about 10% higher than the one without offsets. In the case of the specimens without offsets, the inner leaves underwent very low vertical strains compared to the external leaves, and almost naught horizontal ones. This indicates that no collaboration between the leaves took place, that the load-bearing role was mainly played by the outer leaves, and that the outer leaves exerted a confining action towards the development of transversal deformations of the inner leaf.

Table 3 Results of the compression tests.

Specimen	Type of connection	Strength (N/mm^2)	E_p (N/mm^2) between plates	E_d (N/mm^2) on the wall
PN3	without offsets	5.8	1558	5567
PN5	with offsets	6.4	2068	3862
PS3	without offsets	15.32*	2900	5325
PS5	with offsets	15.32*	2519	4211

**Maximum applied stress, since failure was not reached*

This was confirmed by the crack pattern, where the inner leaf appeared practically undamaged at the end of the test (figures 11 – 14). Nevertheless, at low stress level, the separation on one side of the outer leaf from the inner one started. It is clear that the separation of the leaves at an early stage corresponds to the migration of the load to the outer leaves, which therefore assume the whole load-bearing function.

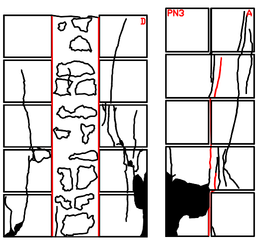


Figure 11 PN3 at the end of test.

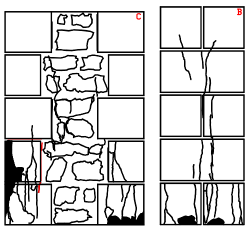


Figure 12 PN5 at the end of test.

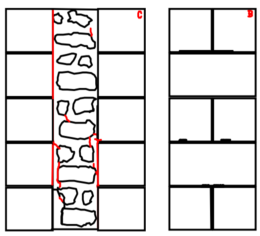


Figure 13 PS3 at the end of test.

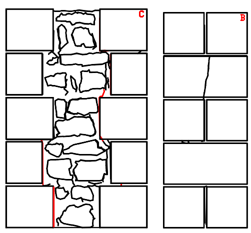


Figure 14 PS5 at the end of test.

The failure of the Noto stone specimens without offset was due to the development of several long vertical cracks in the outer leaves and spalling near the base (Figure 11). In the case of the Serena stone specimens, despite the fact that the peak stress could not be reached, the initial detachment between outer and inner leaves took place (Figure 13). Considering the specimens with offsets, in all cases almost coincident vertical strains of both leaves and higher horizontal strains of the inner leaves developed, indicating a good collaboration between the leaves. A strength increase was observed in comparison to the strength of the inner leaves tested individually,

particularly evident in the case of PS5 (not failed during testing). In the case of the Noto stone specimen with offsets (Figure 14) the outer leaves presented a more severe and diffuse final crack pattern than the ones without offsets. Relatively to the inner leaf, and unlike the specimens without offsets, several vertical cracks developed near the peak load, some of them splitting the stone pebbles. In the case of the Serena stone specimens with offsets and despite the fact that the peak load was not achieved, the development of some cracks in the inner leaf was observed that, however, went around the stone pebbles without breaking them (see Figure 14).

3.3 Shear bond tests

Shear bond tests were carried out according to the testing procedure described in Section 3 and in Figure 3. The aim of the tests was to understand the different mechanisms of failure occurring in specimens with and without offset. The load-displacement curves obtained are represented in Figures 15 and 16.

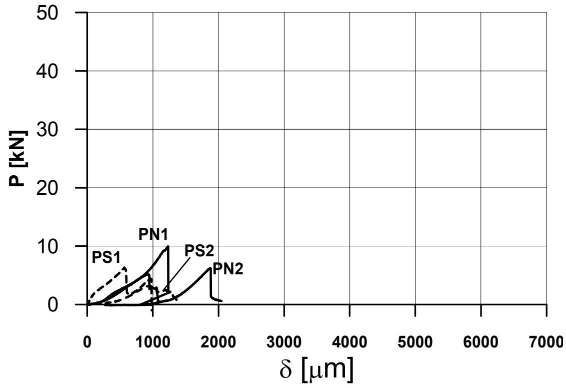


Figure 15 Results of shear-tests obtained on the specimens without offsets.

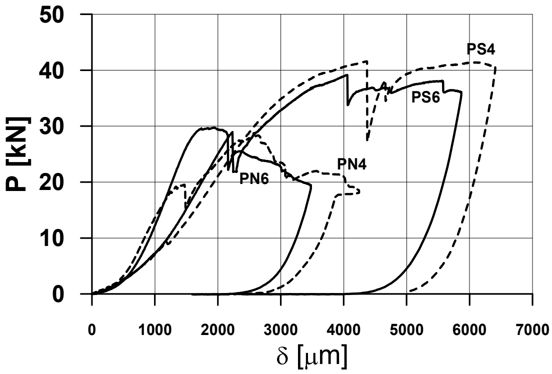


Figure 16 Results of shear-tests obtained on the specimens with offsets.

The peak load and displacement values are presented in Table 4, together with the shear strength (τ). For the specimens with offsets, an “equivalent” shear strength was calculated assuming the same resistant area than in the case without offsets. In the case of specimens without offsets, the failure of one connection occurred first, causing a slight rotation of the two leaves still connected; the failure of the second connection occurred later.

Table 4 Results of shear tests on specimens with and without offsets

Specimen	Type of connection		max. load (kN)	τ (N/mm^2)	displac. (mm)
PN1	Without offsets	fail.1° connect.	61.9	0.566	0.12
		fail.2° connect.	43.4	0.967	-
PN2	Without offsets	fail.1° connect.	97.4	1.230	0.20
		fail.2° connect.	60.2	1.880	-
PN4	With offsets	-	277.7	2.633	0.56
PN6	With offsets	-	291.1	1.949	0.59
PS1	Without offsets	fail.1° connect.	50.4	0.928	0.10
		fail.2° connect.	28.6	0.975	-
PS2	Without offsets	fail.1° connect.	31.7	0.951	0.06
		fail.2° connect.	24.7	1.195	-
PS4	With offsets	-	383.3	4.057	0.77
PS6	With offsets	-	407.6	4.370	0.82

From that point on, the test cannot be intended as a three point shear test since one of the lower point load is not more effective (because of the failure of the connection). This has to be born in mind when considering the shear values related with the secondly failed connection. It is possible to observe that the peak load values for the Noto specimens with offsets are about 3.5 times higher than the ones without offsets; for the Serena stone this relation gains importance assuming a value of 9.5 times. Comparing the specimens with the same type of connections, built with different materials, one can observe that the average peak load for the Serena stone specimens without offsets is about half the ones of Noto, suggesting that the adhesion stone-mortar is better in the last ones probably due to the higher porosity of the stone. Relatively to the specimens with offsets, the Serena stone specimens present an average peak load of about 1.5 times higher than the ones of Noto, due to the higher stone strength. On the specimens without offsets the shear failure is due to the development of vertical cracks in the connections while on the specimens with offsets the development of diagonal cracks in the inner leaf and, in the case of the Serena stone, also in the external leaves was observed. The inner leaf built with the Noto stone behaved more like a homogeneous material than the one built with the Serena stone. The deformations measured in the specimens with offsets are in agreement with the transfer of the load from the inner leaf to the outer leaves.

4 Analytical interpretation of the experimental masonry stiffness

The analysis and interpretation of the mechanical behaviour both of the single leaves and of the three-leaf masonry, which of course have been easily carried out in the laboratory, unfortunately is not likewise possible on the internal leaf of a real wall. Though, the knowledge of its elastic behaviour and the detection of damage onset would be very important to prevent damage diffusion and failure in existing structures.
On site tests like flat jack tests (Binda et al., 2004) usually allow to qualify the external leaves; in some cases they have been applied to internal leaves, but in these cases they could only be carried out after removal of the external leaf, therefore in quite a destructive way (Binda et al., 2003d). An attempt can be made to define the deformability characteristics (elastic modulus) of the masonry leaves simply using the characteristics of the component materials, easily detectable by laboratory testing on the materials sampled from historic buildings (Binda et al., 2003c). The experimental results obtained in the presented research will help in the formulation of reliable hypothesis. In particular, the results showed that within a certain percentage of the peak stress, the stress-strain behaviour of the masonry is rather linear elastic (Binda et al., 2003b). These results suggest adopting the theory of the two-phase composite materials. A general discussion on the determination of the elastic modulus of two-phase materials has been developed in (Fontana and Scirocco, 1983).

4.1 Stiffness of the internal rubble leaf

When the external leaves of a multiple leaf wall is made of well cut stones (or regular bricks), their modulus of elasticity E_e can quite easily be determined from the elastic properties of the component materials. These can be experimentally measured for the stones or bricks and hypothetically assumed for the mortar, taking into account that, in the past, the binder was usually putty lime and the aggregate was a local one (Binda et al., 2003a). Considering the internal rubble leaf or filling, the elastic modulus E_i can be determined, like usually done for concrete, as the elastic modulus of a two-phase composite material; this calculation is possible if the properties of the two components are experimentally known. One of the two phases is the mortar (of modulus E_m); the other one is the stone pebbles, which can be considered as aggregates (of modulus E_a). A stone volume p = 30-35% of the total volume may be reasonably assumed for the tested specimens, which is not far from the real cases.

The average experimental values of E_m for the hydraulic mortar used for the wallettes are those indicated in 2.1. The average experimental values of E_a are respectively:

Noto stone E_a = 9000 N/mm^2
Serena stone E_a = 20800 N/mm^2

The values of the elastic modulus of the rubble material (E_i) was calculated by four different formulations, assuming different hypotheses for two-phases materials deduced from the research of various authors, according to the methodologies proposed by Hashin (1964) and Budianski (1965).
Assuming the same value of the Poisson ratio for the two materials, four equations result as the consequence of the simple models (Kameswara et al., 1974) represented in Fig. 17.

a) b) c) d)

Figure 17 Models for two-phases composites: a) in parallel, b) in series, c) spheres model, d) parallelepiped model (Kameswara et al., 1974).

$E_i = E_m [1 + p (E_a/E_m – 1)]$ material leaves laying in parallel (1)

$E_i = E_m [1 – p (1 – E_m/E_a)]^{-1}$ material leaves laying in series (2)

$E_i = E_m \{[(1 – p) + (1 + p) E_a/E_m)] / [(1 + p) + (1 – p) E_a/E_m]\}$ spheres model (3)

$E_i = E_m \{[1 + r (E_a/E_m – 1)] / [1 + (r – p) (E_a/E_m – 1)]\}$ parallelepiped model (4)

where $r = p^{1/2}$.
In Table 5, the values of E_i are collected, calculated assuming values of p between 0.25 and 0.4, values of the ratio E_a/E_m equals to 2.5, 5.0, 7.5 and the value of E_m = 3389 N/mm^2, corresponding to the mortar after 60 days of curing.

Table 5 Values of E_i calculated according to equations (1), (2), (3), (4), when E_a = 3389 N/mm^2 (after 60 days of curing)

	equation (1)			equation (2)			equation (3)			equation (4)		
p (%)	E_a/E_m			E_a/E_m			E_a/E_m			E_a/E_m		
	2.5	5	7.5	2.5	5	7.5	2.5	5	7.5	2.5	5	7.5
25	4660	6778	8896	3987	4236	4326	4202	4745	4991	4313	5084	5487
30	4914	7456	9998	4133	4459	4580	4389	5084	5407	4501	5432	5921
35	5168	8134	11099	4290	4707	4865	4585	5452	5866	4695	5802	6388
40	5422	8811	12200	4459	4984	5187	4791	5854	6376	4897	6199	6898

It can be easily observed that, for constant values of the ratio E_a/E_m, the value of E_i obtained from the equations (2), (3), (4) when 30% ≤ p ≤ 35%, undergo low variations (10-15%). However, by comparing the results obtained analytically and experimentally when p = 30%, formulation (2) turns out to be the nearest one to the interpretation of the mechanical behaviour of the considered rubble leaf, the difference being anyway not less than 25%. Therefore, equation (2) is chosen; it should be also added that the high percentage variation does not surprise, due to the typical uncertainties of the experimental study of the examined non-homogeneous material. Apparently unexpectedly, the experimental results show that the values of E_i, in the case of the

internal leaf made with Serena sandstone of higher strength, are lower than the composite made with the Noto stone. This is probably due to the poor bond between the mortar and the Serena stones (due to the high strength and low porosity of the stone itself) as it was confirmed by the shear bond tests carried out on the Serena specimens without offsets, by the compression tests on the internal leaves (Budianski, 1965), by the crack pattern developed at the bond between mortar and Serena stones (Fig. 14), opposite to the Noto stones where also the pebbles cracked (Fig. 12). In fact, this aspect is never taken into account by the existing analytical models. In order to account for this unfavourable situation, (i.e. when porosity is as low as 2%), the elastic modulus calculated with the equation (2) has to be conveniently reduced. With the mentioned limitations and corrections, it can be concluded that also the case of rubble fillings, with a tendency to an elastic-brittle behaviour in compression, can be treated as a two-phase material. Therefore a preliminary numerical analysis can make use of elastic modulus deduced in the above-described way, without any direct experimental investigation. The application of software like DEM (e.g. UDEC 2D by ITASCA) requires also the definition of the shear modulus of each leaf, given their compressibility. In fact, the behaviour of the materials to which equation (2) refers is nominally linear-elastic. Therefore $G_i = 0.5\ E_i /(1 + \nu_i)$; $G_e = 0.5\ E_e /(1 + \nu_e)$, where ν_i and ν_e respectively represent the transversal deformability coefficient of the inner and the outer leaves, which may be assumed = 0.1 – 0.2. The determination of the mechanical characteristics of the interface surface (according to its geometry) will be the object of a future paper, given its fundamental importance into the use of software like DEM. The obvious utility of this application becomes more evident, if considering that it is still impossible to know the mechanical characteristics of the internal leaves experimentally, even when using NDT (sonic tests or tomography).

4.2 Stiffness of the masonry wall

For practical constructive reasons, it may be interesting to know the modulus of elasticity value of the entire multiple leaf masonry. When the external leaves of the wall are made of regular stone and regular mortar joints, it can be interesting to define the modulus of elasticity of the entire masonry volume (E), by using again a suitable model concerning two-phase materials (Fig. 17 and equation (1))

The experimental results show that the geometry of the interface between the leaves (with or without offsets) does not influence very much the modulus of elasticity. Offsets only bring a higher load carrying capacity to the wall. In the case of a linear-brittle behaviour of the components if E_e and E_i are respectively the elastic modulus of the external and of the internal leaf (experimentally known), until the bond exists between them:

$$E = (1 - p)\ E_i + p\ E_e \qquad (5)$$

where p is here the percentage volume of the external leaves referred to the total masonry volume.

The presented experimental case can be used as an example. Assuming p=66%, when the external leaves are made with a soft stone (the Noto stone), equation (5) gives a value of E = 3928 N/mm^2; the average experimental value for the same stone is E = 4714 N/mm^2. In the case of a stiff stone (the Serena stone) E = 5120 N/mm^2; the average experimental value for the same stone is E = 4768 N/mm^2. There is a good agreement between theoretical and experimental values, the difference being not more than 20%.

It can be concluded that equation (5) gives a fairly reliable value of E in the case of masonry characterised by regular external leaves, once the elastic properties of the components are known. However, the characteristics of the interface are relevant for the evaluation of the load-bearing capacity of the masonry mass, especially in seismic areas.

5 Conclusions

The objective of the research was to understand the mechanisms of failure of multiple leaf walls in compression and the influence of the bond between the leaves on the movements and on the strength of the masonry. Some concluding remarks are possible, though the results still need further elaboration and perhaps new tests will be required:

- analysing the crack patterns under compression and shear tests is particularly interesting to understand the mechanisms of failure;
- in the case of no or poor connection, the separation under compression between the leaves of different stiffness values starts at very low stress values and the load-bearing function is completely developed by the outer leaves, which start cracking very early;
- the collapse of the three-leaf walls occurs at lower stress values than in the case of outer leaf tested individually. These experimental observations are very similar to those collected on the pillars of ancient Italian churches;
- the shear strength between the leaves is highly influenced by the interface geometry: in fact, better results can be obtained when there is a significant connection between the leaves;
- a two-phase material modelling was satisfactorily applied, capable to calculate analytically the elastic modulus of the internal and of the composite material, like multiple-leaf masonry.

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