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A MICROMECHANICAL DAMAGE MODEL FOR COMPLEX MASONRY STRUCTURES

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Abstract

A micromechanical damage model for the finite element modelling of complex masonry structures is presented. It has been developed in order to model, through the finite element method, complex masonry structures. The model considers masonry material as a continuum medium constituted by an iterative and ordered assembly of bricks (or stones) and mortar. The constitutive equations are based on the homogenisation theory and consider the non linear stress-strain relationship in term of mean stress and mean strain. The damage evolution is governed by specific damage laws, based on an energetic approach derived from Fracture Mechanics. Specific aim of the study is the analysis of the influence of the block assembly on the global behaviour of masonry structures. A simple example is presented in order to demonstrate the effectiveness of the model.

Key Words

Masonry, damage models, historical constructions.

1 Introduction

Masonry constructions are made of a continuum where the resistant structure is not univocally defined. The structure depends on geometry, stiffness and mass distribution, chronological succession of the building works, subsequent alterations. Moreover, it depends on the acting forces and on the damage states of the constituent material. For these reasons, full-scale models are required for the analysis of masonry structures.

One of the main problem in the study of historical constructions is to identify the residual life of the structure. In fact, differently than for new construction, the analyst does not know in which stage the structure is at present. It depends, besides its complex events, on the history of damage. From an experimental point of view, monitoring is the main tool to identify damage evolution, although Analytically, it is well recognized the exigency of numerical tools for evolutive static and dynamic analysis.

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Objective of the present study is to develop a constitutive model for complex masonry structures, able to predict their behaviour from the linear elastic range, through cracking and degradation until complete loss of strength. The finite element method is the adopted as a framework for the numerical implementation.

Masonry is a composite material made of an assembly of blocks connected by a set of joints. It is well evident how, under this definition, a number of different materials can be included. In fact, different can be the size, shape and nature of blocks, such as the pattern of assembly. Moreover, different can be the physical nature of joints (dry or mortar joints). The constitutive model has been developed for a specific and widely diffused class of masonry structures, whose main characteristics are:

- prevalence of bidimensional structural elements (vaults, domes, walls of limited thickness);
- blocks size much lower than structural elements size;
- presence of mortar joints.

On the basis of this assumptions, the model has been formulated with a micromechanical approach, considering the hypothesis of plane stress. In the micromechanical theory, the behaviour of a composite material is described in terms of macro (or average) stress and strain so that it can be assumed, through homogenization techniques, to be a homogeneous material. This approach provides obtain the continuum constitutive properties in terms of properties and structures of the microcostituents.

One of the main objective of this study is to analyse the influence of the block assembly on the global behaviour of masonry structures.

The constitutive model is based on the models presented in Alpa and Monetto (1994) and in Gambarotta and Lagomarsino (1997). In particular, the constitutive equations and the evolution laws of the internal variables are based on the latter, while the damage mechanisms are derived from the former.

2 The constitutive model

Masonry is considered as a composite material, made of a periodically arranged set of blocks, connected by mortar joints. The model considers two different types of mortar joint: bed joints, continuous on their plane and usually set perpendicularly to the compressive principal stresses, and discontinuous head joints (fig. 1).

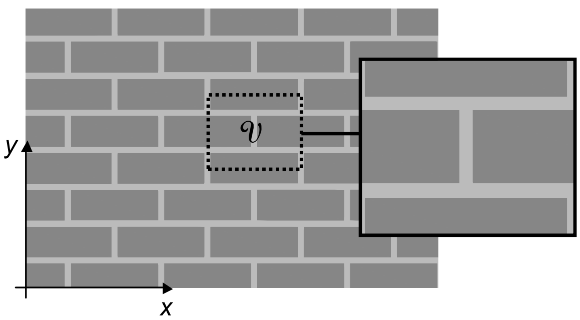

Figure 1: the considered masonry pattern.

The constitutive equations regarding the equivalent continuum damage model are obtained by homogenizing this composite medium. This is carried out by considering a representative volume element (fig. 1-2), having width b+g and height 2(h+g) (b and h are the block width and height respectively, while g is the joint thickness) and by defining the mean strain as a function of the mean stress. The constitutive equations are obtained on the hypothesis of plane stress condition.

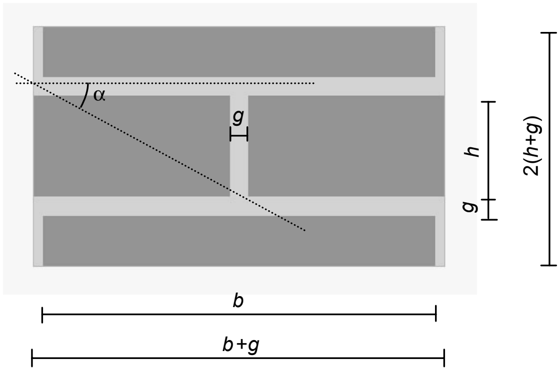

Figure 2: the representative volume element used in the homogenization procedure.

In order to describe the masonry pattern, two geometrical parameters have been introduced:

$$\eta_{mb} = \frac{g}{h+g} \ , \tag{1}$$

$$\eta_{mh} = \frac{g}{b+g} \ . \tag{2}$$

These parameters represents the ratio between the mortar bed thickness and the block height and the ratio between the mortar head joint thickness and the block width. The ratio η_{mb}/η_{mh} represents the shape of the blocks and identifies, in practice, the interlocking of the masonry.

Figure 2 shows different masonry pattern as a function of the ratio η_{mb}/η_{mh}. It is well evident that the limit cases where $\eta_{mb}/\eta_{mh}=0$ and $\eta_{mb}/\eta_{mh} \to \infty$ are not realistic and that, in common practice, $\eta_{mb}/\eta_{mh} > 1$. Figure 4 represents the masonry pattern as a function of η_{mb}. It is worth noting that the case where $\eta_{mb}=0$ represents a dried assembled masonry. For irregular stone masonry, with blocks of different size and shape, the two parameters η_{mb} and η_{mh} assume a statistical meaning.

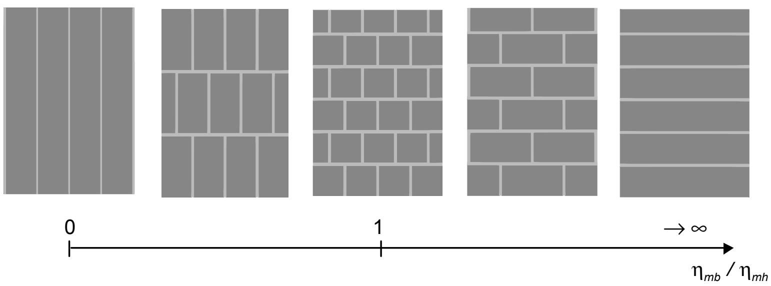

Figure 3: masonry pattern as a function of the ratio η_{mb}/η_{mh}.

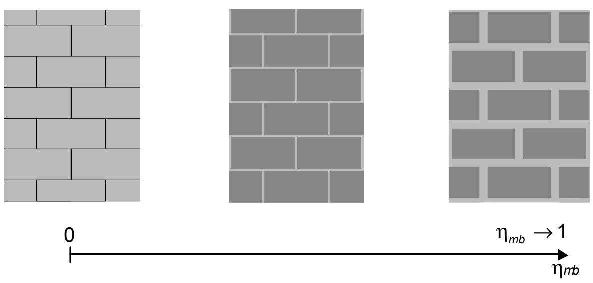

Figure 4: masonry pattern as a function of the parameter η_{mb}.

2.1 Constitutive equations

The model is based on the assumption that the mean strain of masonry is generated by two different contributions: an elastic contribution, associated to an elastic equivalent continuum (obtained through homogenization techniques), and an inelastic contribution, generated by damage phenomena localized in mortar bed and head joints and in blocks. Therefore, the mean strain $\bar{\varepsilon}$ may be expressed as follows:

$$\bar{\boldsymbol{\varepsilon}} = \bar{\mathbf{C}}^{-1}\bar{\boldsymbol{\sigma}} + \bar{\boldsymbol{\varepsilon}}_{mb} + \bar{\boldsymbol{\varepsilon}}_{mh} + \bar{\boldsymbol{\varepsilon}}_{b} \quad , \tag{3}$$

where $\bar{\boldsymbol{\sigma}}$ is the mean stress tensor, $\bar{\mathbf{C}}$ is the mean elastic tensor, $\bar{\boldsymbol{\varepsilon}}_{mb}$ is the inelastic contribute associated to the damage of the mortar bed joints, $\bar{\boldsymbol{\varepsilon}}_{mh}$ is the inelastic contribute associated to the damage of mortar head joints, $\bar{\boldsymbol{\varepsilon}}_{b}$ is the inelastic contribute associated to the damage of blocks. With reference to the natural axis of masonry (fig. 1), considering the hypothesis of plane stress, the mean strain and stress tensors can be expresses as:

$$\bar{\boldsymbol{\varepsilon}} = \left(\varepsilon_x, \varepsilon_y, \gamma\right)^T \quad , \tag{4}$$

$$\bar{\boldsymbol{\sigma}} = \left(\sigma_x, \sigma_y, \tau\right)^T \quad . \tag{5}$$

The definition of the average elastic tensor has not been an aim of this study. Different elastic homogenization techniques has been presented recently in literature (Anthoine1995, Lourenço and Zucchini 2000).

The definition of the inelastic contributions is based on different damage mechanisms. Mechanism *M1* represents damage phenomena in mortar bed joint generated by a tensile stress normal to their plane (σ_y>0). The related strain contribution is the following:

$$\bar{\varepsilon}_y^{M1} = \eta_{mb}\alpha_{mb}c_{mn}H\left(+\sigma_y\right)\sigma_y \quad , \tag{6}$$

where α_{mb}>0 is the bed mortar joint damage variable, c_{mn} is the extensional inelastic compliance parameter characterizing the mortar joint, and H is the Heaviside function taking in account the unilateral response of the interface.

Mechanism *B1* is activated by the analogous tensile force (σ_y>0), but involves blocks. Although mortar joints are usually weaker than blocks, in some particular cases (for example for pozzolanic mortars typical of Southern Italy) blocks could be weaker and, consequentially, subjected to the damage phenomena taken in account from this mechanism. The associated inelastic strain is:

$$\bar{\varepsilon}_y^{B1} = \left(1-\eta_{mb}\right)\alpha_b c_{bnt}H\left(+\sigma_y\right)\sigma_y \quad , \tag{7}$$

where α_b>0 is the blocks damage variable and c_{bnt} is the extensional inelastic compliance parameter characterizing the blocks when they are subjected to a tensile stress.

Mechanism *M2* describes the damage in mortar bed joint due to a tangential stress acting on their plane (τ). Slidings associated to this tangential stress are limited or locked, through friction phenomena, by a compressive stress (σ_y<0). The mechanism produces a sliding dependent on the applied stress τ and on the friction force acting on mortar-block interface f :

$$\bar{\gamma}^{M2} = \eta_{mb}\alpha_{mb}c_{mt}\left(\tau - f\right) \quad , \tag{8}$$

where c_{mt} is the tangential inelastic compliance parameter characterizing the mortar joint.

Mechanism *B2* describes the damage of blocks when they are subjected to an analogous tangential stress. It generates the following inelastic strain contribution:

$$\bar{\gamma}^{B2} = \left(1-\eta_{mb}\right)\alpha_b c_{bt}\tau \quad , \tag{9}$$

where c_{bt} is the inelastic tangential compliance parameter of blocks.

Mechanism *M3* involves simultaneously mortar bed and head joints and represents the typical phenomena of loss of interlocking in the masonry. It is activated by a tensile stress acting on the plane of mortar head joints (σ_x>0) and/or a tangential stress acting on the plane of mortar bed joints (τ). It produces normal extensions of the mortar head joints associated with slidings on mortar bed joints. Analogously to the mechanism *M2*, it is assumed that slidings are limited or locked by the friction force associated to a compressive stress σ_y<0. When both normal and tangential stresses are acting on the reference volume, the damage pattern has a typical stair-shape (fig. 5a). When only a normal stress acts on the reference volume, the damage pattern has the shape illustrated in figure 5b.

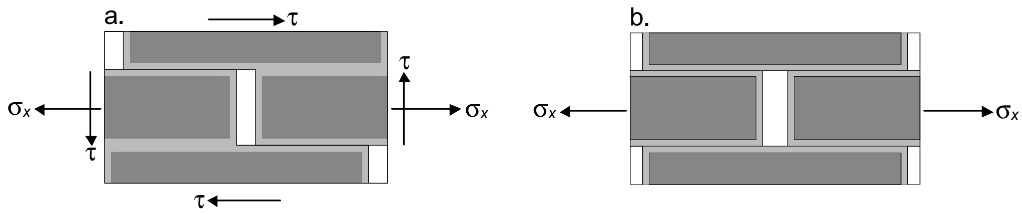

Figure 5: different damage pattern for mechanism M3.

Mechanism *MB* is a mixed mechanism that involves both mortar head joints and blocks. It is activated by a tensile stress acting on the plane of the mortar head joints (σ_x>0) and produces a normal extension of the volume along *x* (fig. 6).

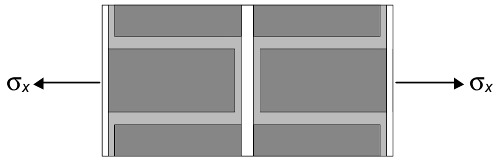

Figure 6: the damage mechanism MB.

Mechanism *M3* and *MB* interacts. In fact, when a tensile stress acts locally on the mortar head joints, depending on the strength ratio between mortar and blocks and on the compressive stress acting normally to the mortar bed joint, the damage can involve either mortar bed and head joints or mortar head joints and blocks. This phenomena has been clearly illustrated by the experimental tests carried out by Backes (1985), as it can be observed in figure 7.

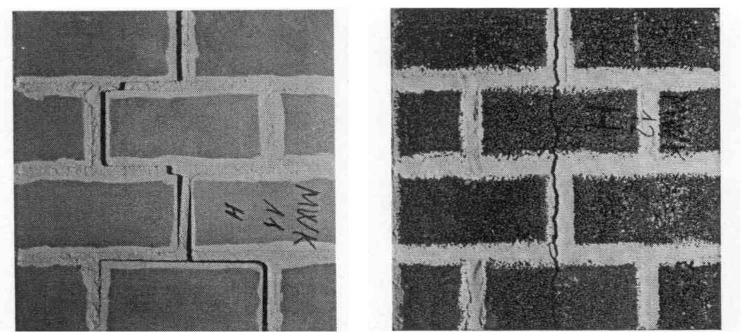

Figure 7: different damage mechanisms in Backes (1985).

The inelastic strain contributes associated to mechanisms *M3* and *MB* are assumed to be dependent on the local normal and tangential stresses acting on mortar joints and blocks (fig. 8):

$$\bar{\gamma}^{M3} = \frac{1}{2}\eta_{mb}\alpha_{mb}c_{mt}\left(\tau_{(h)} - f_{(h)}\right) \quad , \tag{10}$$

$$\varepsilon_x^{M3} = 2\frac{\eta_{mh}}{\eta_{mb}}\bar{\gamma}^{M3} \quad , \tag{11}$$

$$\bar{\varepsilon}_x^{MB} = \eta_{mh}\alpha_b c_{bnt} H\left(+\sigma_{x(b)}\right)\sigma_{x(b)} \quad , \tag{12}$$

$$\bar{\varepsilon}_x^{M3,MB} = \bar{\varepsilon}_x^{M3} + \bar{\varepsilon}_x^{MB} = \eta_{mh}\alpha_{mh}c_{mn}H\left(+\sigma_{x(h)}\right)\sigma_{x(h)} \quad . \tag{13}$$

It can be observed that, for compatibility, the inelastic strain $\bar{\boldsymbol{\varepsilon}}^{MB}$ is a rate of $\bar{\boldsymbol{\varepsilon}}^{M3,MB}$; the total strain of the mortar head joints associated to mechanisms *M3* and *MB*, indeed, is sufficient to describe the stretch of the volume along *x*.

In order to define the entity of the local stresses, an equilibrium and congruence analysis on the reference volume has been carried out:

$$\sigma_{x(h)} = \frac{1}{1+C_h+C_b}\left[(1+2C_b)\sigma_x + \frac{1}{2}\frac{\eta_{mb}}{\eta_{mh}}\left|\tau - f_{(h)}\right|\right] , \tag{14}$$

$$\sigma_{x(b)} = \frac{1}{1+C_h+C_b}\left[(1+2C_h)\sigma_x - \frac{1}{2}\frac{\eta_{mb}}{\eta_{mh}}\left|\tau - f_{(h)}\right|\right] , \tag{15}$$

$$\left|\tau_{(h)}\right| = \frac{1}{1+C_h+C_b}\left[2\frac{\eta_{mh}}{\eta_{mb}}(C_h - C_b)\sigma_x + (C_h + C_b)|\tau| + \left|f_{(h)}\right|\right] , \tag{16}$$

where:

$$C_h = \frac{1}{2}\frac{\eta_{mb}}{\eta_{mh}}\frac{\alpha_{mh}}{\alpha_{mb}}\frac{c_{mn}}{c_{mt}} \quad , \quad C_b = \frac{1}{2}\frac{\eta_{mb}}{\eta_{mh}}\frac{\alpha_b}{\alpha_{mb}}\frac{c_{bnt}}{c_{mt}} \quad , \tag{17}$$

and where f_h is the friction force on mortar bed joints generated by a compressive stress acting on them.

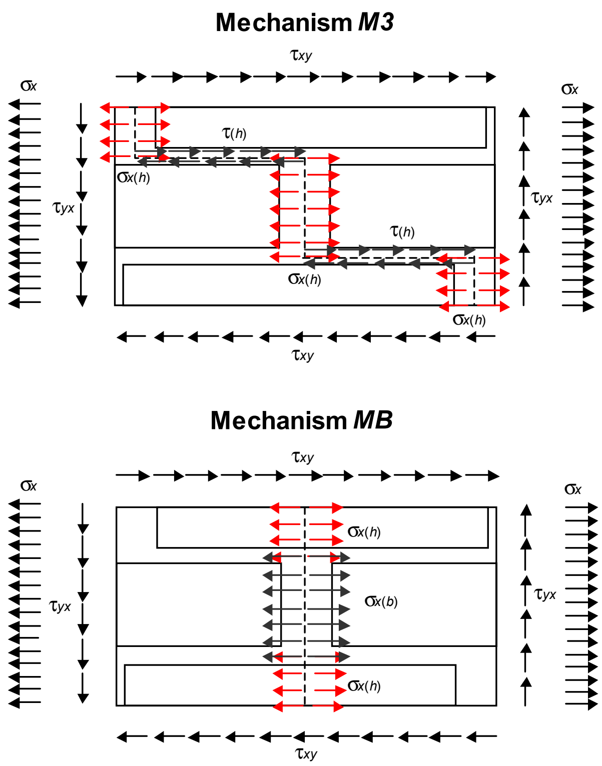

Figure 9: local stresses in mechanisms M3 and MB.

Mechanisms B3 and B4 involves only blocks. They can be activated when a compressive stress acts along x and y axis, respectively. The associated inelastic strains are:

$$\bar{\varepsilon}_x^{B3} = (1-\eta_{mh})\alpha_b c_{bnc} H(-\sigma_x)\sigma_x \quad , \tag{18}$$

$$\bar{\varepsilon}_y^{B4} = (1-\eta_{mb})\alpha_b c_{bnc} H(-\sigma_y)\sigma_y \quad . \tag{19}$$

In Alpa and Monetto (1994) further damage mechanism is considered. It describes an opening of the mortar bed joints associated with slidings on mortar head joints, produced by a shear stress acting on the volume. In the present model, such mechanism has been disregarded since its effects on the failure domain is negligible, at last for standard masonry patterns.

Finally, the anelastic strain tensors have the following form:

$$\bar{\boldsymbol{\varepsilon}}_{mb} = \begin{Bmatrix} 0 \\ \bar{\varepsilon}_y^{M1} \\ \bar{\gamma}^{M2} + \bar{\gamma}^{M3} \end{Bmatrix} , \qquad \bar{\boldsymbol{\varepsilon}}_{mh} = \begin{Bmatrix} \bar{\varepsilon}_x^{M3,MB} \\ 0 \\ 0 \end{Bmatrix} \qquad \bar{\boldsymbol{\varepsilon}}_b = \begin{Bmatrix} \bar{\varepsilon}_x^{MB} + \bar{\varepsilon}_x^{B3} \\ \bar{\varepsilon}_y^{B1} + \bar{\varepsilon}_y^{B4} \\ \bar{\gamma}^{B2} \end{Bmatrix} . \tag{20}$$

2.2 Damage evolution laws and limit domain.

The expressions of the inelastic strain contributions imply that the internal variables α_{mb}, α_{mh} and α_b (besides the friction stresses *f* and f_h) must be known at any step of the loading history through evolution equations. The proposed model assumes two different kinds of evolution laws, accordingly to different damage mechanisms.

The first criterion concerns the damage associated to normal stresses and is based on fracture mechanics theory. The hypothesis is advanced that damage (assumed as a crack growth) may be activated ($\dot{\alpha} > 0$) if the density of damage energy release rate *Y* equals the damage toughness of the material *R*. If *Y*<*R* there is no damage evolution. It follows that admissible states must satisfy the following condition:

$$\phi = Y - R \le 0 \tag{21}$$

Accordingly to this criterion, the following three damage evolution conditions have been introduced in the model:

$$Y_{mb} = \frac{\partial E_{d(mb)}}{\partial \alpha_{mb}} = \frac{\partial}{\partial \alpha_{mb}} \left[\frac{1}{2}\bar{\varepsilon}_y^{M1}\sigma_y + \frac{1}{2}\bar{\gamma}^{M2}(\tau - f) + \frac{1}{2}\bar{\gamma}^{M3}\left(\tau_{(h)} - f_{(h)}\right) \right] . \tag{22}$$

$$Y_{mh} = \frac{\partial E_{d(mh)}}{\partial \alpha_{mh}} = \frac{\partial}{\partial \alpha_{mh}} \left(\frac{1}{2}\bar{\varepsilon}_x^{M3,MB}\sigma_{x(h)} \right) . \tag{23}$$

$$Y_b = \frac{\partial E_{d(b)}}{\partial \alpha_b} = \frac{\partial}{\partial \alpha_b} \left(\frac{1}{2}\bar{\varepsilon}_x^{MB}\sigma_{x(b)} + \frac{1}{2}\bar{\varepsilon}_x^{B3}\sigma_x + \frac{1}{2}\bar{\varepsilon}_y^{B1}\sigma_y + \frac{1}{2}\bar{\varepsilon}_y^{B4}\sigma_y + \frac{1}{2}\bar{\gamma}^{B2}\tau \right) . \tag{24}$$

where E_d is the dissipation in the infinitesimal step. It is worth noting that each damage condition is associated to a specific damage variable α_{mb}, α_{mh} and α_b.

The second criterion regards the friction phenomena on the mortar-block interface. The hypothesis that frictions f and f_h must satisfy the Coulomb condition, conduces to the following additional damage conditions (a *non-associated* flow rule is assumed):

$$|f| + \mu\sigma_y \le 0 \ , \tag{25}$$

$$|f_h| + \mu\left(\sigma_y + 2\frac{\eta_{mh}}{\eta_{mb}}|\tau|\right) \le 0 \ . \tag{26}$$

The terms in round bracket in equation (26) represents a effective normal stress acting on mortar bed joints in mechanisms *M3*. It can be observed that the tangential stress, appearing in absolute value, reduces the effect of the compressive normal stress in limiting slidings.

The three-dimensional limit domain of the constitutive model in the stress-space is represented in figure 9.

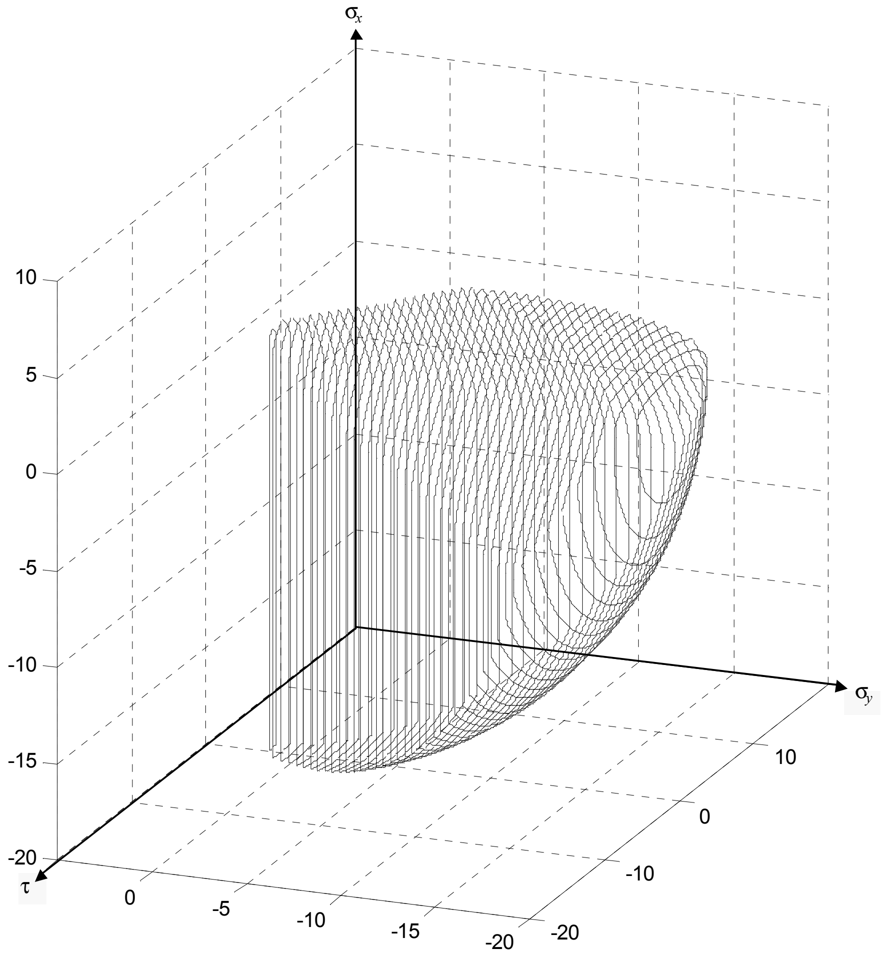

Figure 9: limit domain of the constitutive model

3 Applications

The model has been implemented in a *general purpose* finite element program (ANSYS). The solution of the incremental problem for the definition of the internal state variables has been carried out linearizing the problem though the Newton-Raphson method.

A meaningful application of the model is the analysis of masonry walls subjected to horizontal forces proportional to their own weight. The main objectives of this application is to verify the capability of the model of describing the effect of the masonry pattern on the global behaviour of the structural elements.

The analysis is supported by the experimental tests presented in Giuffrè (1993) for dry block walls. These tests showed that the failure mode of masonry walls depends on the masonry pattern and, in particular, on the interlocking of blocks. For a typical *running pattern,* such as the one considered in the formulation of the constitutive model, the interlocking of blocks in univocally dependent on the ratio *b*/*h* between block width *b* and block height *h*. Figure 10 shows two typical damage mechanisms: in case (a), blocks have a ratio *b*/*h*=4 and the failure is associated to the global overturning of the wall; in case (b), this ratio is *b*/*h*=2 and the failure is associated to the overturning of a single part of the walls. This partial overturning is generated by an opening of the mortar head joints in the left side of the walls associated with sliding on

mortar bed joints. The damage at the basis of the wall (diagonal crack on the right side) represents an initial phase of its global overturning. For a masonry walls of middle slenderness (B/H=1.5, where B is the width of the wall and H is its height), the collapse multiplier is λ_A=0.38 for case (a) and λ_B=0.32 for case (b) (fig. 13).
Figure 11 and 12 shows the results obtained with the proposed model, in terms of the damage variable of the mortar bed joints and mortar head joints, respectively. It can be observed that the two different mechanism are well identifiable. In fact, it can be noted that in case (a), the damage of the mortar bed joints is intense and localized in the right side of the wall, while the damage in mortar head joints is very low. In case (b) the damage is localized in the left side of the walls and involves both mortar bed and head joints. An evident vertical crack lead to the failure of the wall. Moreover, it can be noted the diagonal crack at the basis of the right side of the wall.

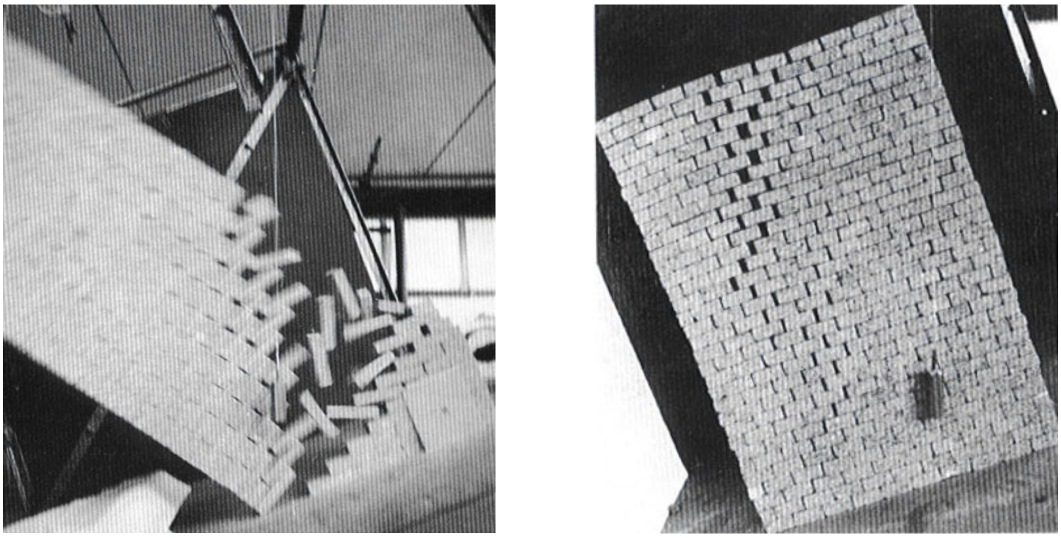

Figure 10: different damage mechanisms for masonry walls with different pattern (Giuffrè 1993).

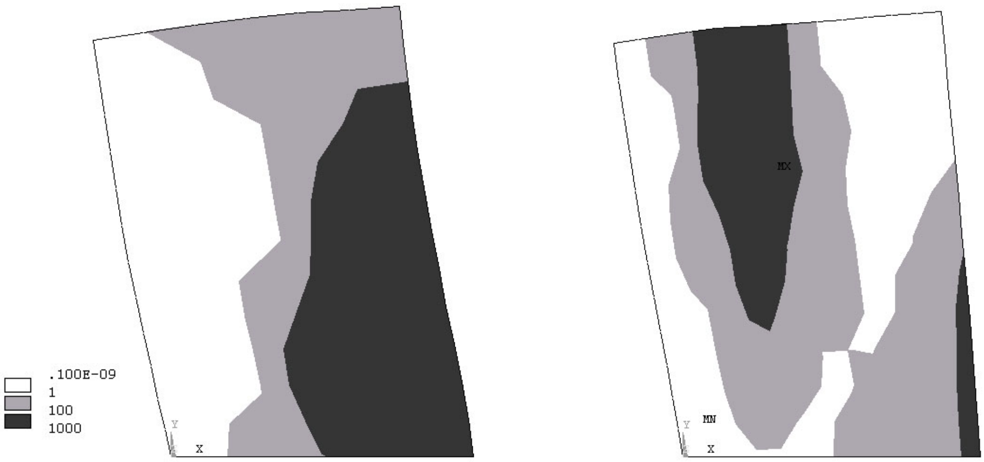

Figure 11: damage in the mortar bed joints at collapse in the numerical model for the case (a) (left) and (b) (right). The mortar bed joints fails at α_{mb}=1.

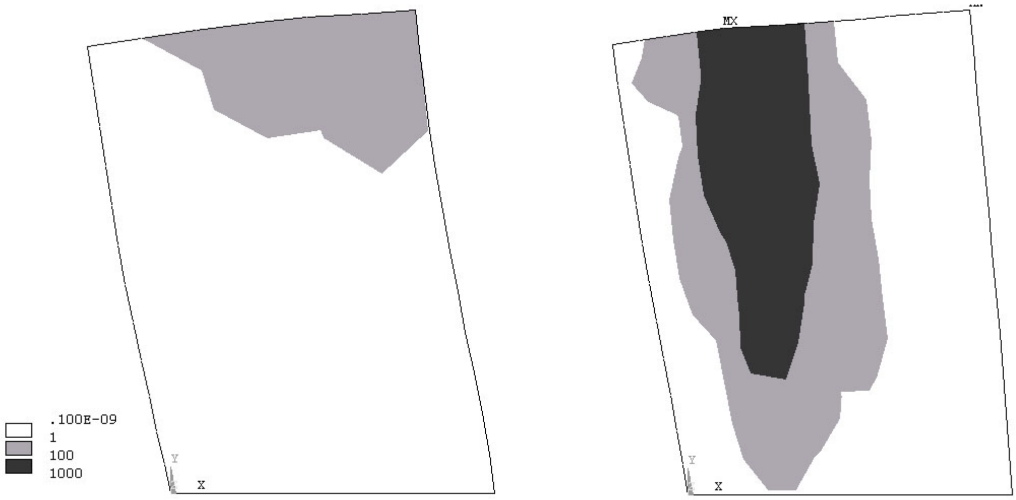

Figure 12: damage in the mortar head joints at collapse in the numerical model for the case (a) (left) and (b) (right). The mortar head joints fails at α_{mh}=1.

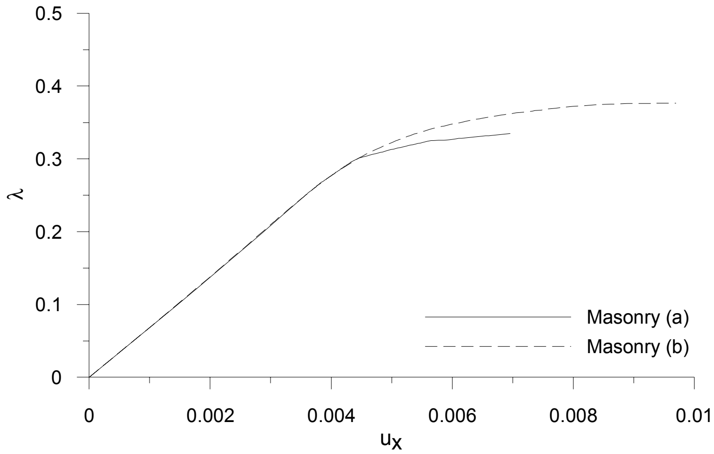

Figure 13: load multiplier as a function of the displacement along x.

4 Conclusions

A constitutive model for masonry structures has been presented. Specific aim of the study has been the analysis of the influence of the block assembly on the global behaviour of masonry structures. The simple example of masonry panels subjected to horizontal forces demonstrated, both from qualitative and quantitative point of view, the effectiveness of the model in describing different masonry behaviours.

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