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FINITE ELEMENT MODEL OF UNREINFORCED CONCRETE BLOCK WALLS SUBJECT TO OUT-OF-PLANE LOADING - A PARAMETRIC STUDY

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Abstract

Finite element modelling of the structural response of hollow concrete block walls subject to out-of-plane loading has become more common given the availability of computers and general-purpose finite element software packages. From the literature, it is apparent that two approaches have been employed to model the masonry, homogeneous models and heterogeneous models. The most common approach uses homogeneous macro models developed either directly from masonry models or via homogenization techniques. Although standard concrete blocks are hollow, these models employ either layered shell elements or three-dimensional solid elements to model the assembly. As such, the actual geometrical representation and corresponding properties are not accurately represented in these models. The other approach is referred to as heterogeneous models where the individual units and mortar joints are modelled separately. Although the heterogeneous models discriminate between the different materials of the assembly, they fail to adequately represent the geometric properties of the block. This study reports on a parametric study conducted to quantify the effects of the modelling technique for hollow blocks on the structural response of the assembly, specifically for out-of-plane bending. Two structural elements with varying span/thickness ratios are considered, a horizontal spanning strip and a vertical spanning strip. Further, both the homogeneous modelling approach and the heterogeneous modelling approach are used. The values computed by means of the homogeneous and heterogeneous finite element models are found to differ significantly depending on the configuration and span/thickness ratio of the wall.

Key Words

Unreinforced masonry, hollow concrete block, finite element, out-of-plane bending, homogeneous versus heterogeneous models.

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1 Introduction

Masonry consists of stone units, blocks and/or bricks that are bonded together via mortar joints. As a result, masonry possesses heterogeneous material properties that depend on the properties of the block, mortar and bond at the interface. The corresponding finite element model, in general, is carried out either by discretizing the individual blocks and mortar joints referred to as a heterogeneous model, or by developing a macroscopic formulation that accounts for the anisotropic properties of the combined assemblage. In the second approach, the individual mechanical properties of the masonry materials are not used and an equivalent homogenous material is derived. Both heterogeneous and homogeneous modelling techniques provide advantages and disadvantages. The heterogeneous approach, which permits the modelling of individual elements including the interface, requires large computational effort in order to analyse masonry structures. The homogeneous models, which are appealing, as they demand less effort for discretizing and analysing the structure, do not adequately account for the anisotropic behaviour of unreinforced masonry, specifically the presence of head and bed joints. Homogeneous and heterogeneous models for the analysis of masonry structures ranging from simple to complex have been proposed in the literature (Bull 2001). The majority of these models include constitutive models for the analysis of masonry before and after cracking occurs.

The geometry of the hollow block presents additional modelling challenges. The focus of previous research has been on the joints without any considerations given to the actual geometry of the unit. In particular, is the geometrical representation of the block a critical parameter when analysing the structural response of concrete block masonry walls? This question and its implications are addressed using data from a numerical experiment on hollow block walls that are subjected to out-of plane bending. Two wall geometries are considered, horizontal spanning strip and vertical spanning strip. And, four different modelling techniques are assessed, homogeneous 3-D solid model, homogeneous detailed model, heterogeneous stack pattern model, and heterogeneous running bond model.

2 Numerical experiment

2.1 Concrete blocks, geometry & construction

Concrete masonry products are classified as solid or hollow depending on their net solid horizontal cross-sectional area. If the net solid horizontal cross-sectional area is less than 75%, the product is classified as hollow. Typically, the percent solid ranges from 50% to 60%. Concrete blocks come in a large variety of sizes and shapes. The 20 cm block is most common and has height, thickness and length dimensions of 190 x 190 x 390 mm, respectively as shown in Figure 1. Consisting of 3 webs joining the face shells, this standard block is used for the parametric study.

Concrete blocks are used to construct various structural elements such as walls, columns, pilasters, beams and lintels. This study focuses on unreinforced single-wythe walls. These walls are normally constructed in running bond as shown in Figure 2, where the head joint is positioned at the middle of the units below and above. Other patterns or bonds are also used, however it is mandated by masonry design codes that a minimum of one-fourth overlap be designated as running bond, as a structural requirement (Drysdale et al 1999). For decorative purposes, stack pattern is used where the units do not overlap as shown in Figure 2. These walls are designed for both loadbearing and nonloadbearing applications. The two types of construction, running bond and stack pattern, are investigated in this study.

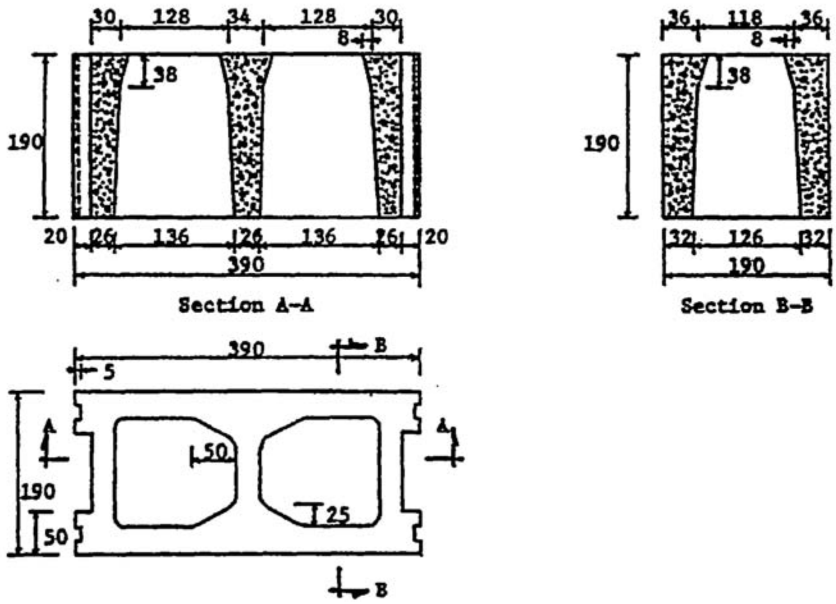

Figure 1 Standard block dimensions (Essawy 1986)

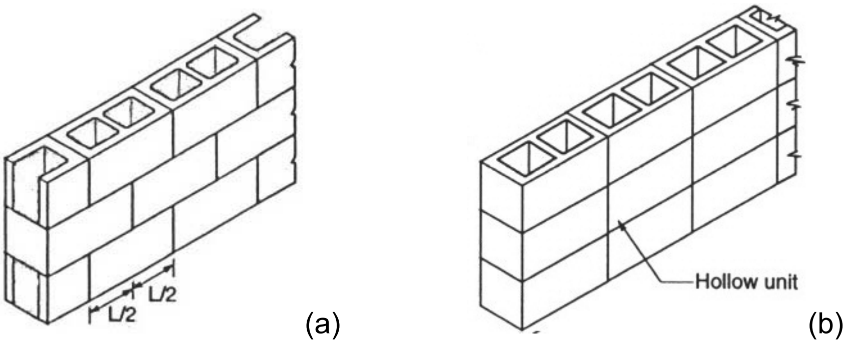

Figure 2 Single-wythe walls, (a) Running bond, (b) Stack pattern (Drysdale et al 1999)

This study examines walls that bend in one direction when subjected to out-of-plane loading. As such, only horizontal spanning walls and vertical spanning walls, as illustrated in Figure 3, are considered. Making use of double symmetry, a quarter model is used to simulate the structural response of a horizontal strip, whereas a full model is used for the vertical strip. For this parametric study, four span/thickness ratios are considered for the two wall strips as given in Table 1.

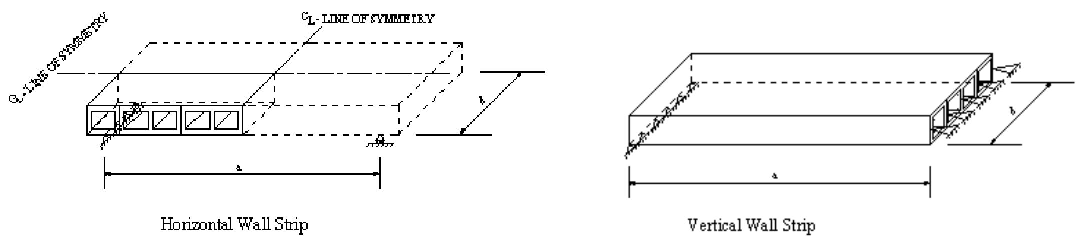

Figure 3 schematic representations of horizontal and vertical wall strips

Table 1 Geometric configuration of the masonry walls.

Configuration	Panel Dimension L x H [mm]	Span/Thickness ratio
Horizontal strip	1000 x 800	5.26
	1800 x 800	9.47
	3400 x 800	17.89
	5800 x 800	30.52
Vertical strip	828 x 800	4.21
	828 x 1600	8.42
	828 x 2800	14.74
	828 x 3200	16.84

2.2 Homogeneous models

Two homogeneous models are used to simulate the structural response of the masonry walls. The first model employs the three-dimensional solid brick element to discretize the structure. Accordingly, a block thickness of 172.5 mm and 176.9 mm instead of 190 mm are used to provide structural properties equivalent to the hollow block for the horizontally spanning walls and vertically spanning walls, specifically the second moment of inertia. The corresponding model for a horizontal strip wall is shown in Figure 4.

The second approach is to use the averaged properties of the masonry according to homogenisation techniques, but to model the actual geometry of the block (Detailed Geometry) as shown in Figure 4. The same level of mesh refinement is used for both models and the mortar joint is given the averaged properties.

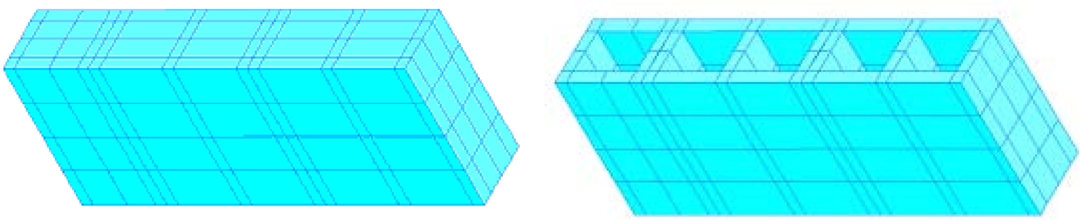

Figure 4 Finite element mesh of horizontal strip wall according to the two homogeneous models

2.3 Heterogeneous models

Two heterogeneous models are used which represent two construction patterns, running bond and stack pattern. Figure 5 shows the corresponding finite element model employed to analyse the response of a horizontal strip. The material properties of the block and the mortar were distinctively defined. Three dimensional brick elements were used to discretize the wall.

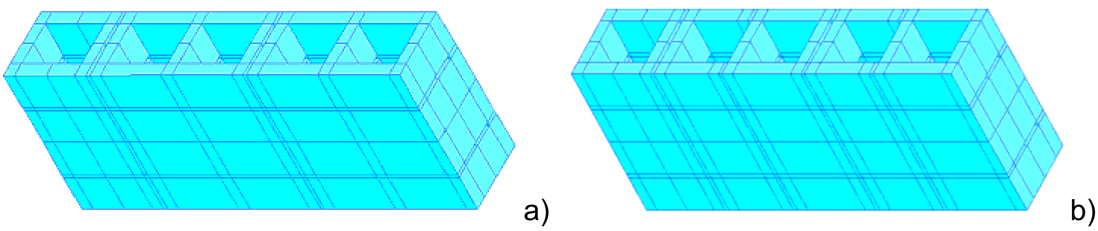

Figure 5 Finite element mesh of horizontal strip wall according to the two heterogeneous models; a) Stack pattern, b) Running bond

2.4 Material properties

Material is assumed to be linear, elastic and to possess orthotropic mechanical characteristics. Representative properties used for this analysis are given in Table 2 (Chahine 1989). It should be noted that the principle objective of this study is not to investigate homogeneous versus heterogeneous constitutive models, but rather to examine the implications of not adequately modelling the geometry of the block on the overall computed response of hollow block walls under out-of-plane bending.

Table 2 Material properties employed in the FEA of masonry walls.

Models	Material	E_1 [MPa]	E_2 [MPa]	E_3 [MPa]	ν_{12}	ν_{13}	ν_{23}
Homogeneous	Masonry	18,490	25,890	25,890	0.213	0.213	0.213
Heterogeneous	Block	23,420	32,790	32,790	0.213	0.213	0.213
	Mortar	10,000	10,000	10,000	0.200	0.200	0.200

3 Analytical results

AFEMS (2003), a general-purpose finite element program, was used to analyse the response of the masonry walls due to out-of-plane uniformly distributed load using the four established models. The results are presented separately for the two directions of bending and are given in the form of maximum and relative values for the out-of-plane displacement, bending stress, first principal stress and Von Mises stress.

3.1 Horizontal strip wall

The computed finite element results for the horizontal strip wall are summarized in Table 3. For the purpose of comparing the results, the computed values are normalized with respect to those obtained from the homogeneous solid model and are listed in Table 4. In order to examine the sensitivity of the results to the mortar properties, an additional run was carried out with a 3,000 MPa for the mortar's modulus of elasticity. The corresponding results as a ratio of the homogeneous solid are listed in Table 5.

3.1.1 Out-of-plane displacement

Examination of the computed results indicate that using an equivalent solid model to simulate the structural response of hollow block yields a stiffer response in comparison to detailed homogeneous and heterogeneous models. For ratio of 5.26, the computed displacement is about half the values of the other three models. As the ratio increases to 30.52, the difference decreases to about 17% for the homogeneous detailed model and is essentially the same for the heterogeneous models. Comparing the results of the two heterogeneous models, it can be seen that the two models differed by about 8% at the 5.26 span/thickness ratio but the difference became very small at span/thickness ratio of 30.52. Another interesting observation is the percent difference between the detailed homogeneous model and the heterogeneous model corresponding to stack pattern is approximately 18% irrespective of the span/thickness ratio. The difference between the detailed homogeneous model and the heterogeneous model corresponding to running bond varies from 10% to 17% as the ratio increases from 5.26 to 30.52. The same observations are noted when the elastic modulus of the mortar was reduced from 10,000 MPa to 3,000 MPa.

3.1.2 Stress

The computed stress values exhibited the same trends as the out-of-plane displacement especially at span/thickness ratio of 5.26. For this wall, the bending stress values from the other three models are about 100% higher than the homogeneous solid model value. As the span / thickness ratio increased to 30.52, the

difference decreases to about 28% which is greater than the difference in displacement at this ratio. The results show good agreement between the bending stress values from the detailed homogeneous model and the two heterogeneous models. The principal stress and effective stress followed patterns similar to the bending stress. From Table 5, it can be observed that only the stresses obtained using the heterogeneous running bond model increased approximately 5% when the stiffness of the mortar joint was changed from 10,000 MPa to 3,000 MPa.

Table 3 Computed max values for the horizontal strip masonry wall.

Model Type	Span/Thick Ratio	Principal stress [MPa]	Bending stress [MPa]	Displacement [mm]	Von Mises stress [MPa]
Homogeneous-Solid	5.26	0.0276	0.0276	0.0014	0.0242
Homogeneous-Detailed		0.0549	0.0536	0.0029	0.0415
Heterogeneous-Stack		0.0556	0.0543	0.0025	0.0434
Heterogeneous-Running		0.0578	0.0564	0.0027	0.0470
Homogeneous-Solid	9.47	0.0920	0.0917	0.0141	0.0813
Homogeneous-Detailed		0.1410	0.1383	0.0211	0.1117
Heterogeneous-Stack		0.1414	0.1389	0.0175	0.1147
Heterogeneous-Running		0.1363	0.1354	0.0183	0.1193
Homogeneous-Solid	17.89	0.3302	0.3291	0.1766	0.2929
Homogeneous-Detailed		0.4581	0.4498	0.2235	0.3712
Heterogeneous-Stack		0.4632	0.4559	0.1872	0.3828
Heterogeneous-Running		0.4471	0.4404	0.1907	0.3803
Homogeneous-Solid	30.52	0.9840	0.9807	1.5260	0.8734
Homogeneous-Detailed		1.3000	1.2270	1.7910	1.0600
Heterogeneous-Stack		1.3150	1.2940	1.5020	1.0920
Heterogeneous-Running		1.2720	1.2540	1.5210	1.0740

Table 4 Normalized values to Homogeneous-Solid for the horizontal strip masonry wall.

Model Type	Span/Thick	Ratio of Homogeneous-Solid			
	Ratio	Principal Stress	Bending Stress	Displacement	Von Mises Stress
Homogeneous-Solid	5.26	1.00	1.00	1.00	1.00
Homogeneous-Detailed		1.99	1.95	2.12	1.71
Heterogeneous-Stack		2.01	1.97	1.78	1.79
Heterogeneous-Running		2.09	2.05	1.92	1.94
Homogeneous-Solid	9.47	1.00	1.00	1.00	1.00
Homogeneous-Detailed		1.53	1.51	1.50	1.37
Heterogeneous-Stack		1.54	1.52	1.25	1.41
Heterogeneous-Running		1.48	1.48	1.30	1.47
Homogeneous-Solid	17.89	1.00	1.00	1.00	1.00
Homogeneous-Detailed		1.39	1.37	1.27	1.27
Heterogeneous-Stack		1.40	1.39	1.06	1.31
Heterogeneous-Running		1.35	1.34	1.08	1.30
Homogeneous-Solid	30.52	1.00	1.00	1.00	1.00
Homogeneous-Detailed		1.32	1.25	1.17	1.21
Heterogeneous-Stack		1.34	1.32	0.98	1.25
Heterogeneous-Running		1.29	1.28	1.00	1.23

Table 5 Normalized values to Homogeneous-Solid for the horizontal strip masonry wall for E_{mortar} = 3,000 MPa.

Model Type	Span/Thick Ratio	Ratio of Homogeneous-Solid			
		Principal Stress	Bending Stress	Displacement	Von Mises Stress
Homogeneous-Solid	5.26	1.00	1.00	1.00	1.00
Homogeneous-Detailed		1.99	1.95	2.12	1.71
Heterogeneous-Stack		2.03	1.99	1.85	1.84
Heterogeneous-Running		2.20	2.17	2.06	2.11
Homogeneous-Solid	9.47	1.00	1.00	1.00	1.00
Homogeneous-Detailed		1.53	1.51	1.50	1.37
Heterogeneous-Stack		1.55	1.53	1.33	1.44
Heterogeneous-Running		1.56	1.57	1.42	1.57
Homogeneous-Solid	17.89	1.00	1.00	1.00	1.00
Homogeneous-Detailed		1.39	1.37	1.27	1.27
Heterogeneous-Stack		1.42	1.40	1.17	1.33
Heterogeneous-Running		1.41	1.41	1.19	1.39
Homogeneous-Solid	30.52	1.00	1.00	1.00	1.00
Homogeneous-Detailed		1.32	1.25	1.17	1.21
Heterogeneous-Stack		1.35	1.34	1.10	1.27
Heterogeneous-Running		1.34	1.34	1.11	1.31

3.2 Vertical strip wall

The computed finite element results for the vertical strip wall are summarized in Table 6. For the purpose of comparing the results, the computed values are again normalized with respect to those obtained from the homogeneous solid model and are listed in Table 7. Table 8 provides the normalized results with respect to those of the homogeneous solid when the mortar's modulus of elasticity is changed to 3,000 MPa.

3.2.1 Out-of-plane displacement

Examination of the computed results indicate that these results exhibit the same trend noted for the horizontal strip but with much smaller difference compared to homogeneous solid model results. Particularly, the computed results using the equivalent solid model tend to be more comparable to the heterogeneous model at the higher slenderness ratios. As was the case with horizontal spanning walls, the detailed homogeneous model predicted larger displacement than the heterogeneous models. All three hollow block models predicted significantly larger displacement for the lower span/thickness ratios. Furthermore, results in Table 8 show that the heterogeneous models gave significantly higher displacements when the mortar's modulus of elasticity changed from 10,000 to 3,000 MPa. This increase is three times greater than the one observed for the horizontally spanning walls.

3.2.2 Stress

The computed maximum bending stress values using the homogeneous detailed model are on average 14% larger than those obtained using the solid model for the four span/thickness ratios. The maximum computed bending stresses using the heterogeneous models were on average about 30% higher than the solid model but somewhat higher at the lowest span/thickness ratio. The two heterogeneous models gave quite similar results but with the running bond model values slightly higher than the stack pattern values. This is the reverse of the situation for the horizontally spanning walls. Similar observations can be made for both the principal stress and the effective stress. Furthermore, no significant changes in any of the three stress values can be noted when the elastic modulus of the mortar joint was changed from 10,000 to 3,000 MPa.

Table 6 Computed max values for the vertical strip masonry wall.

Model Type	Span/Thick ratio	Principal stress [MPa]	Bending stress [MPa]	Displacement [mm]	Von Mises stress [MPa]
Homogeneous-Solid	4.21	0.0137	0.0137	0.0005	0.0173
Homogeneous-Detailed		0.0159	0.0159	0.0006	0.0360
Heterogeneous-Stack		0.0185	0.0184	0.0006	0.0374
Heterogeneous-Running		0.0193	0.0193	0.0006	0.0393
Homogeneous-Solid	8.42	0.0554	0.0554	0.0067	0.0534
Homogeneous-Detailed		0.0624	0.0624	0.0075	0.0827
Heterogeneous-Stack		0.0716	0.0713	0.0069	0.0851
Heterogeneous-Running		0.0733	0.0733	0.0070	0.0910
Homogeneous-Solid	14.74	0.1691	0.1691	0.0604	0.1637
Homogeneous-Detailed		0.1908	0.1908	0.0647	0.1891
Heterogeneous-Stack		0.2177	0.2168	0.0594	0.1971
Heterogeneous-Running		0.2222	0.2219	0.0599	0.2007
Homogeneous-Solid	16.84	0.2208	0.2208	0.1026	0.2138
Homogeneous-Detailed		0.2490	0.2490	0.1094	0.2466
Heterogeneous-Stack		0.2840	0.2828	0.1003	0.2571
Heterogeneous-Running		0.2899	0.2896	0.1010	0.2618

Table 7 Normalized values to Homogeneous-Solid for the vertical strip masonry wall.

Model Type	Span/Thick Ratio	Ratio of Homogeneous-Solid			
		Principal stress	Bending stress	Displacement	Von Mises Stress
Homogeneous-Solid	4.21	1.00	1.00	1.00	1.00
Homogeneous-Detailed		1.16	1.16	1.30	2.08
Heterogeneous-Stack		1.36	1.35	1.19	2.16
Heterogeneous-Running		1.41	1.41	1.23	2.27
Homogeneous-Solid	8.42	1.00	1.00	1.00	1.00
Homogeneous-Detailed		1.13	1.13	1.12	1.55
Heterogeneous-Stack		1.29	1.29	1.03	1.59
Heterogeneous-Running		1.32	1.32	1.04	1.70
Homogeneous-Solid	14.74	1.00	1.00	1.00	1.00
Homogeneous-Detailed		1.13	1.13	1.07	1.16
Heterogeneous-Stack		1.29	1.28	0.98	1.20
Heterogeneous-Running		1.31	1.31	0.99	1.23
Homogeneous-Solid	16.84	1.00	1.00	1.00	1.00
Homogeneous-Detailed		1.13	1.13	1.07	1.15
Heterogeneous-Stack		1.29	1.28	0.98	1.20
Heterogeneous-Running		1.31	1.31	0.98	1.22

3.3 Comparative analysis

In general, the four models produced similar trends for the horizontal and vertical spanning walls. For the vertically spanning walls, displacements were adequately predicted using the homogeneous solid model compared to the heterogeneous model at larger span/thickness ratios. At low span/thickness ratios, the comparatively larger shear forces caused large shear stresses in the webs of the block connecting the block face shells. Displacements resulting from this type of deformation are underestimated in the equivalent solid model, which has much lower shear stresses. In addition, the effects of the smaller cross-sectional area at the mortar bed joints and the lower modulus of elasticity of the mortar are not modelled in the homogeneous solid model. The detailed homogenous model, which uses an averaged modulus of elasticity, seems to overestimate the effects of the mortar. For face shell mortar bedding, use of the averaged property for the entire block (i.e., including the block webs) is not

consistent but is difficult to avoid. Changing the mortar's modulus of elasticity from 10,000 to 3,000 MPa has resulted a 30% increase in displacements. These results show that the predictions obtained using the heterogeneous models are sensitive to the material properties.

Table 8 Normalized values to Homogeneous-Solid for the vertical strip masonry wall for E_{mortar} = 3,000 MPa.

Model Type	Span/Thick Ratio	Ratio of Homogeneous-Solid			
		Principal stress	Bending stress	Displacement	Von Mises Stress
Homogeneous-Solid	4.21	1.00	1.00	1.00	1.00
Homogeneous-Detailed		1.16	1.16	1.30	2.08
Heterogeneous-Stack		1.29	1.28	1.53	2.16
Heterogeneous-Running		1.33	1.31	1.58	2.23
Homogeneous-Solid	8.42	1.00	1.00	1.00	1.00
Homogeneous-Detailed		1.13	1.13	1.12	1.55
Heterogeneous-Stack		1.25	1.24	1.37	1.56
Heterogeneous-Running		1.27	1.25	1.39	1.65
Homogeneous-Solid	14.74	1.00	1.00	1.00	1.00
Homogeneous-Detailed		1.13	1.13	1.07	1.16
Heterogeneous-Stack		1.25	1.24	1.32	1.16
Heterogeneous-Running		1.27	1.25	1.33	1.19
Homogeneous-Solid	16.84	1.00	1.00	1.00	1.00
Homogeneous-Detailed		1.13	1.13	1.07	1.15
Heterogeneous-Stack		1.25	1.24	1.32	1.16
Heterogeneous-Running		1.27	1.25	1.33	1.18

For the vertically spanning walls, the larger bending stresses for the nonsolid models compared to the homogeneous solid walls are predictable because the equivalent solid wall was based on moment of inertia, not section modulus. The greater difference at low span/thickness ratios may reflect the effects of shear deformation in the block webs. In contrast to displacement predictions, the detailed homogeneous model predicted lower bending stresses than the heterogeneous models. This is attributed to reduced local effects of the mortar joints and effects of using an overall lower modulus of elasticity for the combined material. This trend is also observed when the properties of the mortar joint were changed.

For horizontally spanning walls, the webs of the blocks act as webs between the face shells in a Vierendeel truss type of configuration. The bending and shear deformations in these comparatively flexible webs reduce the coupling between the face shells. Also, because the webs run perpendicular rather than parallel to the direction of bending, they add to the effective cross-section area over only a small fraction of the span. The softer mortar in the head joints acting as extensions of the face shells also contribute to reduced shear and flexural stiffness.

At low span/thickness ratios, the larger displacement predicted by the three nonsolid models compared to the solid model is understandable based on the significance of the higher shear forces. When the shear effects are much lower for larger span/thickness ratio, the solid model provides a reasonable estimate of heterogeneous model displacements. The higher displacement from the detailed homogeneous model seems to overestimate deflection again by decreasing the effectiveness of the block webs as connectors between the face shells.

The solid model underestimates the stress in the hollow masonry represented by the other three models. In this case, equating moment of inertia of a solid to moment of inertia of the face shells assuming plane section behaviour is radically incorrect because of the shear deformations. In addition, there is the effect of section modulus

versus moment of inertia. Also, the thickness of the solid wall model based on moment of inertia for vertical bending should be different for horizontal bending because the moments of inertia of the hollow block are different in the two directions. The effect of non-planar behaviour decreases as shear forces decrease with longer spans. Moreover, the detailed homogeneous model and the heterogeneous models predict reasonably similar bending stresses as the influence of mortar is less pronounced in this case. Deformation of the block webs is the dominant factor.

4 Conclusions

A linear elastic finite element analysis of a horizontal strip wall and a vertical strip wall was carried using four different models; two homogeneous models and two heterogeneous models. Representative orthotropic material properties were employed in this study. Based on these results, the following conclusions can be drawn:

- A proper geometric representation of the hollow block is important when calculating the structural response of concrete block walls subject to out-of-plane bending.
- Using a solid model to represent hollow concrete masonry can result in erroneous computed values for both the displacements and stresses. The magnitude of the error ranges up to 100% depending on the wall configuration and its span/thickness ratio.
- The detailed homogenous model yielded similar stress values to the heterogeneous models for horizontal strip walls only.

These results clearly show that further investigation is needed to assess the adequacy of these models in the elastic range and sensitivity to quantification of material properties. In the longer term, analysis into the nonlinear range of behaviour (cracking and nonlinear material response) is desirable.

Acknowledgments

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