AN INVESTIGATION OF OUT-OF-PLANE LOADED UNREINFORCED MASONRY WALLS DESIGN CRITERIA

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SUMMARY

Different approaches on how to estimate the ultimate failure of an unreinforced masonry panel are compared: the yield-line method used in the British Code, the fracture-line method used in the Canadian Code, the failure-line method and a mechanical approach method reported in the literature. Great differences are found among the strength predictions of each method. A comparison of the methods for the case of a panel supported simply on all four edges is reported.

INTRODUCTION

Several tests have shown that the failure pattern of a masonry panel subject to out-of-plane loading is somewhat like that observed in reinforced concrete slabs, with diagonal, horizontal and/or vertical crack lines dividing the panel into smaller portions at failure. Even though the plastic behaviour of the materials is known to be different, the RC-slab yield-line approach was first used to design masonry panels. Subsequently, other theories have been developed specifically regarding masonry composition and behaviour. Here, we compare a number of different methods to design masonry panels: the yield-line (Haseltine et al.1978), failure-line (Baker et al. 2005), and fracture-line methods (Sinha, 1978), together with the diagonal bending strength method of Griffith et al. (2005).

YIELD-LINE, FRACTURE-LINE and FAILURE-LINE ANALYSES

Yield-line theory was first developed by K. W. Johansen (as per Jones and Wood, 1967) as a plastic method to predict the ultimate load capacity of reinforced concrete slabs. The method was incorporated into the British Code for the design of masonry (BS5628) following the work of Haseltine et al. (1978) who assessed the ability of the method to predict the cracking pattern and strength of masonry panels.

While yield-line theory takes into account flexural strength orthotropy it does not assess stiffness orthotropy. The solution for the maximum load depends on the failure mechanism chosen (e.g. the failure line pattern). In this paper results from the yield-line theory are related to those presented in the BS5628 code. Sinha (1978) indicated that the yield-line approach overestimates the failure load and suggested a fracture-line approach in which this second characteristic is considered. Since the elastic moduli parallel and normal to the bed joint are different in most known types of masonry, he argued that stiffness orthotropy should be taken into account appropriately when designing masonry. The unsafe gap between yield-line analysis and the real wall will depend on how far the stiffness orthotropy is from a ratio of 1.0. For the case of uniform lateral pressure on a panel, analytical solutions for various boundary conditions can be found in Hendry at al. (1997).

Sinha et al. (1997) showed that if the yield-line theory is applied to a masonry panel, the length of one side can be changed virtually in the analysis to take stiffness orthotropy into account. This change should be: Ly' = Ly / $\sqrt[3]{k}$, where k = Ex/Ey (the stiffness orthotropy), and Ly' is the modified Y-side length

In order to understand this relationship, one must imagine that the panel will behave as a series of cross-beams with vertical and horizontal spans, and the total load P will be carried by each of these beams as:

- $P = P_x + P_y$ Equation 1

For compatibility, the deflections of the beams in each direction at a crossing point must be equal:

- $$\frac{P_x (L_x)^3}{48 E_x I_x} = \frac{P_y (L_y)^3}{48 E_y I_y}$$ Equation 2

Since $I_x = I_y$

- $$P_y = \frac{P_x (L_x)^3}{(L_y)^3} \frac{E_y}{E_x}$$ Equation 3

Therefore, from Eq.1 and 3:

- $$P = P_x \left[1 + \left(\frac{L_x}{L_y} \right)^3 \frac{E_y}{E_x} \right]$$ Equation 4

The modified isotropic panel must have the same load distribution as the actual orthotropic panel. Hence:

- $$P = P_x\left[1+\left(\frac{L_x}{L_y}\right)^3\frac{E_y}{E_x}\right]_{\substack{actual\\orthotropic}} = P_x\left[1+\left(\frac{L_x}{L_{y'}}\right)^3\frac{E_x}{E_x}\right]_{\substack{orthotropic\\modified\ to\\equivalent\\isotropic}} \qquad \text{Equation 5}$$
$$\Rightarrow \left(\frac{L_x}{L_{y'}}\right)^3\frac{E_x}{E_x} = \left(\frac{L_x}{L_y}\right)^3\frac{E_y}{E_x} \;\Rightarrow\; L_{y'} = L_y\sqrt[3]{\frac{E_x}{E_y}}$$

Another approach to predicting the strength of laterally loaded masonry panels is the failure-line method, which has been developed over the past few years and recently added to the Canadian Code (Baker et al. 2005; Drysdale and Hamid, 2005; CAN-CSA S304.1-04). The method is similar to the others, except that the moment at the first yield (or fracture)-line is taken to be equal to zero. According to the references, the zero-moment at the first failure line was observed in laboratory tests, leading to the refinement in the approach. The difference between this method and the others is illustrated in Figure 1 and Table 1.

Table 1: comparison of different plastic methods

METHOD	Consider flexural orthotropy?	Consider stiffness orthotropy?	Consider moment at first crack line equal to zero?
Yield-line	YES	NO	NO
Fracture-line	YES	YES	NO
Failure-line	YES	NO	YES

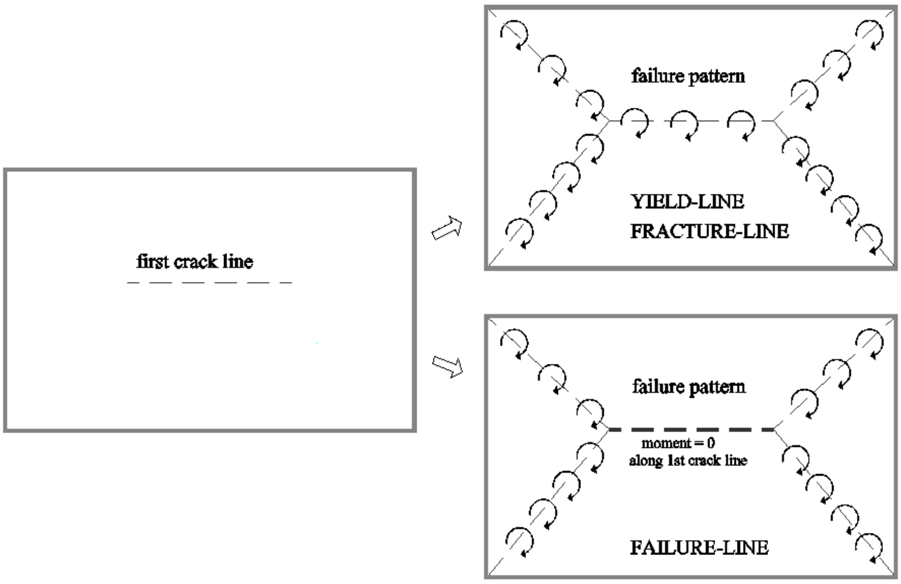

Figure 1: Difference between failure-line and yield/fracture-line analyses

In all analyses a yield/fracture/failure-line pattern is assumed that divides the panel into sections at failure. A virtual displacement is then applied, usually to the intersection of the lines, and the internal and external work calculated. The external work, done by the external load working through the virtual displacement, must equal the internal work, done on the yield/fracture/failure line.

An example of this analysis for a simply-supported case is presented below. In this example it is assumed that the panel length (L) is sufficiently greater than its height (H) that the first straight crack line will be horizontal rather than vertical. For the general case, both conditions

must be checked. The notations for the panel and crack dimensions are shown in Figure 2.

There are different approaches in the literature/codes regarding the flexural strength notation, many times leading to great confusion. Sometimes the parallel strength is related to as the "crack parallel to the bed joint" or sometimes as the "stresses parallel to the bed joint". The convention for Mx is that it can be the moment around the X-axis causing stresses perpendicular to bed joint or sometimes the bending moment in the horizontal-X direction. Also different authors refer to μ in inverted ratios. The notation used in this paper is presented in Figure 2 and below:

- m = the ultimate moment / unit length along the bed joint (My → vertical span);
- im = the ultimate negative moment / unit length along the bed joint (M'y);
- μm = the ultimate moment / unit length normal to bed joint (Mx → horizontal span);
- i = negative / positive moment ratio (M'y / My);
- μ = normal / parallel to bed joint flexural strength ratio (My / Mx);
- k = Ex/Ey;
- w = force / area (kN/m^2).

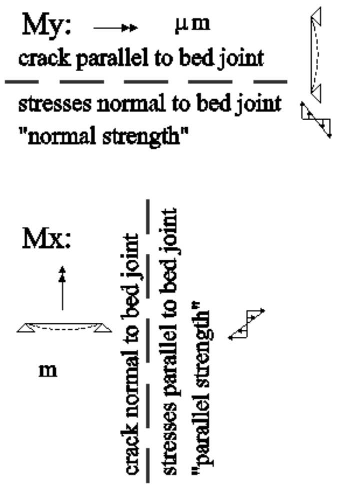

Figure 2: Notation for flexural strength

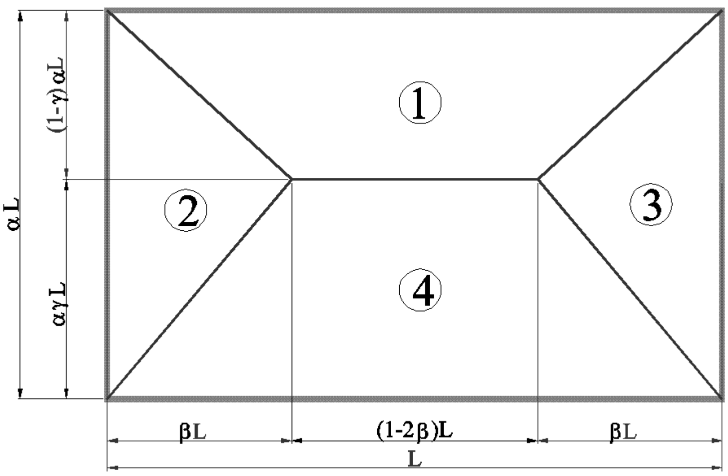

Figure 3: Panel notation for dimensions and crack lines

The internal work done by all yield/fracture/failure lines can be expressed as:

- $$\frac{\mu m}{\alpha(1-\gamma)} + \frac{\mu m}{\alpha\gamma} + \frac{2m\alpha}{k\beta} - \frac{\mu m(1-2\beta)}{\alpha(1-\gamma)} - \frac{\mu m(1-2\beta)}{\alpha\gamma}$$ Failure line only
- Equation 6

where the highlighted minus terms are with regard to the work at the first horizontal crack and should be subtracted in the failure-line case. k=1 for the yield-line and failure-line cases and k=2 in the fracture-line method.

The external work can be expressed as:

- $w \cdot \alpha \cdot L^2 (3-2\beta) / 6$ Equation 7

The solution for the minimum collapse moment is obtained by taking internal work equal to external work and solving $\partial(m/P) / \partial\beta, \partial\gamma = 0$.

GRIFFITH ET AL. MECHANICAL APPROACH

Griffith et al. (2005) presented a mathematical model to calculate the diagonal bending moment capacity based on the moment of resistance perpendicular to the bed joint and the joint torque capacity.

According to the authors, adopting a linear interaction between bending and torsion, the diagonal moment capacity can be calculated as:

- $M_d = n_b(\alpha_1 \cdot T_b \cdot \sin\theta + \alpha_2 \cdot M_b \cdot \cos\theta)$ Equation 8
- $\alpha_1 = l_1 / (l_1 + l_2)$; Equation 9
- $\alpha_2 = l_2 / (l_1 + l_2)$. Equation 10

Where:

- M_d = diagonal moment capacity;
- T_b = torque developed at each joint;
- M_b = perpendicular moment developed at each joint;
- n_b = number of bed joints;
- θ = crack inclination;
- α_1 and α_2 = reduction factors considering linear interaction;
- l_1 and l_2 = projected lengths of the perpend and bed joints onto the axis of the applied diagonal bending moment (Figure 4).

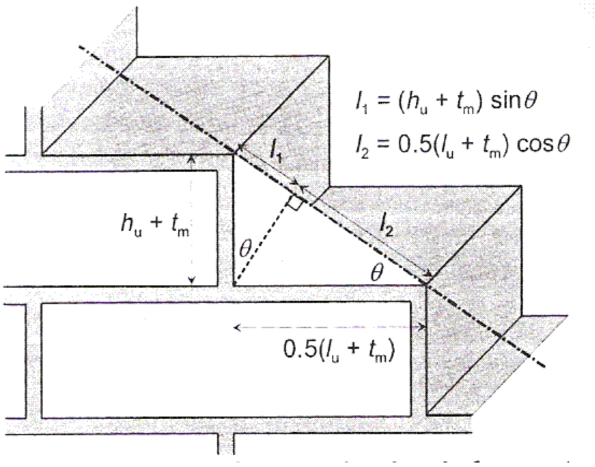

Figure 4: Determination of interaction levels for mortar joints (Griffith et al., 2005)

Griffith et al. (2005) presented expressions to calculate Tb and Mb for brick masonry with

full-bedded mortar. In the case of face-shell bedded block masonry, the expression can be adjusted to calculate the moment capacity per bed joint length, and is presented below:

- $M_b = (f_{mt} + \sigma_v + f_{sw}) \cdot I \cdot {}^{2}\!/_{t} \times l_b$ Equation 11
- $T_b = v_m \cdot {}^{2}\!/_{3} \cdot a^2 \times l_b$ Equation 12

where:
- f_{mt} = flexural tensile strength of masonry perpendicular to bed joint;
- σ_v = compressive stress;
- f_{sw} = compressive stress due to self-weight;
- I = second moment of area of the face-shell masonry (per unit length);
- t = masonry unit depth;
- l_b = bed joint length;
- v_m = shear strength of masonry; and
- a = face-shell thickness.

Using the same virtual work approach as before, the external work is the same as in Equation 7 and the internal work can be expressed as below. The first term of the equations represents the work done by the diagonal cracks and the second term represents the work done by the horizontal central crack.

- $4M_d + M_b(1-2\beta)\left(\dfrac{1}{\alpha\gamma} + \dfrac{1}{1-\alpha\gamma}\right)$, for horizontal crack line, or
- $4M_d + M_b\alpha(1-2\gamma)\left(\dfrac{1}{\beta} + \dfrac{1}{1-\beta}\right)$, for vertical crack line Equation 13

COMPARISON OF RESULTS FOR SIMPLY SUPPORTED FACE-SHELL BEDDED BLOCKWORK PANELS

Maluf (2007) presented a comparison among yield/failure/fracture line methods for different boundary conditions. Here, only simply supported panels are discussed. Results for the parameter used to calculate the panel moments, called α' here in order to distinguish it from other notation, are summarized in Table 2. The stiffness orthotropy parameter (k) in the fracture-line method was taken equal to 2.0 as is usually the case with masonry. Also, the flexural strength orthotropy parameter (μ) was always taken as the perpendicular to parallel strength ratio, with the ratio being adapted when the inverted relation was found in references.

In order to compare methods, face-shell bedded 190-mm hollow concrete block masonry is considered, with flexural strengths of 0.45 and 0.90 MPa, for the vertical and horizontal spanning cases respectively. For such masonry, the moment strengths are 1.92 and 3.84 kNm/m respectively. From the α'-value in Table 2 the maximum horizontal moment is $M_h = \alpha' w \gamma_f L^2$: and the failure load can then be calculated as $wL^2 = 3.84/\alpha'$.

Table 2: Yield/failure/fracture line α' values for panels with simple supports on 4 edges.

	h/L							
μ	**0.3**	**0.5**	**0.75**	**1**	**1.25**	**1.5**	**1.75**	
1	0.008	0.018	0.030	0.042	0.051	0.059	0.066	yield
	0.011	0.026	0.037	0.045	0.060	0.075	0.091	failure
	0.009	0.021	0.038	0.056	0.073	0.088	0.101	fracture
	91%	**86%**	**78%**	**74%**	**70%**	**67%**	**65%**	**Yield/fracture**
	125%	**125%**	**96%**	**80%**	**82%**	**85%**	**90%**	**Failure/fracture**
0.9	0.009	0.019	0.032	0.044	0.054	0.062	0.068	yield
	0.012	0.028	0.038	0.048	0.064	0.080	0.096	failure
	0.010	0.023	0.041	0.060	0.077	0.093	0.106	fracture
	93%	**84%**	**77%**	**73%**	**70%**	**67%**	**64%**	**Yield/fracture**
	124%	**124%**	**92%**	**80%**	**83%**	**86%**	**91%**	**Failure/fracture**
0.8	0.010	0.021	0.035	0.046	0.056	0.064	0.071	yield
	0.014	0.029	0.040	0.052	0.069	0.086	0.103	failure
	0.011	0.025	0.045	0.065	0.082	0.098	0.111	fracture
	93%	**85%**	**78%**	**71%**	**68%**	**65%**	**64%**	**Yield/fracture**
	131%	**117%**	**89%**	**81%**	**84%**	**88%**	**93%**	**Failure/fracture**
0.7	0.011	0.023	0.037	0.049	0.059	0.067	0.073	yield
	0.016	0.031	0.042	0.057	0.075	0.093	0.112	failure
	0.012	0.028	0.049	0.070	0.088	0.103	0.117	fracture
	92%	**84%**	**75%**	**70%**	**67%**	**65%**	**63%**	**Yield/fracture**
	133%	**113%**	**86%**	**82%**	**85%**	**90%**	**96%**	**Failure/fracture**
0.6	0.012	0.025	0.040	0.053	0.062	0.070	0.076	yield
	0.019	0.033	0.044	0.063	0.082	0.102	0.113	failure
	0.014	0.031	0.054	0.076	0.094	0.110	0.123	fracture
	88%	**81%**	**74%**	**70%**	**66%**	**64%**	**62%**	**Yield/fracture**
	139%	**107%**	**81%**	**83%**	**87%**	**93%**	**92%**	**Failure/fracture**
0.5	0.014	0.028	0.044	0.057	0.066	0.074	0.080	yield
	0.022	0.035	0.049	0.070	0.092	0.104	0.113	failure
	0.016	0.035	0.061	0.083	0.102	0.118	0.131	fracture
	88%	**79%**	**73%**	**68%**	**65%**	**63%**	**61%**	**Yield/fracture**
	138%	**99%**	**81%**	**84%**	**90%**	**88%**	**86%**	**Failure/fracture**
0.4	0.017	0.032	0.049	0.062	0.071	0.078	0.084	yield
	0.025	0.038	0.056	0.080	0.104	0.113	0.113	failure
	0.019	0.041	0.069	0.093	0.112	0.127	0.140	fracture
	89%	**77%**	**71%**	**67%**	**63%**	**61%**	**60%**	**Yield/fracture**
	131%	**92%**	**81%**	**86%**	**93%**	**89%**	**81%**	**Failure/fracture**
0.35	0.018	0.035	0.052	0.064	0.074	0.081	0.086	yield
	0.027	0.040	0.061	0.087	0.113	0.113	0.113	failure
	0.021	0.045	0.074	0.098	0.118	0.133	0.145	fracture
	84%	**77%**	**70%**	**65%**	**63%**	**61%**	**59%**	**Yield/fracture**
	127%	**88%**	**82%**	**88%**	**96%**	**85%**	**78%**	**Failure/fracture**
0.3	0.020	0.038	0.055	0.068	0.077	0.083	0.089	yield
	0.029	0.042	0.067	0.095	0.125	0.125	0.125	failure
	0.024	0.050	0.081	0.105	0.124	0.139	0.151	fracture
	83%	**76%**	**68%**	**65%**	**62%**	**60%**	**59%**	**Yield/fracture**
	121%	**84%**	**83%**	**90%**	**101%**	**90%**	**83%**	**Failure/fracture**

In the diagonal bending method Griffith et al. (2005) assumed that the crack will be stepped at every vertical joint and therefore should respect the geometry of the masonry unit. Thus, the diagonal crack slope for running bond with half block-length overlap, will be equal to:

- 0.5 x (unit length + mortar joint thickness) / (unit height + mortar joint thickness)

In the case of 190 x 390-mm blocks, the resulting diagonal crack angle lies at 45^o to the horizontal. Since the crack line inclination is known, the values of α, β and γ are also known and the solution for the failure load can be found by simply making the virtual internal work (Equation 13) equal to the virtual external work (Equation 7). Moment capacities per unit length are:

- Mb = 1.92 kNm/m;
- Tb = 0.44 kNm/m assuming v_m = 0.64 MPa;
- Diagonal moment: $\theta=45^o$, l1= 0.71, l2= 0.71, α1= 0.5, α2= 0.5, Md= 2.07 kNm/m.

RESULTS AND DISCUSSION

The failure loads per square unit of panel according to each method are shown in Table 3. A comparison is presented in Table 4.

Table 3: Failure load for each method

H/L	Griffith et al. 2005							yield		failure		fracture	
	Mb	Md	γ	β	VIW	α (3−2β)/6	**w L²**	α'	**w L²**	α'	**w L²**	α'	**w L²**
0.30	1.92	2.07	0.50	0.15	18.8	0.128	**148**	0.014	**274**	0.022	**175**	0.016	**240**
0.50	1.92	2.07	0.50	0.25	13.4	0.188	**71**	0.028	**137**	0.035	**110**	0.035	**110**
0.75	1.92	2.07	0.50	0.38	10.3	0.234	**44**	0.044	**87**	0.049	**78**	0.061	**63**
1.00	1.92	2.07	0.50	0.50	8.3	0.250	**33**	0.057	**67**	0.070	**55**	0.083	**46**
1.25	1.92	2.07	0.40	0.50	10.2	0.313	**33**	0.066	**58**	0.092	**42**	0.102	**38**
1.50	1.92	2.07	0.33	0.50	12.1	0.375	**32**	0.074	**52**	0.104	**37**	0.118	**33**
1.75	1.92	2.07	0.29	0.50	14.0	0.438	**32**	0.080	**48**	0.113	**34**	0.131	**29**

Table 4: Failure load comparison

H/L	**yield**	**Griffith et al. 2005**	**failure**	**fracture**
0.30	100%	54%	64%	88%
0.50	100%	52%	80%	80%
0.75	100%	50%	90%	72%
1.00	100%	49%	81%	69%
1.25	100%	56%	72%	65%
1.50	100%	62%	71%	63%
1.75	100%	67%	71%	61%

Looking at the yield/failure/fracture line results in Table 24 one can observe that the fracture method leads to smaller failure loads in all cases compared to the yield method. For the panels with higher height to width ratios greater than one, the load predicted by the fracture line method is less than two-thirds that predicted by the yield line.

Comparing the fracture and failure methods differences of up to 30% occur, depending on the H/L aspect ratio. In this comparison, there is a tendency for smaller failure loads to be predicted by the failure method for smaller aspect ratios, a trend which can also be observed in Table 4. This is probably due to stiffness orthotropy, which is considered in the failure method but not the fracture. Since the vertical stiffness is taken to be half the horizontal stiffness, the failure load will be comparatively smaller when the vertical dimension (H) is increased.

Comparing results from the failure method to those from the yield method one can observe that differences are greater for more rectangular aspect ratios. Since the failure method does not consider the moment at the first horizontal or vertical crack, it will lead to smaller failure loads when these crack lines are longer, which is the case with more rectangular panels.

Comparing the results of all methods (Table 4), the Griffith et al. (2005) method in general gives the smallest failure load and is thus the most conservative, with the yield-line method being the most unconservative.

CONCLUSIONS

Results of methods to calculate the maximum out-of-plane loads for unreinforced masonry panels were compared. From this comparison it is possible to conclude;

- The failure-line method gives comparatively lower results when the panel dimensional aspect ratio tends to an elongated rectangle;
- The fracture-line methods tends to be more conservative than the failure-line method for H/L greater than 0.5;
- The Griffith et al. (2005) method in general gives the smallest failure load, being the most conservative;
- The yield-line method gives the most unconservative values.

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