PRELIMINARY ANALYSIS ON MASONRY – RC COMBINED SYSTEMS: STRUCTURAL ASSESSMENT

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SUMMARY

In this paper a theoretical analysis on seismic behaviour of masonry – Reinforced Concrete (RC) combined systems is carried out. The following elementary models are preliminarily examined: (i) solid wall – opened masonry wall; (ii) solid wall – RC frame; (iii) opened masonry wall – RC frame. Subsequently, the analysis is extended to the systems composed by a generic number of multi-storey (frame or wall) bearing elements, reaching to the formulation of a matrix algorithm based on equilibrium and kinematic compatibility equations written for a generic distribution of lateral forces applied to the structure.

INTRODUCTION

In the first half of the 20th Century, a particular type of mixed structures spread in the Italian territory: the masonry – RC combined systems. Initially, only industrial facilities were built based on this new structural model, while its maximum diffusion took place for civil buildings, when external masonry bearing walls and internal RC frame structures were constructed. In the 70's some interventions on existing buildings were executed and they consisted in demolition of internal masonry walls and their substitution with RC frames. The external masonry walls were left unchanged resulting in internal spaces larger than the original ones and with a flexible planimetric distribution, at a parity of vertical load-bearing capacity and thermo-acoustic isolation. Sometimes RC walls or cores were built up around both vertical architectural connections (stairs or elevators) and technical spaces, so they increased vertical load-bearing capacity and modified completely the lateral behaviour of the structure. Other combined systems were obtained through RC planimetric enlargements.
Although the masonry – RC combined systems has had a great diffusion in the building stock of many Countries, only few researchers have focused their attention on them, as instead it has been done for other types of mixed structures. On the other hand, both national and internationals codes impose to take into account mutual interaction between frames and walls, but they do not provide any indication on modelling and analysis of these structures. The present note focuses its attention on these aspects, looking to provide an analytical tool able to describe the performance peculiarities of combined systems.

THE ELEMENTARY COMBINED MODELS

This paper does not concern infilled frames or framed masonry walls. Rather it investigates the behaviour of structural systems composed by masonry walls and RC walls or frames, that are divided apart in vertical direction but connected by rigid diaphragms.
The theoretical analysis of combined systems starts with examination of three elementary planar models: (i) solid wall – opened masonry wall; (ii) solid wall – RC frame; (iii) opened masonry wall – RC frame. Then the formulation is generalised to the *s*-storey combined systems composed by *w* parallel bearing elements.
The structural analysis have been conducted under these hypotheses:
- lateral forces applied to the level of each diaphragm;
- prismatic solid (masonry or RC) wall with flexural and shear flexibility;
- opened masonry wall modelled through the macro-elements approach with RAN method (Augenti 2004);
- RC frame with inextensible beams and columns;
- parallel bearing elements connected themselves by inextensible pendulums;
- equal height storeys;
- solid walls fixed to the base perfectly.

In the following treatment storeys and bearing elements are progressively numbered from the top to the base and from the left to the right, respectively.
The analysis under lateral forces is carried out by using the flexibility method, also known as force method because the redundant forces applied after the cutting of the statically indeterminate system are defined as primary unknowns and the structure is solved by setting the relative displacements at the cuts to zero.

The solid wall – opened masonry wall combined system

Let us consider a three-storey structural system formed by a prismatic solid wall and an opened masonry wall, connected themselves by inextensible pendulums and loaded by lateral forces at several levels (see Figure 1). Solid wall and opened masonry wall are numbered as 1 and 2, respectively.

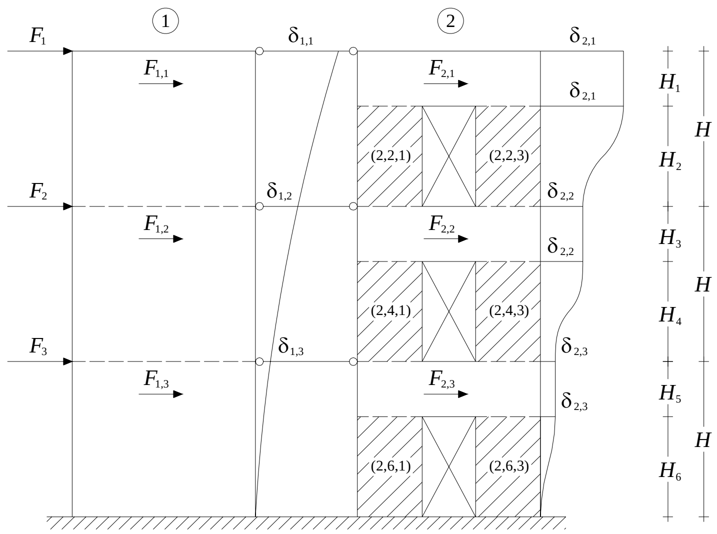

Figure 1. Solid wall – opened masonry wall combined system

In this case, the distribution of lateral forces applied to the structure between solid wall and opened masonry wall is characterized by six unknowns. In order that algebraic problem is

determinate the writing of a system composed by as many equations is required, so compatibility of absolute lateral displacements and translational equilibrium in horizontal direction must be imposed at each level.
Let us denote by:

- k the generic bearing element;
- p the generic storey;
- $E_{k,p}$ the generic static or kinematic parameter referred to the p storey of k wall;

hence our problem is based on the following system of equations:

$$\begin{cases} \delta_{1,1} = \delta_{2,1} \\ \delta_{1,2} = \delta_{2,2} \\ \delta_{1,3} = \delta_{2,3} \\ F_{1,1} + F_{2,1} = F_1 \\ F_{1,2} + F_{2,2} = F_2 \\ F_{1,3} + F_{2,3} = F_3 \end{cases} \tag{1}$$

In order to explain the absolute lateral displacements of solid wall in terms of lateral forces applied on itself, let us remember the equations related to a prismatic cantilever beam of height H, with flexural and shear flexibility and subjected to a lateral force applied to the top:

$$\begin{aligned} v(z) &= \frac{FH}{2EJ} \cdot z^2 - \frac{F}{6EJ} \cdot z^3 + \chi \cdot \frac{F}{GA} \cdot z \\ v'(z) &= \frac{FH}{EJ} \cdot z - \frac{F}{2EJ} \cdot z^2 + \chi \cdot \frac{F}{GA} \end{aligned} \tag{2}$$

For a prismatic cantilever beam of height 3H subjected to lateral forces applied at H, 2H e 3H heights the mutual effects must be considered, so let us apply the principle of effects superposition to three separate schemes (see Figure 2).

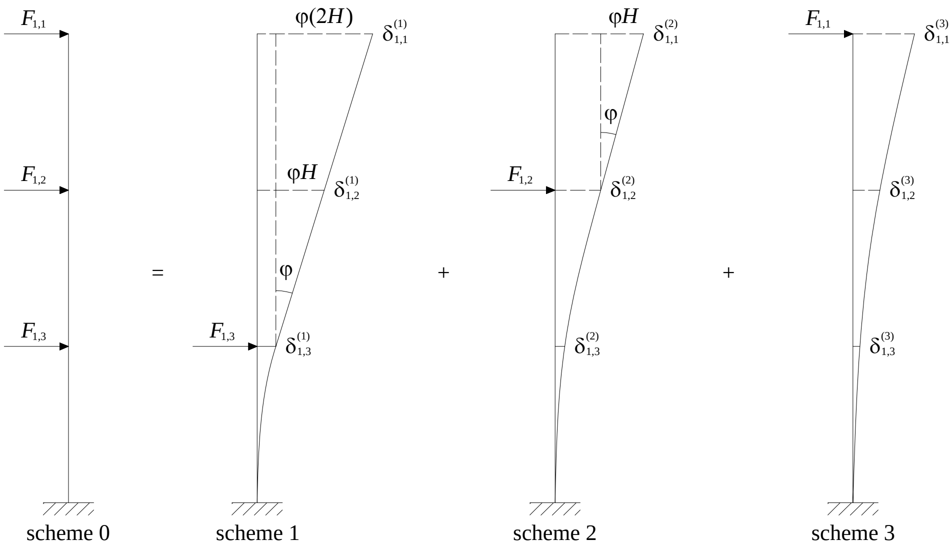

Figure 2. Effects superposition for a cantilever beam of height 3H

The absolute lateral displacement $\delta_{1,p}$ at generic level of the solid wall can be expressed as the sum of three rates: the first one caused by lateral forces applied to the levels below the one held in consideration; the second one caused by lateral force applied to the level held in consideration; the last one caused by lateral forces applied to the levels above the one held in consideration. Thus, it is possible to write:

$$\delta_{1,1} = \delta_{1,1}^{(1)} + \delta_{1,1}^{(2)} + \delta_{1,1}^{(3)}; \qquad \delta_{1,2} = \delta_{1,2}^{(1)} + \delta_{1,2}^{(2)} + \delta_{1,2}^{(3)}; \qquad \delta_{1,3} = \delta_{1,3}^{(1)} + \delta_{1,3}^{(2)} + \delta_{1,3}^{(3)} \tag{3}$$

in which the superscripts are referred to the schemes that are superposed.
Taking into account that deformed configurations of schemes (1) and (2) are composed by an elastic part and a rigid part, the following relations can be written:

$$\begin{aligned}
\delta_{1,3}^{(1)} &= \frac{F_{1,3}H^3}{3EJ} + \chi \cdot \frac{F_{1,3}H}{GA} \\
\delta_{1,2}^{(1)} &= \frac{F_{1,3}H^3}{3EJ} + \chi \cdot \frac{F_{1,3}H}{GA} + \frac{F_{1,3}H^2}{2EJ} \cdot H \\
\delta_{1,1}^{(1)} &= \frac{F_{1,3}H^3}{3EJ} + \chi \cdot \frac{F_{1,3}H}{GA} + \frac{F_{1,3}H^2}{2EJ} \cdot 2H
\end{aligned} \tag{4}$$

$$\begin{aligned}
\delta_{1,3}^{(2)} &= \frac{F_{1,2}\,2H}{2EJ} \cdot H^2 - \frac{F_{1,2}}{6EJ} \cdot H^3 + \chi \cdot \frac{F_{1,2}H}{GA} \\
\delta_{1,2}^{(2)} &= \frac{F_{1,2}(2H)^3}{3EJ} + \chi \cdot \frac{F_{1,2}\,2H}{GA} \\
\delta_{1,1}^{(2)} &= \frac{F_{1,2}(2H)^3}{3EJ} + \chi \cdot \frac{F_{1,2}\,2H}{GA} + \frac{F_{1,2}(2H)^2}{2EJ} \cdot H
\end{aligned} \tag{5}$$

$$\begin{aligned}
\delta_{1,3}^{(3)} &= \frac{F_{1,1}\,3H}{2EJ} \cdot H^2 - \frac{F_{1,1}}{6EJ} \cdot H^3 + \chi \cdot \frac{F_{1,1}H}{GA} \\
\delta_{1,2}^{(3)} &= \frac{F_{1,1}\,3H}{2EJ} \cdot (2H)^2 - \frac{F_{1,1}}{6EJ} \cdot (2H)^3 + \chi \cdot \frac{F_{1,1}\,2H}{GA} \\
\delta_{1,1}^{(3)} &= \frac{F_{1,1}(3H)^3}{3EJ} + \chi \cdot \frac{F_{1,1}\,3H}{GA}
\end{aligned} \tag{6}$$

The relationships between the absolute lateral displacements that the opened masonry wall experiences and the lateral forces are:

$$\delta_{2,3} = \frac{F_{2,1} + F_{2,2} + F_{2,3}}{k_{2,3}}; \qquad \delta_{2,2} = \frac{F_{2,1} + F_{2,2}}{k_{2,2}} + \delta_{2,3}; \qquad \delta_{2,1} = \frac{F_{2,1}}{k_{2,1}} + \delta_{2,2} \tag{7}$$

denoting by $k_{2,p}$ the lateral stiffness of its p level. The system of equations (1) can be rewritten in terms of unknown lateral forces, that are applied on the walls owing to the distribution of external forces, by substituting lateral displacements through the equations from (3) to (7).
Supposing to define the flexibility coefficients of the walls, it turns out to be (8):

$$\begin{cases} F_{1,1}D_{1,1} + F_{1,2}\left(D_{1,2} + \Phi_{1,2}H\right) + F_{1,3}\left(D_{1,3} + \Phi_{1,3}2H\right) - F_{2,1}\sum_{p=1}^{3}\frac{1}{k_{2,p}} - F_{2,2}\sum_{p=2}^{3}\frac{1}{k_{2,p}} - F_{2,3}\frac{1}{k_{2,3}} = 0 \\ F_{1,1}d_{2,1} + F_{1,2}D_{1,2} + F_{1,3}\left(D_{1,3} + \Phi_{1,3}H\right) - F_{2,1}\sum_{p=2}^{3}\frac{1}{k_{2,p}} - F_{2,2}\sum_{p=2}^{3}\frac{1}{k_{2,p}} - F_{2,3}\frac{1}{k_{2,3}} = 0 \\ F_{1,1}d_{3,1} + F_{1,2}d_{3,2} + F_{1,3}D_{1,3} - F_{2,1}\frac{1}{k_{2,3}} - F_{2,2}\frac{1}{k_{2,3}} - F_{2,3}\frac{1}{k_{2,3}} = 0 \\ F_{1,1} + F_{2,1} = F_1 \\ F_{1,2} + F_{2,2} = F_2 \\ F_{1,3} + F_{2,3} = F_3 \end{cases}$$

The flexibility coefficients of the solid wall are expressed by:

$$\begin{aligned} D_{1,1} &= \frac{(3H)^3}{3EJ} + \chi\cdot\frac{3H}{GA} & D_{1,2} &= \frac{(2H)^3}{3EJ} + \chi\cdot\frac{2H}{GA} \\ D_{1,3} &= \frac{(3H)^3}{3EJ} + \chi\cdot\frac{3H}{GA} & \Phi_{1,2} &= \frac{(2H)^2}{2EJ} \\ \Phi_{1,3} &= \frac{H^2}{2EJ} & d_{2,1} &= \frac{3H\cdot(2H)^2}{2EJ} - \frac{(2H)^3}{6EJ} + \chi\cdot\frac{2H}{GA} \\ d_{3,1} &= \frac{3H\cdot H^2}{2EJ} - \frac{H^3}{6EJ} + \chi\cdot\frac{H}{GA} & d_{3,2} &= \frac{2H\cdot H^2}{2EJ} - \frac{H^3}{6EJ} + \chi\cdot\frac{H}{GA} \end{aligned} \tag{9}$$

while the ones of the opened masonry wall have the form:

$$\sum_{p=1}^{3}\frac{1}{k_{2,p}} \tag{10}$$

The formulation developed for a three-storey combined model may be generalised to the case of an *s*-storey structural system.
Therefore, for a solid wall let us indicate with:

- $\delta_{1,p,r}^{(1)}$ (with $r > p$) the absolute lateral displacement at generic level p due to the lateral force applied at level r below the first one;
- $\delta_{1,p}^{(2)}$ the absolute lateral displacement at generic level p due to the lateral force applied to it;
- $\delta_{1,p,t}^{(3)}$ (with $t < p$) the absolute lateral displacement at generic level p due to the lateral force applied at level t above the first one.

These quantities represent the separate contribution to the absolute lateral displacement at generic level p of the solid wall and the superscripts (1), (2) and (3) are referred at three groups of schemes whose the effects are superposed:

- the first one comprises the walls loaded by the lateral forces acting to the p+1,…, s levels;
- the second one includes an unique wall subjected to the lateral force acting to the p level;
- the last one comprises the walls loaded by the lateral forces acting to the 1,…, p–1 levels.

Thus, the following generalised relations can be written:

$$\delta_{1,p,r}^{(1)} = \frac{F_{1,r} \cdot \left[(s-r+1) H \right]^3}{3EJ} + \chi \cdot \frac{F_{1,r} \cdot (s-r+1) H}{GA} + \frac{F_{1,r} \cdot \left[(s-r+1) H \right]^2}{2EJ} \cdot (r-p) H \qquad (11)$$

$$\delta_{1,p}^{(2)} = \frac{F_{1,p} \cdot \left[(s-p+1) H \right]^3}{3EJ} + \chi \cdot \frac{F_{1,p} \cdot (s-p+1) H}{GA} \qquad (12)$$

$$\delta_{1,p,t}^{(3)} = \frac{F_{1,t} \cdot (s-t+1) H \cdot \left[(s-p+1) H \right]^2}{2EJ} - \frac{F_{1,t} \cdot \left[(s-p+1) H \right]^3}{6EJ} + \chi \cdot \frac{F_{1,t} \cdot (s-p+1) H}{GA} \qquad (13)$$

The absolute lateral displacement at p level due to lateral forces applied at p+1,…, s levels is given by:

$$\delta_{1,p}^{(1)} = \sum_{r=p+1}^{s} \delta_{1,p,r}^{(1)} \qquad (14)$$

while the one due to lateral forces applied at 1,…, p–1 levels is:

$$\delta_{1,p}^{(3)} = \sum_{t=1}^{p-1} \delta_{1,p,t}^{(3)} \qquad (15)$$

so the global lateral displacement at p level of the solid wall results:

$$\delta_{1,p} = \delta_{1,p}^{(1)} + \delta_{1,p}^{(2)} + \delta_{1,p}^{(3)} \qquad (16)$$

The absolute lateral displacement at p level of the opened masonry wall is defined by:

$$\delta_{2,p} = \frac{T_{2,p}}{k_{2,p}} + \delta_{2,p+1} \qquad (17)$$

being $T_{2,p}$ the shear at level considered and taking into account a shear-type behaviour for the opened masonry wall.
In order to express the lateral displacements as the product of lateral forces for respective flexibility coefficients, let us define with:

$$D_{1,r} = \frac{\left[(s-r+1) H \right]^3}{3EJ} + \chi \cdot \frac{(s-r+1) H}{GA} \qquad (18)$$

the lateral flexibility of the solid wall referred to the r level;

$$\Phi_{1,r} = \frac{\left[(s-r+1) H \right]^2}{2EJ} \qquad (19)$$

the rotational flexibility of the solid wall referred to the r level, that is intended as the rotation experienced by the wall at p level owing to a unit lateral force applied to the r level;

$$D_{1,p} = \frac{\left[\left(s-p+1\right)H\right]^3}{3EJ} + \chi \cdot \frac{\left(s-p+1\right)H}{GA} \tag{20}$$

the lateral flexibility of the solid wall referred to the *p* level;

$$d_{p,t} = \frac{\left(s-t+1\right)H \cdot \left[\left(s-p+1\right)H\right]^2}{2EJ} - \frac{\left[\left(s-p+1\right)H\right]^3}{6EJ} + \chi \cdot \frac{\left(s-p+1\right)H}{GA} \tag{21}$$

the local lateral flexibility of *p* level of the solid wall with respect to the *t* level below.
Therefore, the equations (11), (12) and (13) become:

$$\delta_{1,p,r}^{(1)} = F_{1,r} \cdot D_{1,r} + F_{1,r} \cdot \Phi_{1,r} \cdot \left(r-p\right)H \tag{22}$$

$$\delta_{1,p}^{(2)} = F_{1,p} \cdot D_{1,p} \tag{23}$$

$$\delta_{1,p}^{(3)} = F_{1,t} \cdot d_{p,t} \tag{24}$$

and remembering the equations (14) and (15), global lateral displacement at *p* level is:

$$\delta_{1,p} = \sum_{r=p+1}^{s} F_{1,r}\left[D_{1,r} + \Phi_{1,r}\left(r-p\right)H\right] + F_{1,p} \cdot D_{1,p} + \sum_{t=1}^{p-1} F_{1,t} d_{p,t} \tag{25}$$

where all unknown forces $F_{1,1},\ldots, F_{1,s}$ applied on the solid wall appear.
Proceeding in terms of sub-matrices, the external lateral forces vector must be defined and, for both walls, it is necessary to construct the lateral flexibility matrix and the unknown lateral forces vector. The flexibility matrices of solid and opened masonry walls are respectively:

$$\mathbf{D}_1 = \begin{bmatrix} D_{1,1} & D_{1,2} + \Phi_{1,2}H & \cdots & D_{1,2} + \Phi_{1,s}\left(s-1\right)H \\ d_{2,1} & D_{1,2} & \cdots & D_{1,2} + \Phi_{1,s}\left(s-2\right)H \\ \vdots & \vdots & \ddots & \vdots \\ d_{s,1} & d_{s,2} & \cdots & D_{1,s} \end{bmatrix} \tag{26}$$

$$\mathbf{D}_2 = \begin{bmatrix} \sum_{p=1}^{s} \frac{1}{k_{2,p}} & \sum_{p=2}^{s} \frac{1}{k_{2,p}} & \cdots & \frac{1}{k_{2,s}} \\ \sum_{p=2}^{s} \frac{1}{k_{2,p}} & \sum_{p=2}^{s} \frac{1}{k_{2,p}} & \cdots & \frac{1}{k_{2,s}} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{1}{k_{2,s}} & \frac{1}{k_{2,s}} & \cdots & \frac{1}{k_{2,s}} \end{bmatrix} \tag{27}$$

and by writing the equation:

$$\left[\begin{array}{c:c} \mathbf{D}_1 & -\mathbf{D}_2 \\ \hdashline \mathbf{I} & \mathbf{I} \end{array}\right] \left\{\begin{array}{c} \mathbf{F}_1 \\ \hdashline \mathbf{F}_2 \end{array}\right\} = \left\{\begin{array}{c} \mathbf{0} \\ \hdashline \mathbf{F} \end{array}\right\} \tag{28}$$

the system (8) assumes the following form:

$$\begin{cases} \mathbf{D}_1\,\mathbf{F}_1 - \mathbf{D}_2\,\mathbf{F}_2 = \mathbf{0} \\ \mathbf{F}_1 + \mathbf{F}_2 = \mathbf{F} \end{cases} \tag{29}$$

that is composed by two matrix equations: the former establishes the congruence of the lateral displacements experienced by the two walls; the latter sets up the translational equilibrium.
The solution of the previous algebraic system is represented by:

$$\mathbf{F}_1 = \left(\mathbf{D}_2^{-1}\,\mathbf{D}_1 + \mathbf{I}\right)^{-1}\mathbf{F} \qquad \text{and} \qquad \mathbf{F}_2 = \left(\mathbf{D}_1^{-1}\,\mathbf{D}_2 + \mathbf{I}\right)^{-1}\mathbf{F} \tag{30}$$

where the distribution factors matrices of the walls turn up:

$$\boldsymbol{\Gamma}_1 = \left(\mathbf{D}_2^{-1}\,\mathbf{D}_1 + \mathbf{I}\right)^{-1} \qquad \text{and} \qquad \boldsymbol{\Gamma}_2 = \left(\mathbf{D}_1^{-1}\,\mathbf{D}_2 + \mathbf{I}\right)^{-1} \tag{31}$$

The physical meaning of these matrices is extremely important, because they indicate the participation of the individual structural elements at the lateral behaviour of the entire combined system at each level.
The previous equations show that the distribution of external lateral forces in a multi-storey combined model is very different from the one related to a one-storey combined model. In fact in the former case, apart the major computational effort, the following physical concept must be taken into account: the lateral force applied to the generic level of a bearing element depends on the lateral stiffnesses of all levels. In other terms, there is a mutual interaction between the parallel structural elements in both horizontal and vertical directions.

The solid wall – RC frame combined system

The distribution of lateral forces applied at *s* levels of a structural system composed by a solid (masonry or RC) wall and a RC frame can be executed by using the formulation proposed for the solid wall – opened masonry wall combined system. In fact, the lateral flexibility matrix $\mathbf{D}_t$ can be defined for the RC frame too, by inverting the lateral stiffness matrix $\mathbf{K}_t$ expressed by the well-known equation:

$$\mathbf{K}_t = \mathbf{K}_{tt} - \mathbf{K}_{t\varphi}\,\mathbf{K}_{\varphi\varphi}^{-1}\,\mathbf{K}_{\varphi t} \tag{32}$$

so we obtain:

$$\mathbf{D}_t = \mathbf{K}_t^{-1} = \left(\mathbf{K}_{tt} - \mathbf{K}_{t\varphi}\,\mathbf{K}_{\varphi\varphi}^{-1}\,\mathbf{K}_{\varphi t}\right)^{-1} \tag{33}$$

denoting by:

- $\mathbf{K}_{\varphi\varphi}$ the flexural stiffness matrix related to the nodal rotations;
- $\mathbf{K}_{\varphi t}$ the flexural stiffness matrix related to the nodal translations;
- $\mathbf{K}_{t\varphi}$ the shear stiffness matrix related to the nodal rotations;

- $\mathbf{K}_{tt}$ the shear stiffness matrix related to the nodal translations.

The opened masonry wall – RC frame combined system

The matrix algorithm that has been previously discussed is applicable to models composed by an opened masonry wall and a RC frame too, because it is sufficient to construct the lateral flexibility matrices of the individual bearing elements.

THE COMBINED SYSTEMS COMPOSED BY MORE PARALLEL ELEMENTS

The distribution's problem of lateral forces applied to an *s*-storey combined model composed by *w* bearing elements (solid walls, opened masonry walls or RC frames) presents $w \times s$ unknowns. At each level, *w*–1 linearly independent compatibility equations and one translational equilibrium equation can be written, so that the algebraic system written for the entire multi-storey combined model is composed by $s \times (w–1)$ compatibility equations and *s* translational equilibrium equations. Moreover, the unknown forces can be included in the column vectors $\mathbf{F}_1$, $\mathbf{F}_2$,…, $\mathbf{F}_w$. The factors matrix of the algebraic system is still squared and its order is equal to $w \times s$, because it is composed by *w* square sub-matrices that have an order equal to *s*. The $w \cdot s \times 1$ column vector of given terms (embodying the vector of external forces **F**) is obtained by multiplying the factors matrix by the $w \cdot s \times 1$ column vector of unknown forces.
For this structural model the equation (28) assumes the generalised form:

$$\left[\begin{array}{c|c|c|c|c|c} \mathbf{D}_1 & -\mathbf{D}_2 & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} \\ \hline \mathbf{0} & \mathbf{D}_2 & -\mathbf{D}_3 & \cdots & \mathbf{0} & \mathbf{0} \\ \hline \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ \hline \mathbf{0} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{D}_{w-1} & -\mathbf{D}_w \\ \hline \mathbf{I} & \mathbf{I} & \mathbf{I} & \cdots & \mathbf{I} & \mathbf{I} \end{array}\right] \left\{\begin{array}{c} \mathbf{F}_1 \\ \hline \mathbf{F}_2 \\ \hline \vdots \\ \hline \mathbf{F}_{w-1} \\ \hline \mathbf{F}_w \end{array}\right\} = \left\{\begin{array}{c} \mathbf{0} \\ \hline \mathbf{0} \\ \hline \vdots \\ \hline \mathbf{0} \\ \hline \mathbf{F} \end{array}\right\} \tag{34}$$

at which correspond the following algebraic system of *w* matrix equations:

$$\begin{cases} \mathbf{D}_1\,\mathbf{F}_1 - \mathbf{D}_2\,\mathbf{F}_2 = \mathbf{0} \\ \mathbf{D}_2\,\mathbf{F}_2 - \mathbf{D}_3\,\mathbf{F}_3 = \mathbf{0} \\ \cdots \\ \mathbf{D}_{w-1}\,\mathbf{F}_{w-1} - \mathbf{D}_w\,\mathbf{F}_w = \mathbf{0} \\ \mathbf{F}_1 + \mathbf{F}_2 + \ldots + \mathbf{F}_{w-1} + \mathbf{F}_w = \mathbf{F} \end{cases} \tag{35}$$

so that the solution is obtained by the relations:

$$\begin{aligned} &\mathbf{F}_1 = \left(\mathbf{I} + \mathbf{D}_2^{-1}\,\mathbf{D}_1 + \ldots + \mathbf{D}_w^{-1}\,\mathbf{D}_1 \right)^{-1} \mathbf{F} \\ &\mathbf{F}_2 = \left(\mathbf{D}_1^{-1}\,\mathbf{D}_2 + \mathbf{I} + \ldots + \mathbf{D}_w^{-1}\,\mathbf{D}_2 \right)^{-1} \mathbf{F} \\ &\cdots \\ &\mathbf{F}_w = \left(\mathbf{D}_1^{-1}\,\mathbf{D}_w + \mathbf{D}_2^{-1}\,\mathbf{D}_w + \ldots + \mathbf{I} \right)^{-1} \mathbf{F} \end{aligned} \tag{36}$$

After the lateral flexibility matrices of the individual bearing elements have been constructed, the lateral forces applied on them can be calculated through the equations (36). The form of the last ones is very simple, just because the model's behaviour is analyzed by using the flexibility method and because the bearing elements are substantially coupled two by two. In this case, the distribution factors matrices of the parallel structural elements are expressed by:

$$\begin{aligned} \mathbf{\Gamma}_1 &= \left(\mathbf{I} + \mathbf{D}_2^{-1}\,\mathbf{D}_1 + \ldots + \mathbf{D}_w^{-1}\,\mathbf{D}_1 \right)^{-1} \\ \mathbf{\Gamma}_2 &= \left(\mathbf{D}_1^{-1}\,\mathbf{D}_2 + \mathbf{I} + \ldots + \mathbf{D}_w^{-1}\,\mathbf{D}_2 \right)^{-1} \\ &\ldots \\ \mathbf{\Gamma}_w &= \left(\mathbf{D}_1^{-1}\,\mathbf{D}_w + \mathbf{D}_2^{-1}\,\mathbf{D}_w + \ldots + \mathbf{I} \right)^{-1} \end{aligned} \tag{37}$$

related between themselves by the algebraic condition:

$$\mathbf{\Gamma}_1 + \mathbf{\Gamma}_2 + \ldots + \mathbf{\Gamma}_w = \mathbf{I} \tag{38}$$

CONCLUSIONS

The algorithm of matrix calculus carried out in this paper represents a first approach for the analysis of masonry – RC mixed structures and particularly for that ones composed by frame-wall combined systems subjected to lateral forces.
The expressions that define the flexibility coefficients of the solid (masonry or RC) wall and opened masonry wall enable to construct easily the lateral flexibility matrices, that characterize the lateral behaviour of these structural elements.
Matrix notation makes the resolution of the equilibrium and compatibility equations very simple, allowing to the use of spreadsheets. After construction of the lateral flexibility matrices of the bearing elements, the vectors of lateral forces applied on them owing to the distribution of the external lateral forces can be determined directly.
Furthermore, the definition of the distribution factors matrices lets the exact individuation of the bearing elements that participate, with greater measure than the others, to equilibrate the external lateral forces. It is convenient to emphasize that knowledge of stiffness coefficients' distribution along the height does not make possible, deductively, the evaluation of the lateral stiffness of both frames and walls, because their values depend on the particular distribution of given external lateral forces. Consequently, judgment on the regularity of structural response in elevation is subordinate to the calculation of lateral forces applied on each bearing element. In the case of combined models, the values of these forces are sensibly conditioned by the mutual interaction between the parallel structural elements. This interaction is caused by the great difference of lateral behaviour between solid walls, opened masonry walls and frames and mainly by the very significant variability of their lateral stiffness along the height. All these considerations allow to conclude that, in general, no structural element can be neglected in the masonry – RC combined systems.

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