STOCHASTIC MODELLING OF MODERN MASONRY

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Summary: Since probabilistic calculations to assess the reliability of structural elements and buildings are gaining a higher importance, the estimation of statistical parameters of material properties plays a major role. Whereas for steel and concrete structures the required information based on extensive data of test results is existent, there is a lack for masonry structures. This is valid especially for modern masonry. In this paper the main statistical parameters for the assessment of the reliability of big sized masonry are provided.

INTRODUCTION

Within the scope of the harmonization of the European design codes the partial safety concept is recommended also for masonry structures. Thus the question of the required value of the partial safety factors has to be discussed. The comparison of the international masonry codes (see Table 1) shows, that especially the safety factors on the resistance for unreinforced masonry are contradictory. Possibly the provided safety factors were calibrated on basis of experience and traditions of the different countries. Up to now only some reliability studies on masonry have been conducted (Galambos et. al 1982, Holicky & Markova 2002, Schueremanns 2002). The reason for this can be found in the lack of required statistical data especially for industrially produced masonry. Due to the great amount of different unit-brick combinations with different properties of modern masonry the estimation of the statistical parameters is also difficult.
In the following sections some required statistical parameter for the assessment of the reliability of masonry, made of big sized units (in the following called big sized masonry) will be given. The focus will be set on axial loading. The main parameter for the assessment of the reliability of masonry structures is its compressive strength. Other important material parameters, e.g. the modulus of elasticity, are correlated with the compressive strength and may be defined on this basis. Hence the compressive strength needs to be analyzed carefully. Beside the influence of the material parameters on the reliability of masonry structures, uncertainties due to the model used to calculate the load capacity play a major role and will be analyzed. Please note that all presented partial safety factors in following table have been converted into the format corresponding to (1).

$$\gamma_f \cdot E_k \leq \frac{R_k}{\gamma_R} \tag{1}$$

Where: E_k is characteristic value of the actions
R_k is the characteristic value of the resistance
γ_f is the partial safety factor on the actions
γ_m is the partial safety factor on the resistance

Table 1. Summary of safety factors for the ultimate limit state design of unreinforced masonry in different countries

Country	Masonry	Action[1)]	
		permanent	variable
	γ_M	γ_G	γ_Q
Germany	1.50	1.35	1.50[2)]
Austria	2.20	1.35	1.50
Switzerland	2.00	1.30	1.50[2)]
U.K.	2.50-3.50[5)]	1.20-1.40[2)]	1.40-1.60[2)4)]
Netherlands	1.80	1.20-1.35[2)]	1.50[2)]
Sweden	1.50-2.76[3)5)]	1.00-1.15	1.30[2)]
Australia	1.67-2.22[6)]	1.20-1.35[2)]	1.50[2)]
USA	1.25-2.5[6)]	1.20	1.30-1.60[2)4)]
Canada	1.67	1.25-1.40	1.50[2)]

1) Unfavourable effect
2) Depending on the load combination
3) Depending on the structural class an kind of building stones
4) Depending on the effect
5) Depending on the quality of building stones and the building inspection
6) Depending on the analysed design situation (e.g. shear, axial, bending)

SPECIFICATION OF BIG SIZED MASONRY

Big sized masonry is made of big sized units in combination with thin layer mortar and is common in Germany, as it saves on construction time and so reduces the cost of the structure. In the following big sized units are defined as stones with a height of 248mm and more, while the length of the stones reaches 998mm in special cases. The most common materials for this kind of masonry are porous concrete and calcium silicate unit. In the case of calcium silicate unit more or less solid stones are used for big sized masonry.

The units provide a plane surface and geometrical deviations from the nominal size are negligible. The units are stone walled in stretcher course with the aid of small chain hoists if the weight of the units exceeds 25kg. The compressive strength and the weight of units made of porous concrete are significantly smaller in comparison to calcium silicate units. In the following the most important statistical parameters for the determination of the reliability of big sized masonry made of porous concrete and calcium silicates units will be analyzed.

COMPRESSIVE STRENGTH

DEFINITION OF THE CALCULATION MODEL

The compressive strength of masonry is affected by the compressive strength of the units and the mortar used. Direct measurements of the compressive strength of masonry are expensive and thus reduced to minimum. However, the properties of units and mortar are subjected to quality control, so that the statistical parameters of the basic materials can be obtained on basis of extensive data. These reasons make clear that a calculation model should be defined which uses all sources of information for the estimation of the compressive strength of masonry. The model according to equation (2) was proposed by Mann 1983 and provides a good adjustment with experimental investigations. It is also used in Eurocode 6 for the calculation of the compressive strength of masonry:

$$f_m = a \cdot f_b^b \cdot f_{mo}^c \tag{2}$$

Where: f_m is the compressive strength of masonry
f_b is the compressive strength of the unit (brick)
f_{mo} is the compressive strength of the mortar
a, b, c are factors which characterize the kind of masonry

The factors *a, b* and *c* may be obtained using a regression analysis based on experimental investigations. It should be mentioned that for $b+c \neq 1$ equation (2) is restricted to a special dimension. For the estimation of the required statistical parameter of the compressive strength of masonry the model uncertainties due to the transformation of experimental data to a mathematical model have to be considered. Thus model uncertainties *M* must to be added to equation (2).

$$f_m = M \cdot a \cdot f_b^b \cdot f_{mo}^c \tag{3}$$

Where: *M* is a variable with a mean value of 1.0 which considers model uncertainties

In the probabilistic model *a*, *b* and *c* can be regarded as deterministic parameters, since the uncertainties due to the definition of these factors will be included in the model uncertainty *M*. Besides the model uncertainty the compressive strength of the bricks and the mortar will be treated as random variables. Prior values for the factors a, b and c can be found at Schubert 2005. However it should be mentioned, that the values provided by Schubert 2005 are valid for a slenderness of the masonry specimens the defined as height-to-thickness ratio of 10, whereas in Eurocode 6 these factors are related to a (theoretical) slenderness of zero.
For thin layer mortar, the influence on the compressive strength of masonry is negligible and thus the factor *c* can be set to zero (see Schubert 2005), so that equation (3) becomes:

$$f_m = M \cdot a \cdot f_b^b \text{ for masonry with thin mortar layers} \tag{4}$$

PARAMETER ESTIMATION

The required parameters *a* and *b* in equation (4) for big sized masonry are calculated on basis of a regression analysis of experimental investigations, where the compressive strength of the units and the masonry were analyzed. Note that all variables in equation (4) are treated as deterministic values within this stage of the analysis. Also the parameter *M* remains constant with value of 1.0.

Since the height of the units has a significant influence on the compressive strength a reference value for the slenderness has to be defined. The specimens used for the following calculation had a slenderness λ defined as height-to-thickness ratio of 3 up to 10. The compressive strengths provided by the tests were converted into a slenderness of zero using an approach of Mann 1983 so that a standardized basis for the parameter estimation was provided. On these basis the parameter *a* and *b* are estimated to determine the mean value of f_m in such a way that the squared mean error is minimized.

Table 2 shows the results of the analysis. These parameters may be used to calculate the mean value of the compressive strength of big sized masonry. The parameter *a* also considers also the slenderness on which the compressive strength of masonry is related to.

Table 2. Parameters for the calculation of the compressive strength

Masonry	Parameter	
	a	*b*
Calcium silicate unit	0.23	1.31
Porous concrete	0.96	0.88

DETERMINATION OF THE MODEL UNCERTAINTIES

Since no calculation model exactly fits to experimental results model uncertainties have to be regarded in a probabilistic calculation. In Figure 1 the adjustment of the model to the data using the parameters according to Table 2 is shown. The statistical properties of the model uncertainties *M* are determined by the comparison of the compressive strength provided by the model and experimental data. Due to the fact that the model uncertainties are considered multiplicative in equation (4), the following relationship was used to calculate the mean value and the coefficient of variation of the model uncertainties:

$$m = \frac{f_{m,experiment}}{f_{m,model}} \tag{5}$$

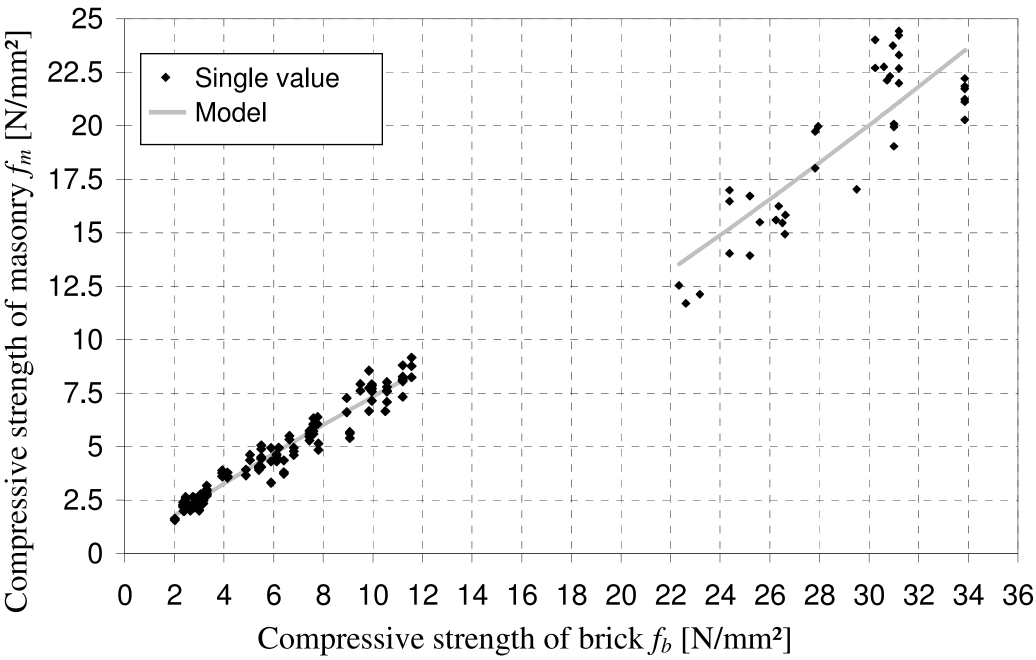

Figure 1. Compressive strength of masonry against compressive strength of brick

Considering that these values come from a limited amount of data the confidence provided by the estimation may be obtained. Table 3 shows the results.

Table 3. Parameters for the model uncertainties of the compressive strength of masonry

Masonry	Parameter		Model uncertainty M			Model uncertainty M considering the amount of data[1)]		
	a	b	m_M	σ_M	V_M	m_M[2)]	σ_M[3)]	V_M
Calcium silicate unit	0.23	1.31	1.00	0.10	0.10	0.97	0.13	0.13
Porous concrete	0.96	0.88	1.00	0.10	0.10	0.98	0.11	0.11
1) α = 95 % (confidence) 2) Lower limit 3) Upper limit								

RESULTING STATISTICAL PROPERTIES

Due to the fact that the compressive strength of the units and the model uncertainties are random variables the compressive strength of masonry is random variable, too. Since a log-normal distribution is assumed for the units and the model uncertainty the resulting mean value and the coefficient of variation of the compressive strength of masonry may be calculated by equation (6) and (7):

$$E[f_m] = E[M] \cdot a \cdot E[f_b]^b \tag{6}$$

$$V[f_m] = \sqrt{V_M^2 + \left(1 + V_{f_b}^2\right)^{b^2} - 1} \tag{7}$$

On the basis of investigations in the context of quality control the required statistical properties of the bricks are obtained. For calcium silicate units the coefficient of variation of the compressive strength is about 7% whereas for porous concrete it is about 9%. Using all information the statistical properties of the compressive strength of masonry can be calculated. The results are summarized in Table 4. According to Galambos 1982, Holicky and Markova 2002, Schueremans 2002 and Schubert 2001 the compressive strength of masonry is (approximately) assumed to be lognormal.

Table 4. Statistical Parameters of the compressive strength of masonry

<table>
<tr><th rowspan="2">Masonry</th><th colspan="2">Parameter</th><th colspan="3">Mean value</th><th colspan="3">Coefficient of Variation</th><th rowspan="2">Class[2)]</th></tr>
<tr><th>a</th><th>B</th><th>f_b[1)]</th><th>M</th><th>f_m[1)]</th><th>f_b</th><th>M</th><th>f_m</th></tr>
<tr><td rowspan="3">Calcium silicate unit</td><td rowspan="3">0.23</td><td rowspan="3">1.33</td><td>22.9</td><td>0.97</td><td>13.5</td><td rowspan="3">0.07</td><td rowspan="3">0.13</td><td rowspan="3">0.16</td><td>16</td></tr>
<tr><td>29.5</td><td>0.97</td><td>18.8</td><td>20</td></tr>
<tr><td>38.9</td><td>0.97</td><td>27.0</td><td>28</td></tr>
<tr><td rowspan="3">Porous concrete</td><td rowspan="3">0.96</td><td rowspan="3">0.88</td><td>2.9</td><td>0.98</td><td>2.4</td><td rowspan="3">0.09</td><td rowspan="3">0.12</td><td rowspan="3">0.14</td><td>2</td></tr>
<tr><td>5.3</td><td>0.98</td><td>4.1</td><td>4</td></tr>
<tr><td>7.5</td><td>0.98</td><td>5.5</td><td>6</td></tr>
<tr><td colspan="10">1) [N/mm²]
2) Characteristic compressive strength of the brick (common values)</td></tr>
</table>

In comparison to the variations of other types of masonry, big sized masonry provides a smaller coefficient of variation, as Table 5 shows. These values are considered for various materials of the units (e.g. clay, calcium silicate unit, porous concrete etc.) and thus represent a mean value for the compressive strength of the masonry.

Table 5. Parameters for the calculation of the compressive strength

Author	V	Basic variable
Galambos et al. 1982	0.18	Resistance
Holicky and Markova 2002	0.20	Compressive strength
Schueremans[1)] 2002	0.19	Compressive strength
Kirtschig and Kasten 1980	0.17	Compressive strength
Graubner and Glowienka 2007	0.20	Compressive strength
1) For historical masonry made of bricks (clay)		

Using the estimated parameter according to Table 4 the characteristic values for the compressive strength can be calculated. These are usually defined as 5%-quantile of the distribution of the dependent parameter. Figure 2 shows the results in comparison to the characteristic values according to Eurocode 6. While for porous concrete a good conformance is provided, there are significant variations between the values according to Eurocode 6 and the results on the basis of experimental investigations for calcium silicate units.

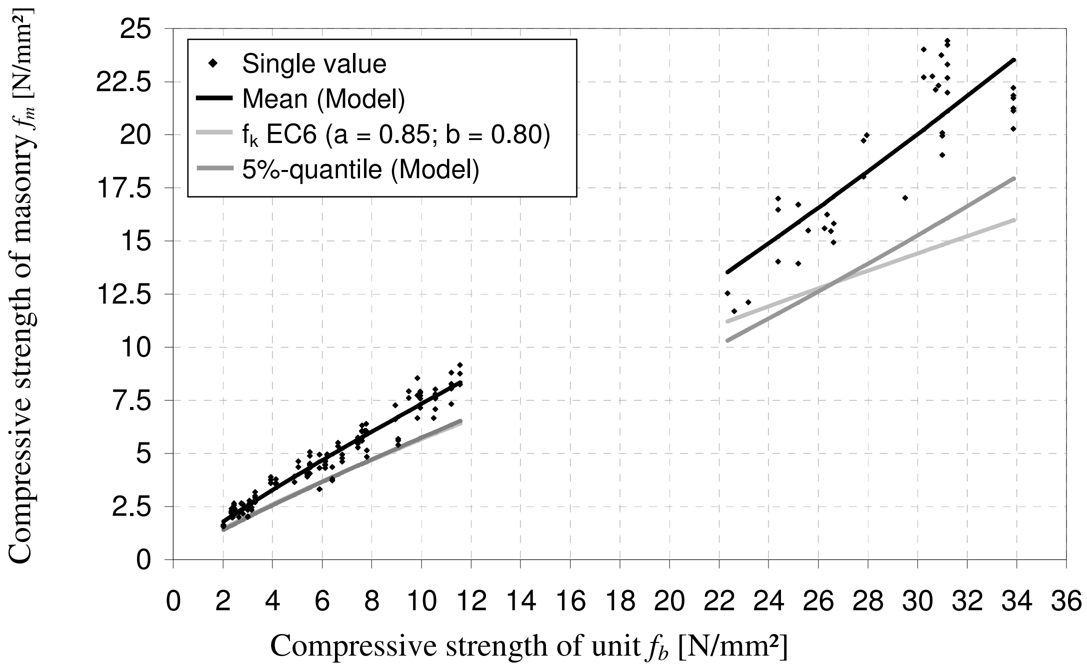

Figure 2. Compressive strength of masonry against compressive strength of unit and characteristic values according to EC 6 and the model according to equation (4)

MODEL UNCERTAINTIES FOR THE CARRYING CAPACITY OF UNREINFORCED MASONRY WALLS

For the reliability assessment and for the calculation of the probability of failure, calculation models are required. For the bearing capacity under compression the model developed by Glock 2004 is recommended. It was found that Glock's 2004 model provides a good conformance to the bearing capacity measured in experiments (see also Figure 4 and Figure 5). It considers the material properties and the nonlinear behaviour of each material, as well as the bending tensile strength. It may also be used for the calculation of steel, timber or unreinforced concrete.
The model developed by Glock 2004 is slightly simplified for the application on big sized masonry. Equations (8) to (15) are needed to calculate the carrying capacities in dependence of the slenderness of the analyzed wall.

$$N_R = \Phi_i \cdot b \cdot d \cdot f_m \tag{8}$$

$$\Phi_I = V \cdot \left(1 - 2 \cdot \frac{e_I}{d}\right) \text{ for the analysis of sections, } e_I \text{ according to (14)} \tag{9}$$

$$V = 1 + \frac{\exp\left(-6 \cdot (\eta_u \cdot (k_0 + 1) + 1) \cdot \frac{e_I}{d}\right) - 1}{\frac{k_0 + 2}{k_0} \cdot (\eta_u \cdot (k_0 + 1) - 1)^2 + 1} \text{ in general} \tag{10}$$

For big sized masonry the variable V can be chosen to $V = 1.0$ in any case, as the comparison to experimental investigations show (see Figure 4 and Figure 5).

$$\Phi_{II} = \Phi_{II,cr} \geq \Phi_{II,sb} \text{ for the analysis of slender walls} \tag{11}$$

$$\Phi_{II,cr} = \frac{\Phi_I}{0{,}3 \cdot \left(\dfrac{\lambda}{1 - 2 \cdot \dfrac{e_I}{d}} \right)^{\frac{3,1}{k_0^{\,0,3}}} + 1} \tag{12}$$

$$\Phi_{II,sb} = -\frac{1}{2} \cdot \left| \frac{f_t}{f_m} \right| \cdot \left(1 - \frac{k_0}{50}\right) + \frac{k_0}{3} \cdot \left(1 - \frac{k_0}{50}\right) \cdot \frac{1}{\lambda^2} \cdot \left(1 - 6 \cdot \frac{e_I}{d} + \sqrt{ \left(1 - 6 \cdot \frac{e_I}{d} - \frac{3 \cdot \left| \frac{f_t}{f_m} \right|}{2 \cdot k_0} \cdot \lambda^2 \right)^2 + 6 \cdot \frac{\left| \frac{f_t}{f_m} \right|}{k_0} \cdot \lambda^2 } \right) \tag{13}$$

Approximation valid for $e_I/d \geq 0.2$ and $|f_t/f_m| \leq 0.10$

$$\text{Where } e_I = \frac{M_E}{N_E} \tag{14}$$

$$\text{Where } k_0 = \frac{E_0 \cdot \varepsilon_f}{f} \quad 1 \leq k_0 \leq \infty \tag{15}$$

$$\text{Where } \lambda = \frac{h_{ef}}{d} \cdot \sqrt{\varepsilon_f} \tag{16}$$

In Figure 3 the definition of the required parameters is shown.

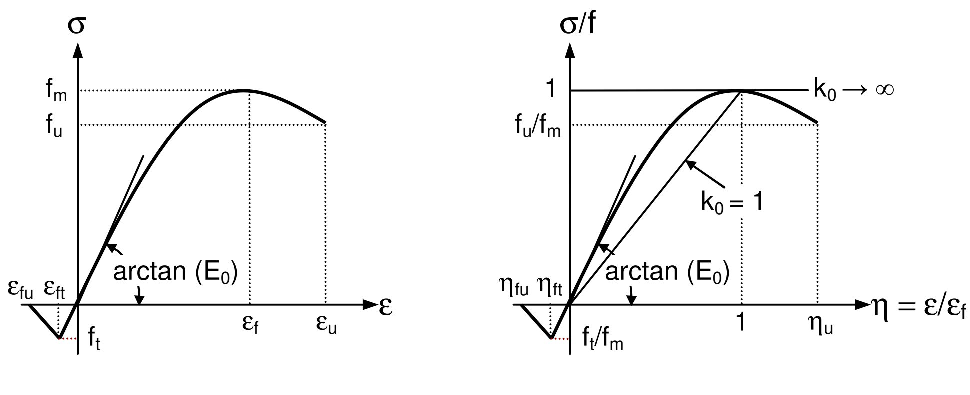

Figure 3. Definition of the parameters of Glock's 2004 model

For a probabilistic assessment of structures, uncertainties due to the calculation models used have to be considered, too. The resulting model uncertainties are estimated by comparison with the results of experimental tests. In this approach, it is approximately assumed that the tests provide the exact load carrying capacity. Figure 4 and Figure 5 give information about the quality of the model by comparing the provided carrying capacities of the model to the outcome of experimental tests.

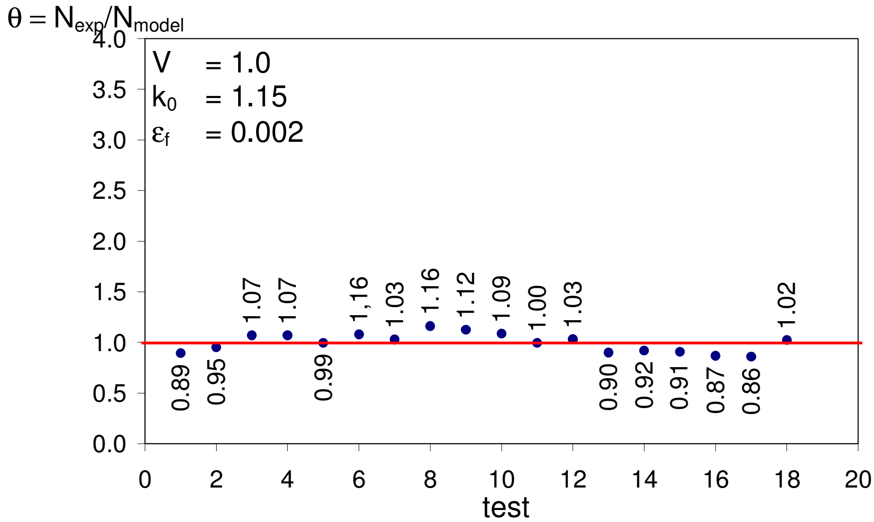

Figure 4. Bearing capacity measured in the test N_{exp} compared to bearing capacity of the model N_{model} for porous concrete

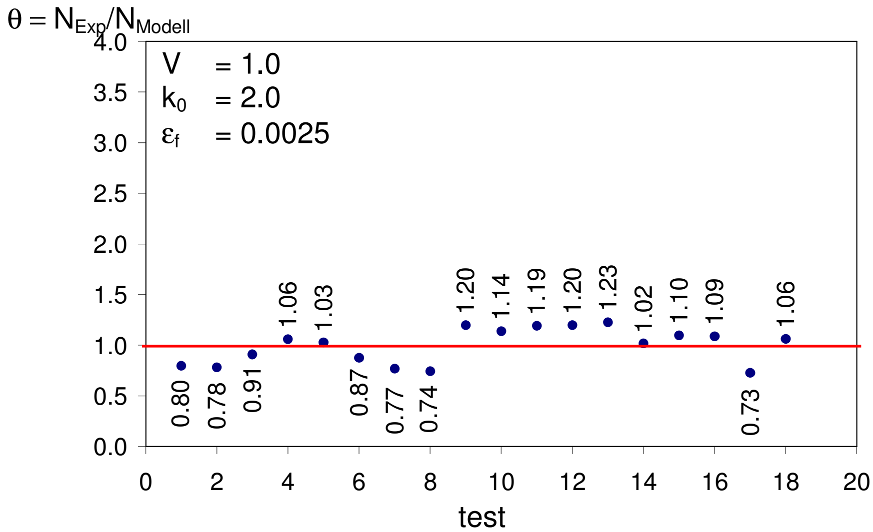

Figure 5. Bearing capacity measured in the test N_{exp} compared to bearing capacity of the model N_{model} for calcium silicate unit

However, in many cases experimental investigations are rare, as tests are expensive. Thus an update procedure (see JCSS 2001) was carried out to reduce the uncertainty due to the amount of experimental results. Table 7 summarizes the main results. As the results in Table 7 show the model uncertainties of calcium silicate units have a higher coefficient of variation compared to porous concrete.

Table 6. Parameters for the calculation model according to Glock 2004 and related model uncertainties for selected materials for big sized masonry

Masonry	Modified model according to Glock					
	Parameter[1)]			Moments[2)]		
	V	k_0	ε_f	$m_{\theta R}$	$\sigma_{\theta R}$	$V_{\theta R}$
KS	1.00	2.00	2.5‰	0.99	0.16	0.16
PB	1.00	1.15	2.0‰	1.01	0.11	0.11
parameters are based on Jäger et al. 2002 and Schubert 2004 parameters of the model uncertainty θ_R after update						

Besides, the compressive strength and the model uncertainty, the statistical parameters of the bending tensile strength are required. Experimental tests show that the bending tensile strength basically depends on the mortar properties. For porous concrete the bending tensile strength is limited by the tensile strength of the units in some cases. The required statistical parameters for masonry build with thin layer mortar are presented in Table 8.

Table 7. Statistical parameters of the bending tensile strength for masonry build with thin layer mortar

Masonry	m_x	V_x	n[1)]	Source
KS	0.61	0.35	77	Anstötz 1990, Schubert 2005
PB	0.34	0.30	17	Schubert 2001, Schubert 2004
[1)] amount of statistical series [2)] N/mm²				

RELIABILITY ANALYSIS

After a stochastic model is defined and the statistical properties of the main basic variables are determined, a reliability analysis can be carried out. For the calculation of the failure probability different methods were developed in the past. In general is has to be differentiated between exact methods as numerical integration or Monte Carlo-Simulation and mathematical approximation such as FORM and SORM. For details of these calculation methods it is referred to the JCSS 2003.

Using these methods partial safety factors may be determined on probabilistic basis. The most common method for the calibration of safety coefficients is the direct probabilistic optimization. First a design using safety factors and characteristic values of the design parameters is conducted in such a way, that the utilization of the analyzed structural component reaches 100%, since this is the worst case. After this the mean values of the random variables are calculated. After this the failure probability or the reliability index β is calculated and compared to the target value. If the target value is reached, an optimization of the safety factors is obsolete. Otherwise the partial safety factors should be changed. This should be carried out for a representative amount of structural elements.

Based on extensive probabilistic calculations, it was found that the reliability of unreinforced masonry is dominated by the material properties and model uncertainties for small and medium eccentricities of the load whereas the probability of failure is determined by the actions respectively the bending moments for larger eccentricities. This especially holds for bending moments due to time variant actions, such as wind or live load. Thus an accurate modelling of

the load eccentricity is very important to ensure a realistic assessment of the probability of failure of unreinforced masonry structures.

It was found that the partial safety factors for the design of unreinforced masonry according to the current German design codes provide neither sufficient safety nor efficiently designed structures. On basis of an extensive reliability analysis, optimized safety factors for the design of big sized masonry made of porous concrete and calcium silicate unit were established and recommendations for the practical application were given. For porous concrete a value of $\gamma_M = 1.25$ for the material and for calcium silicate of $\gamma_M = 1.35$ are sufficient to meet the demands of DIN 1055-100 (2001) and DIN EN 1990 (2002) if the model according to chapter 5 is used but neglecting the influence of the bending tensile strength in the design. For high ratios of time dependent loads on the total load, the partial safety factor γ_Q should be increased to $\gamma_Q = 1.80$ in cases where the material utilisation is high, too. This especially holds for high eccentricities of the load. In contrast to the old approach (used in the current German design codes), the new approach provides sufficiently safe and efficiently designed structures. For details of the reliability of unreinforced masonry it is referred to Glowienka 2007.

CONCLUSION

This paper provides statistical parameters and models for a probabilistic assessment of masonry structures made of big sized units, focusing on axial loading. Beside the required material parameters, model uncertainties have to be regarded in a probabilistic calculation and are quantified in this paper, too. The estimated values are based on experimental tests of masonry walls and if test results are missing on basis of expert opinions and literature research. The determined parameters can be used as prior parameters when additional tests become available. In comparison to other kinds of masonry, the statistical parameters of big sized masonry provide a smaller scatter.

For high eccentricities the reliability of unreinforced masonry is dominated by the scatter of the load, especially if the ration of time dependent loads on the total load is high. Thus an accurate stochastic modelling of the eccentricity is very important. To increase the reliability in such cases the partial safety factor γ_Q should be raised, if the material utilization is high, too. Basing on probabilistic calculations the partial safety factor for the material may be decreased in general compared to the recommendation in the German design codes.

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